Table 1 Maximum-mass models

Equation of state	$M_{max}/M_\odot$	% Increase	$M_0/M_\odot$	$\rho_c(10^{15}\,g\,cm^{-3})$	$\Omega(10^4\,s^{-1})$	T/W
L	3.53	30	3.68	0.84	0.69	0.108
C	2.16	17	2.47	2.7	1.11	0.108
F	1.72	17	1.96	3.9	1.23	0.091
G	1.55	14	1.73	5.5	1.53	0.101

The first column identifies the equation of state. The remaining columns exhibit the following quantities for the rotating model with maximum gravitational mass: gravitational mass; its percentage increase over the maximum gravitational mass of nonrotating models; baryon mass; central mass-energy density; angular velocity of rotation; ratio of rotational kinetic energy to gravitational potential energy.

Table 1 also provides an estimate of the upper limiting rotation rate of a neutron star. The softest EOS permit the largest rotation rates, and for a given equation of state we find that the largest possible uniform angular velocity occurs at a termination point with Ω close to that of the maximum mass configuration. Consequently, it is evident from Table 1 that the fast pulsar is close to being able to rule out the stiffest EOS. Further, the maximum uniform Ω allowed by the softest equation of state is $\Omega_{max} \sim 1.5\times10^4\,s^{-1}$, corresponding to a minimum allowed period of $\sim 0.4\times10^{-3}$ s. The existence of a uniformly rotating neutron star with a smaller period would rule out all proposed EOS. This result agrees with an earlier conclusion of Shapiro *et al.*[35] based on relativistic homogeneous configurations and Roche models.

Several checks on the accuracy of our models are available. Agreement with the Butterworth-Ipser polytrope code was obtained to six places by using a polytrope in place of tabulated EOS; in turn, the polytropic code produced $n = 3/2$ relativistic polytropes in which the values of the angular velocity of dragging agreed to within 1% with the values for Hartle's corresponding slowly rotating models[36]. The largest angular velocity for which an equilibrium model could be constructed agreed to within 3% with the corresponding Keplerian angular velcoity at the equator of the model. Finally, our results for Ω versus T/W were compared with approximate results[4] obtained previously by using rotating newtonian polytropes to estimate the changes rotation induces in the energies and moments of inertia of spherical relativistic configurations. The comparison yielded agreement to within ~15%, about the best that could be expected.

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Operational approach to phase-space measurements in quantum mechanics

R. F. O'Connell

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA

D. F. Walls*

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA

Wódkiewicz[1] has derived an operational formula for a positive phase-space distribution function in quantum mechanics (see also ref. 2). Here we point out that the proposed formula is actually a special case of a two-particle Wigner distribution function in which correlations have been neglected. We present a new operational formula which includes correlations. Also, we incorporate a brief description of the role and significance of quantum distribution functions in general.

Realizing that one of the most powerful approaches to classical mechanics is through the concept of phase-space, Wigner[3] introduced a quantum distribution function of positional and momentum coordinates, which provides a framework for an exact reformulation of non-relativistic quantum mechanics in terms of classical concepts. A generalization to the case of spin one-half particles has also been presented[4].

In particular, the use of Wigner's distribution function permits one to replace a quantum-mechanical ensemble average by a classical phase-space integration. Also, as distinct from Schrödinger-Heisenberg quantum mechanics, a development of various results in powers of $\hbar$ is relatively straightforward, the limit $\hbar \to 0$ leading immediately to classical mechanics. Thus, it is not only a useful calculational tool but it also provides insights into the nature of quantum mechanics and its role in such areas as measurement theory. Similar remarks apply to other quantum-distribution functions which are in common use in many areas of non-equilibrium statistical mechanics, most particularly in quantum optics[5]. (These functions and their inter-relationships are reviewed in ref. 6.)

Wódkiewicz[1] defines his function thus:

$$P(q,p) = \iint dq'\,dp'\; W_\psi(q+q', p+p')\, W_\phi(q', p') \qquad (1)$$

where q, q' and p, p' refer to position and momentum coordinates, respectively, and W denotes the Wigner quantum distribution function[3], W_ψ describing the state of a laser pulse with which a detected particle interacts and W_ϕ describing the state of the particle after the interaction. Note that $P(q,p)$ is always positive (in contrast to $W(q,p)$ which can sometimes be negative), and that the so-called "... deficiencies of Wigner's calculation ... have been explained"[2].

Our main purpose here is to point out that $P(q,p)$ is not a new phase-space distribution function. It is, in fact, a special case of a two-particle Wigner distribution function in which correlations have been neglected. (Here 'particle' should be interpreted in the general sense of incorporating the systems ψ and ϕ referred to above.) The W functions appearing in equation (1) are single-particle Wigner functions and these are the kind most often discussed. However, Wigner's original paper actually introduced the general n-particle function, $W^{(n)}$ say,

* Permanent address: Department of Physics, University of Waikato, Hamilton, New Zealand.

whose properties are discussed in ref. 7. In particular, these functions obey a quantum BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) hierarchy of equations[8], the rate of change of $W^{(n)}$ depending not only on $W^{(n)}$ but also on the higher-order function $W^{(n+1)}$.

The two-particle Wigner function for a pure state, $W^{(2)}$ say, may be written in the form (equation (5) of ref. 3, with $n=2$):

$$W^{(2)}(q_1, q_2; p_1, p_2) = (\pi\hbar)^{-2} \iint dy_1\, dy_2\, \Phi^*(q_1+y_1, q_2+y_2) \times \Phi(q_1-y_1, q_2-y_2) \exp\left[\frac{2i}{\hbar}(p_1 y_1 + p_2 y_2)\right] \quad (2)$$

where Φ is the wave function for the system. If we now assume that there are no correlations between particles 1 and 2, we may write

$$\Phi(1, 2) = \psi(1)\phi(2) \quad (3)$$

from which it follows that

$$W^{(2)}(q_1, q_2; p_1, p_2) = W_\psi(q_1, p_1) W_\phi(q_2, p_2) \quad (4)$$

that is the two-particle function may be written as a product of single-particle functions. Hence, from equations (1) and (4), we deduce that

$$P(q, p) = \iint dq'\, dp'\, W^{(2)}(q+q', q'; p+p', p') \quad (5)$$

In other words, the operational phase-space distribution function $P(q, p)$ is obtained by starting with the two-particle Wigner function $W^{(2)}(q_1, q_2; p_1, p_2)$ and then (1) assuming that there is no correlation between particles 1 and 2, (2) that $q_1 \equiv q+q' = q+q_2$ and $p_1 \equiv p+p' = p+p_2$, and (3) integrating over all q' and p'.

We conclude that $P(q, p)$, as defined in equation (1), is a special case of a two-particle Wigner distribution function. In addition, we may regard $P(q, p)$, as defined in equation (5), as a new operational formula for a distribution function, which includes correlations.

We are not decrying the work of Wódkiewicz[1]—which has the merit of deriving $P(p, q)$ from a dynamical process involving the detected particle, the detector, and the filtering device—but we are rather putting it in its proper context. In essence, it does not highlight '... deficiencies of Wigner's calculation ...'[2] but rather it displays the wide scope and power of the results presented by Wigner[3].

In the same context, we point out that Wódkiewicz's function $P(q, p)$ is the same as the function $J(p, q)$ introduced by R.F.O'C. and Rajagopal[9], in their demonstration of the fact that the '... new interpretation of the scalar product in Hilbert space ...' presented by Aharonov *et al.*[10]) is essentially equivalent to Wigner's exact reformulation of non-relativistic quantum mechanics using distribution functions. R.F.O'.C. and Rajagopal also demonstrated explicitly that $P(q, p) \equiv J(q, p) \geq 0$ (equation (5) of ref. 9) but it is perhaps useful to point out that such a result is also contained in the earlier work of R.F.O'C. and Wigner[11]) (see their equations (2) and (5)), who used such a result to prove that the Husimi distribution[12] (which is obtained by smoothing a Wigner distribution function with the Wigner distribution function for the ground-state of a harmonic oscillator) is always positive. The Aharonov *et al.* interpretation can also be regarded as the one underlying the stochastic phase-space formulation of non-relativistic quantum mechanics, as demonstrated by Prugovečki[13].

All previous discussions treated pure states, but a generalization to mixed states is readily obtained. In fact, using the general result (equation (2.11) of ref. 7) that

$$\iint dq\, dp\, A(q, p) B(q, p) = (2\pi\hbar) \mathrm{Tr}(\hat{A}\hat{B}) \quad (6)$$

where Tr denotes the trace and $\hat{A}$ and $\hat{B}$ are operators in Hilbert space and where $A(p, q)$ and $B(p, q)$ are the corresponding Wigner phase-space functions, and also using the fact that the phase-space function corresponding to the density matrix $\hat{\rho}$ is $(2\pi\hbar)$ times the distribution function[4], it follows that (taking $\hat{A} = \hat{\rho}_\psi$ and $\hat{B} = \hat{\rho}_\phi$)

$$\iint dq\, dp\, W_\psi(q, p) W_\phi(q, p) = (2\pi\hbar)^{-1} Tr(\hat{\rho}_\psi \hat{\rho}_\phi) \geq 0 \quad (7)$$

a result also reported by Iagolnitzer[14] in work on the S-matrix description of particle states and of their measurements. In other words, the operational phase-space distribution function in the absence of correlations is always non-negative, even for mixed states.

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The North Equatorial Countercurrent and heat storage in the western Pacific Ocean during 1982–83

Gary Meyers

CSIRO Division of Oceanography, GPO Box 1538, Hobart, Australia

Jean-Rene Donguy

ORSTOM Groupe SURTROPAC, BP A5, Noumea, New Caledonia

A sequence of extreme climate anomalies known as an El Niño Southern Oscillation episode was observed in the tropical Pacific Ocean during 1982–83[1,2]. Models suggest that oceanic circulation has an important role in the formation of such anomalies[1,3,4] by altering the pattern of sea-surface temperature through advection. The baroclinic structure of the upper 500 m of the ocean has been monitored routinely in the western Pacific since 1979, providing indirect current measurements by the geostrophic method. These observations, reported here, show large changes in the near-equatorial currents during 1982–83 which are consistent with currents observed in the central and eastern Pacific[5,6]. In particular, the North Equatorial Countercurrent (NECC) flowed with 25–50% increased strength during the early phase, then weakened almost to zero flow. The West Pacific heat pool cooled by more than 1 °C. Observed changes in circulation were large enough to alter surface heat storage by advection, although other potentially important processes may not be negligible.

Baroclinic currents in the tropical ocean are largely determined by the topography of the thermocline (that is, the temperature field), in the same way that wind is determined by a map of pressure. Temperature structure has been monitored using expendable bathythermographs (XBTs) launched from volunteer observing (merchant) ships in a programme operated jointly by France and the United States since 1979[7]. The programme involves soundings to 450 m depth at intervals of ~60