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ON THE OPTIMUM METHOD OF ANALYSIS OF FARADAY ROTATION AND ELLIPTICITY MEASUREMENTS IN A METAL–OXIDE–SEMICONDUCTOR SYSTEM

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Abstract—We have previously calculated the Faraday rotation θ and ellipticity δ due to the two-dimensional electron gas at the oxide-semiconductor interface of a metal–oxide–semiconductor system. The results depend on such parameters of the system as the effective mass m^*, the relaxation time τ, and the electron surface concentration N, and in fact the motivation is to enable one to determine such parameters from measurements of θ and δ. Here we discuss the optimum method for the determination of these parameters from the data. In particular, we argue that it is more desirable to carry out measurements at fixed magnetic field B and variable photon frequency ω, rather than at fixed ω and variable B. We also demonstrate how our analysis can be used to determine the predicted dependence of τ on magnetic field, without explicit knowledge of m^* or N.

We have recently calculated the Faraday rotation θ and the ellipticity δ due to the two-dimensional electron gas at the oxide–semiconductor interface of a metal–oxide–semiconductor (MOS) system[1]. The incident radiation of frequency ω, and the external dc magnetic field B are oriented normal to the interface and we considered the transmission of the right and left circularly polarized components of a linearly polarized wave to obtain, in the case of the parameters of interest experimentally,

$$\theta = \frac{2\pi}{c(n_0+n_s)}(\sigma''_- - \sigma''_+), \tag{1}$$

and

$$\delta = \frac{2\pi}{c(n_0+n_s)}(\sigma'_- - \sigma'_+), \tag{2}$$

where $\sigma'_\pm$ and $\sigma''_\pm$ refers to the real and imaginary parts, respectively, of the conductivity $\sigma_\pm$, and $\pm$ refers to right and left components, respectively. Also, n_0 and n_s are the indices of refraction of the oxide and semiconductor, respectively. Equations (1) and (2) have been obtained from more exact—but also considerably more complicated—expressions and they are valid if $(4\pi/c)|\sigma_\pm| \ll (n_0+n_s)$, which is equivalent to the assumption that $\omega_{ps} \ll [(\omega \pm \omega_c)^2 + \nu^2]^{1/2}$, where $\omega_c = eB/m^*c$ is the cyclotron frequency, m^* is the effective mass, ν is the collision frequency, and

$$\omega_{ps} = 4\pi Ne^2/m^*c(n_0+n_s), \tag{3}$$

where N is the electron surface concentration.

The starting-point for the derivation of these results was the model of Chiu *et al.*[2], who treated the inversion layer electrons as strictly two-dimensional in the sense that they took $j\delta(z)$ as the current density induced by the field, where the z axis is normal to the oxide–semiconductor interface (located at $z=0$). We have recently treated, the case of transmission through an inversion layer of finite thickness d[3], taking into account *boundary effects at the interface* as well as *multiple reflection effects in the inversion layer* (see Fig. 5 of Ref.[1] or Fig. 1 of Ref.[3]), and we showed that for $\omega d/c \ll 1$ our results agreed with those of Chiu *et al.*[2]. Since the condition $(\omega d/c) \ll 1$ is equivalent to $2\pi d \ll \lambda$ (λ being the wavelength of the light in vacuum) and since d is typically $\approx 10^{-6}$ cm and $\lambda \approx 3\times 10^{-2}$ (far infra-red) it is clear that the condition is well fulfilled in practice. A noteworthy feature resulting from the use of this model is that it enables one to go beyond the so-called "single-pass" model (which gives results depending only on the properties of the inversion layer) and incorporate also the properties of the oxide layer (via n_0 in eqns 1 and 2).

The next stage in the investigation is the choice of σ. Our choice was the Drude-like expression

$$\sigma_\pm = \frac{iNe^2/m^*}{\omega \pm \omega_c + i\nu} \tag{4}$$

but we will briefly discuss below other models. Then, from eqns (1) to (4) we obtained

$$\theta = \frac{\omega_{ps}\omega_c(\omega^2-\Omega^2)}{[(\omega+\omega_c)^2+\nu^2][(\omega-\omega_c)^2+\nu^2]}, \tag{5}$$

$$\delta = \frac{2\omega\omega_c\nu\omega_{ps}}{[(\omega+\omega_c)^2+\nu^2][(\omega-\omega_c)^2+\nu^2]}. \tag{6}$$

In eqns (5) and (6), we are dealing with two known external quantities—ω and ω_c—and we would like to make use of the observational results of θ and δ to obtain information on the *three parameters of the inversion layer viz.* m^*, ν *and* N. The purpose of this communication is to discuss the optimum method for the determination of these parameters from the data. Our method of approach will be to isolate specific data points or combination thereof for which there is dependence only on one or at the most two of the three quantities m^*, ν, and N.

First of all, by taking the derivative of θ with respect to ω we find that θ has maximum and minimum values, $\theta_{\max}$ and $\theta_{\min}$ say (collectively denoted by θ_m), at values of ω which obey the relation

$$\omega^4 - 2\omega^2(\omega_c^2 + \nu^2) + (\omega_c^4 - 3\nu^4 - 2\nu^2\omega_c^2) = 0. \quad (7)$$

This is a quadratic equation in ω^2 whose solutions, which we denote by ω_m^2, are given by

$$\omega_m^2 = \Omega^2 \pm 2\nu\Omega \quad (8)$$

where

$$\Omega \equiv (\omega_c^2 + \nu^2)^{1/2}, \quad (9)$$

and where the $\pm$ signs give $\omega_{\max}$ and $\omega_{\min}$, respectively. It immediately follows that the minimum in the θ–ω curve disappears for values of ω_c less than 1.73ν (this was not the case for the parameters selected in Ref.[1] so that the θ–ω curve therein displays both a maximum and a minimum).

In addition, it follows, either from eqn (7) or (8), that

$$\nu^2 = \tfrac{1}{3}\{-(\omega_c^2 + \omega_m^2) + 2\sqrt{\omega_c^4 + \omega_m^4 - \omega_m^2\omega_c^2}\}. \quad (10)$$

We note the absence of a minus sign in front of the square root, occasioned by the requirement that ω^2 (and also ν) is positive. Thus, from a knowledge of ω_c (which of course depends on m^* whose determination we will discuss below) and the photon frequency at the max or min of the θ–ω curve, we can determine ν. We note that this expression for ν is independent of N. The same value of ν will ensue by use of either $\omega_{\max}$ or $\omega_{\min}$ for ω_m in eqn (10). In the case where $\nu \ll \omega_c$, which often occurs in practice, then eqn (10) simply reduces to $\nu \approx \pm(\omega_m - \omega_c)$ where the + sign refers to the max and the — sign to the min. In other words, $\nu \approx \frac{1}{2}(\omega_{\max} - \omega_{\min})$, i.e. it is independent of both N and m^*. Returning to eqn (8), we can deduce an alternative and even simpler expression for ν^2 viz.

$$\nu^2 = \tfrac{1}{2}(\omega_{\max}^2 + \omega_{\min}^2) - \omega_c^2. \quad (11)$$

Using eqn (8) in eqn (5) we can also obtain

$$\theta_m = \pm\frac{\omega_{ps}\omega_c}{4\nu(\Omega \pm \nu)} \quad (12)$$

from which it follows that

$$|\theta_{\min}| - \theta_{\max} = \frac{\omega_{ps}}{2\omega_c}. \quad (13)$$

This is a very simple result. It is independent of ν and m^* and depends only on N. Thus we have a means of determining N. Alternatively, if N is determined from the gate voltage in the usual manner[4], we have a simple and clear-cut test of the model used for σ. We also remark that the asymmetry in the θ–ω curve about $\theta = 0$, given by eqn (13), is manifest in Fig. 2 of Ref.[1].

One can also make use of eqn (12) to derive another relation from which ν can be determined viz.

$$\nu^2 = [\{|\theta_{\min}| - \theta_{\max}\}^2/4|\theta_{\min}|\theta_{\max}]\omega_c^2. \quad (14)$$

Now making use of eqn (13), this result can be put in the simpler form:

$$\nu^2 = \frac{\omega_{ps}^2}{16|\theta_{\min}|\theta_{\max}}. \quad (15)$$

Alternatively, solving eqn (11) for ω_c^2 and substituting in eqn (14) leads to the result

$$\nu^2 = \tfrac{1}{2}(\omega_{\max}^2 + \omega_{\min}^2)(|\theta_{\min}| - \theta_{\max})^2/(|\theta_{\min}| + \theta_{\max})^2. \quad (16)$$

This is our most striking result because it enables one to *determine* ν directly from the measurements of θ at the turning points, along with the corresponding frequencies, *without explicit knowledge of* m* *or* N.

In the case of the ellipticity, there is not the same variety of interesting results available. We simply find that δ has a maximum at ω values given by

$$\omega^2 = \tfrac{1}{3}\{(\omega_c^2 - \nu^2) + 2\sqrt{\nu^4 + \omega_c^4 + \nu^2\omega_c^2}\} \quad (17)$$

or alternatively

$$\nu^2 = (\omega^2 - \omega_c^2) + 2\omega\sqrt{\omega^2 - \omega_c^2} \quad (18)$$

where ω refers to the photon frequency for which δ is a max.

We now turn to a brief consideration of more sophisticated models for σ. For example, Ando[5] has discussed a magnetic field dependence of τ. Such a dependence could be investigated experimentally by plotting θ-ω curves for various B fields and using eqn (14) to determine ν as a function of B, since our analysis also holds in this case. On the other hand, our equations are not valid in the case where m^* and τ are functions of ω, which is a result of using a memory-function approach[6] toward the calculation of the conductivity[7]. However, it appears that the frequency dependent shift in m^* is very small[7] and so we feel that the use of the Drude model is at least a good first step.

We now ask the question if additional information of a similar type may be obtained by taking the derivative of θ with respect to ω_c. Unfortunately, this leads to a cubic equation in ω_c^2, in contrast to the quadratic equation for ω^2 given by eqn (7). An investigation of the cubic equation led to no simple conclusions like those given above. As a result, we conclude that more information can be gleaned from a θ-ω plot at fixed B values than from a θ-ω_c plot at fixed ω values. It turns out that data has now been obtained in the latter case[8] and so we could urge that corresponding data also be obtained in the former case.

Note added in proof. Recently, we have studied the effect of a finite semiconductor substrate (which is relevant when the semiconductor does not have a wedge to prevent multiple reflections from the semiconductor–vacuum interface) on the Faraday rotation and ellipticity in a MOS system[9]. In addition, we have presented a brief survey of magneto-optical experiments in the general area of two-dimensional systems in solid-state and surface physics[10].

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