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EFFECT OF A FINITE SEMICONDUCTOR SUBSTRATE ON THE FARADAY ROTATION AND ELLIPTICITY IN A METAL–OXIDE–SEMICONDUCTOR SYSTEM

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In this work we study the effect of a finite semiconductor substrate on the Faraday rotation and ellipticity in a metal–oxide–semiconductor system. We find that the multiple reflections within the semiconductor substrate can give an enhancement by factors of up to 2.5 over the case where the semiconductor is considered semi-infinite. In addition, the ellipticity is markedly changed and, in particular, null values may be obtained in contrast with the results obtained without multiple reflections within the semiconductor.

Recently we calculated the Faraday rotation θ and ellipticity δ due to a two-dimensional electron gas (2DEG), in the case where the directions of both the incident radiation of frequency ω and the external dc magnetic field $B>0$ are oriented normal to the oxide–semiconductor interface ($z=0$) containing the 2DEG in a metal–oxide–semiconductor (MOS) system and where the oxide and semiconductor were assumed semi-infinite [1], the latter being appropriate when a wedge is used to eliminate multiple reflections within the semiconductor. However, various authors [2, 3] have considered the effect of a finite (i.e. unwedged) semiconductor substrate on cyclotron resonance. Thus we are motivated to study other magneto-optical phenomena under such conditions. We show below that the introduction of the semiconductor–vacuum interface at $z=d$ (see the inset to fig. 1) has a strong influence on both the rotation and the ellipticity. Further, we obtain a unique decomposition of the "multiple pass" rotation Θ and ellipticity Δ into the θ and δ of ref. [1] and a correction term due purely to multiple internal reflections within the semiconductor substrate.

We consider the propagation of right and left circularly polarized light through the SiO_2–Si–vacuum system. The electric fields in the three regions are

$$z<0: \qquad \boldsymbol{E}_{i\pm} = E_{i\pm}\,e^{i(k_o z-\omega t)}\hat{\varepsilon}_{\pm}\,, \qquad \boldsymbol{E}_{r\pm} = E_{r\pm}\,e^{-i(k_o z+\omega t)}\hat{\varepsilon}_{\pm}\,; \tag{1a}$$

$$0<z<d: \qquad \boldsymbol{E}_{u\pm} = E_{u\pm}\,e^{i(k_s z-\omega t)}\hat{\varepsilon}_{\pm}\,, \qquad \boldsymbol{E}_{v\pm} = E_{v\pm}\,e^{-i(k_o z+\omega t)}\hat{\varepsilon}_{\pm}\,; \tag{1b}$$

$$z>d: \qquad \boldsymbol{E}_{t\pm} = E_{t\pm}\,e^{i[k_v(z-d)-\omega t]}\hat{\varepsilon}_{\pm}\,, \tag{1c}$$

so that $\boldsymbol{E}_i = \boldsymbol{E}_{i+} + \boldsymbol{E}_{i-}$, $\boldsymbol{E}_r = \boldsymbol{E}_{r+} + \boldsymbol{E}_{r-}$, etc., k is the wave number in the corresponding medium and is given by

$$k_i = \frac{\omega}{c}\,n_i\,, \qquad i = \mathrm{o, s, v} \quad \text{or} \quad 1, 2, 3\,, \tag{1d}$$

where o, s, v is the oxide, semiconductor, or vacuum, respectively, n_i is the index of refraction and

$$\hat{\varepsilon}_{\pm} \equiv (\hat{x} \mp i\hat{y})/\sqrt{2}\,. \tag{1e}$$

The boundary conditions at $z=0$ and $z=d$ are (a) continuity of the tangential component of the ac electric fields and (b) equality of the discontinuity of the ac magnetic fields and $(4\pi/c)j_{\pm} = (4\pi/c)\sigma_{\pm}E_{\pm}$, where j and σ are the surface current and conductivity of the 2DEG located at the interface. These conditions imply, respectively, the following results (suppressing the "$\pm$" sign):

$$z=0: \quad E_{\mathrm{i}} + E_{\mathrm{r}} = E_{\mathrm{u}} + E_{\mathrm{v}}\,, \tag{2a}$$

$$k_{\mathrm{o}}(E_{\mathrm{i}} - E_{\mathrm{r}}) - k_{\mathrm{s}}(E_{\mathrm{u}} - E_{\mathrm{v}}) = \frac{4\pi\omega}{c^2}\,\sigma(E_{\mathrm{u}} + E_{\mathrm{v}})\,, \tag{2b}$$

$$z=d: \quad E_{\mathrm{u}}\,\mathrm{e}^{\mathrm{i}k_{\mathrm{s}}d} + E_{\mathrm{v}}\,\mathrm{e}^{-\mathrm{i}k_{\mathrm{s}}d} = E_{\mathrm{t}}\,, \tag{2c}$$

$$k_{\mathrm{s}}(E_{\mathrm{u}}\,\mathrm{e}^{\mathrm{i}k_{\mathrm{s}}d} - E_{\mathrm{v}}\,\mathrm{e}^{-\mathrm{i}k_{\mathrm{s}}d}) - k_{\mathrm{v}}E_{\mathrm{t}} = 0\,. \tag{2d}$$

After some algebra, we find that the solution of eq. (2) for the transmission coefficient $t_{\pm} \equiv E_{\mathrm{t}\pm}/E_{\mathrm{i}\pm}$ is, using eq. (1d),

$$t_{\pm} = \frac{4n_{\mathrm{o}}n_{\mathrm{s}}}{(n_{\mathrm{v}} + n_{\mathrm{s}})\left(n_{\mathrm{s}} + n_{\mathrm{o}} + \frac{4\pi}{c}\sigma_{\pm}\right)\mathrm{e}^{-\mathrm{i}k_{\mathrm{s}}d} - (n_{\mathrm{s}} - n_{\mathrm{v}})\left(n_{\mathrm{s}} - n_{\mathrm{o}} - \frac{4\pi}{c}\sigma_{\pm}\right)\mathrm{e}^{\mathrm{i}k_{\mathrm{s}}d}}\,. \tag{3}$$

Eq. (3) may be written as

$$t_{\pm} = \frac{t_{12\pm}t_{23}\,\mathrm{e}^{\mathrm{i}k_{\mathrm{s}}d}}{1 - r_{21\pm}r_{23}\,\mathrm{e}^{2\mathrm{i}k_{\mathrm{s}}d}}\,, \tag{4}$$

where [4]

$$t_{ij\pm} = 2n_i\Big/\left[n_i + n_j + \frac{4\pi}{c}\sigma_{\pm}\right], \tag{5a}$$

$$r_{ij\pm} = \left[n_i - n_j - \frac{4\pi}{c}\sigma_{\pm}\right]\Big/\left[n_i + n_j + \frac{4\pi}{c}\sigma_{\pm}\right] \tag{5b}$$

are the Fresnel transmission and reflection coefficients at a surface containing a 2DEG and $t_{ij\pm} - r_{ij\pm} = 1$. In eqs. (4) and (5), the pair of indices (ij) denote propagation from medium i in the direction of medium j. At the semiconductor–vacuum (23) interface $\sigma_{\pm}$ is zero.

It is convenient to write eqs. (5) as

$$t_{ij\pm} = |t_{ij\pm}|\,\mathrm{e}^{\mathrm{i}\xi_{ij\pm}}\,, \tag{6a}$$

$$r_{ij\pm} = |r_{ij\pm}|\,\mathrm{e}^{\mathrm{i}\zeta_{\pm}}\,. \tag{6b}$$

We now define $\eta_{\pm}$ by

$$(1 - r_{21\pm}r_{23}\,\mathrm{e}^{2\mathrm{i}k_{\mathrm{s}}d})^{-1} = |1 - r_{21\pm}r_{23}\,\mathrm{e}^{2\mathrm{i}k_{\mathrm{s}}d}|^{-1}\,\mathrm{e}^{\mathrm{i}\eta_{\pm}}\,. \tag{7}$$

Using eq. (6b), we may write

$$1 - r_{21\pm}r_{23}\,\mathrm{e}^{2\mathrm{i}k_{\mathrm{s}}d} = 1 - |r_{21\pm}|r_{23}\,\mathrm{e}^{\mathrm{i}(2k_{\mathrm{s}}d + \zeta_{\pm})} \tag{8}$$

so that

$$\eta_\pm = \tan^{-1}\left(\frac{r_{23}|r_{21\pm}|\sin(2k_s d + \zeta_\pm)}{1 - r_{23}|r_{21\pm}|\cos(2k_s d + \zeta_\pm)}\right). \tag{9}$$

From eqs. (6a) and (7) we have

$$\frac{t_-}{t_+} = \left|\frac{t_-}{t_+}\right| e^{i(\xi_- - \xi_+)} e^{i(\eta_- - \eta_+)}. \tag{10}$$

It is easy to show that the Faraday rotation Θ and the ellipticity Δ are related to the transmission coefficients by

$$\frac{t_-}{t_+} = \left(\frac{1-\Delta}{1+\Delta}\right) e^{-2i\Theta}. \tag{11}$$

Thus, we obtain a unique decomposition of Θ as

$$\Theta = \theta + \theta_{\mathrm{MR}}, \tag{12a}$$

where

$$\theta = \tfrac{1}{2}(\xi_+ - \xi_-) \tag{12b}$$

is the rotation studied in ref. [1] and

$$\theta_{\mathrm{MR}} = \tfrac{1}{2}(\eta_+ - \eta_-) \tag{12c}$$

is the correction term for the finite thickness of the semiconductor and is due purely to multiple reflections.

Note that θ_{MR} and hence Θ is a periodic function of the thickness d of period $d_p \equiv \pi c/\omega n_s$. Thus, following Abstreiter et al. [3], we define a dimensionless thickness or interference parameter Y by

$$Y = y \bmod [\![y]\!] \tag{13}$$

where $y \equiv d/d_p$ and $[\![\cdot\cdot\cdot]\!]$ denotes the greatest integer part of $\cdot\cdot\cdot$, so that $0 \leq Y < 1$.

Using the parameters of fig. 1 for ω and n_s and taking Si substrate thicknesses [2] of 250–500 μm, we obtain y values from 5.894 to 11.788, respectively. Since an integer value for y corresponds to $Y = 0$, we see that any Y value $(0 \leq Y < 1)$ is realizable experimentally.

We explicitly point out that the results so far obtained are independent of the model used for the surface conductivity $\sigma_\pm$. We will now choose a Drude-type model, taking the surface conductivity [4] as

$$\sigma_\pm = \frac{iNe^2/m^*}{\omega \pm \omega_c + i\nu}, \tag{14}$$

where N is the electron surface concentration, m^* the effective mass, $\omega_c = eB/m^*c$ is the cyclotron frequency and ν the collision frequency $(=\tau^{-1}$, τ being the collision time). Thus, we obtain

$$\xi_\pm = -\tan^{-1}\left(\frac{\omega_{ps}(\omega \pm \omega_c)}{(\omega \pm \omega_c)^2 + \nu(\nu + \omega_{ps})}\right) \tag{15a}$$

and

$$\zeta_\pm = -\tan^{-1}\left(\frac{(n+1)\omega_{ps}(\omega\pm\omega_c)}{n(\omega\pm\omega_c)^2+(n\nu-\omega_{ps})(\nu+\omega_{ps})}\right), \tag{15b}$$

where

$$n \equiv \frac{n_s - n_o}{n_s + n_o}, \tag{16}$$

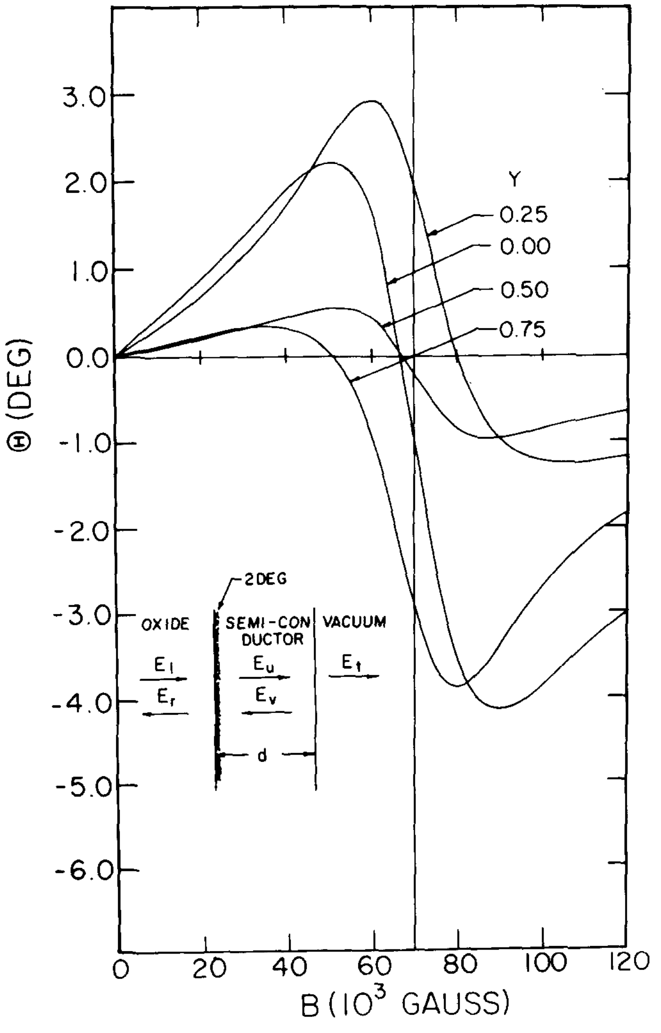

Fig. 1. Plot of the Faraday rotation Θ vs. the magnetic field B. The parameters used are $N = 2.3\times10^{12}\,\mathrm{cm}^{-2}$, $m^* = 0.19m_e$, where m_e is the rest electron mass, $\tau = 6\times10^{-13}$ s, $\omega = 6.455\times10^{12}\,\mathrm{s}^{-1}$, $n_0 = 1.95$, $n_s = 3.44$, and for the Y values indicated on the curves. The vertical line corresponds to the value $\omega = \omega_c$ or $B = 6.97\times10^4$ G.

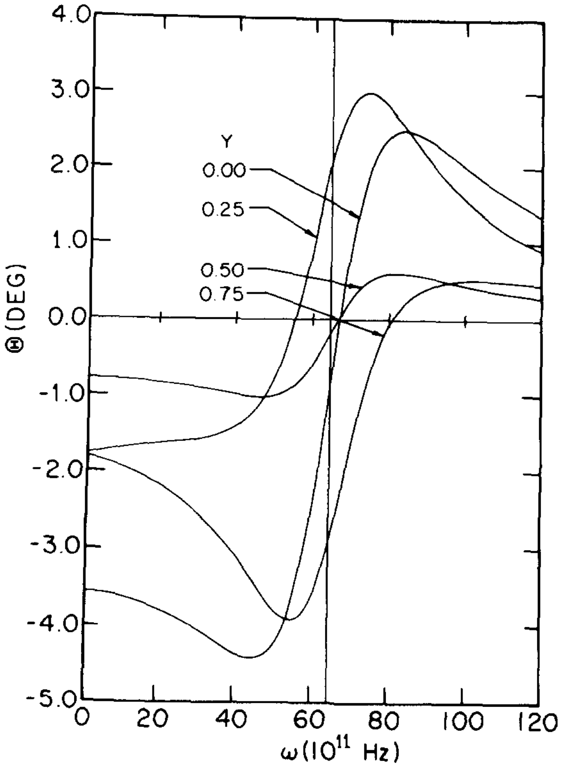

Fig. 2. Plot of the Faraday rotation Θ vs. the photon frequency ω using the same parameters as in fig. 1.

and, in the notation of ref. [4],

$$\omega_{ps} \equiv 4\pi N e^2 / m^* c (n_o + n_s)\,. \tag{17}$$

Using eq. (12), we present, in figs. 1 and 2, plots of Θ versus B and ω, respectively, for various values of the interference parameter Y. From the curves, we see that multiple reflections within the semiconductor substrate have a strong influence on the rotation values. In particular, null rotation can now be obtained for an ω_c value such that $\omega_c > \omega$, in marked contrast with the results of the rotation θ of ref. [1]. Also, corresponding to $Y \approx 0.15$ we attain maximum enhancement (not shown in curve), such that the maximum value of Θ is 2.5 times the maximum value of θ.

Similar to the decomposition in eq. (12a), we obtain for the ellipticity Δ

$$\Delta = \frac{|t_+| - |t_-|}{|t_+| + |t_-|}$$

$$= \tanh(\delta_s + \delta_{MR})\,, \tag{18a}$$

where

$$\delta = \tanh \delta_s \tag{18b}$$

is the ellipticity δ of ref. [1] and

$$\delta_{MR} = \frac{1}{2} \ln \left| \frac{1 - r_{21-} r_{23}\, e^{2ik_s d}}{1 - r_{21+} r_{23}\, e^{2ik_s d}} \right| \tag{18c}$$

is the correction term due purely to multiple reflections. We note that δ_{MR} and hence Δ are periodic

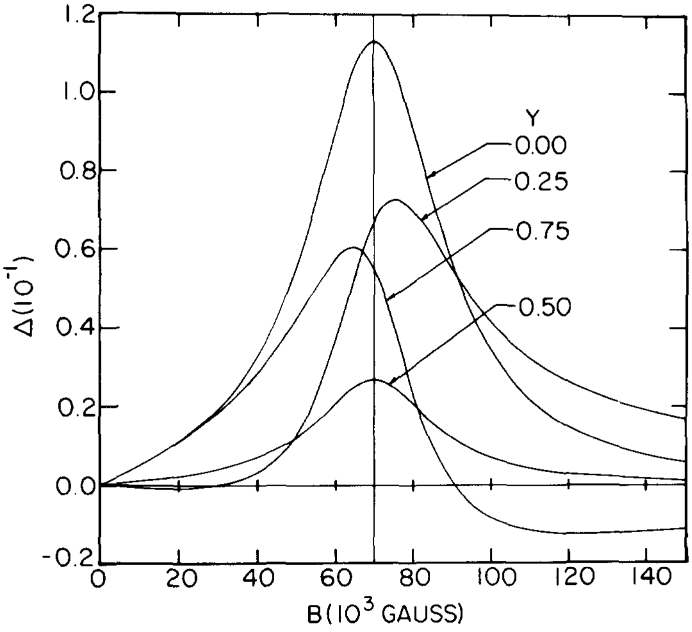

Fig. 3. Plot of the ellipticity Δ vs. the magnetic field B using the same parameters as in fig. 1.

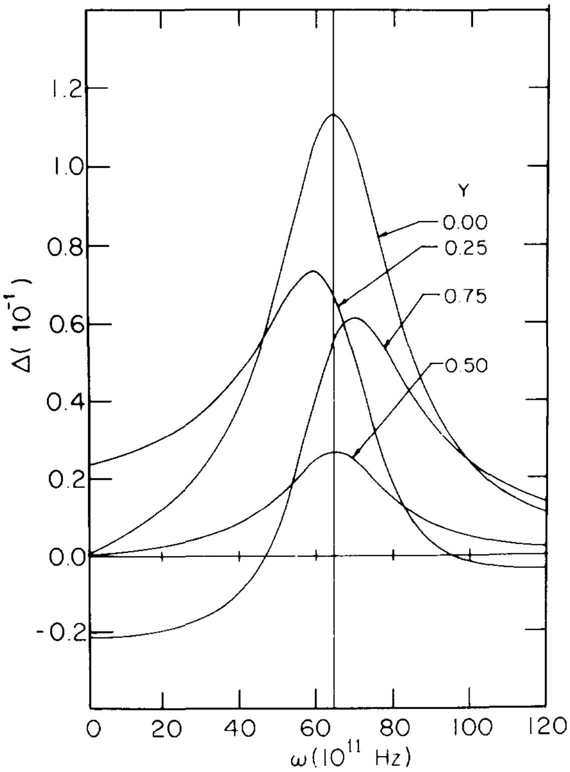

Fig. 4. Plot of the ellipticity Δ vs. the photon frequency ω using the same parameters as in fig. 1.

functions of d with period d_p. Using eq. (18a), we present, in fig. 3, a plot of Δ versus B for various values of Y. Again, multiple reflections have a strong influence on ellipticity values. The maximum value of the ellipticity is shifted to the right and downward as Y goes from 0.0 to 0.5 and reverses this trend for $0.5 < Y < 1$. Also, null ellipticity can now be achieved, in marked contrast with the results of the ellipticity δ of ref. [1]. Fig. 4 is a corresponding plot of Δ versus ω.

In conclusion, we have calculated the Faraday rotation and ellipticity produced by the electron gas in the inversion layer of a MOS system in the case where we have a plane parallel semiconductor substrate causing multiple internal reflections which have a strong influence on both the rotation and the ellipticity.

Acknowledgments

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