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~~hearings held by the Joint Committee on Atomic Energy of the Congress of the United States.~~

~~The exact pattern of deposition of fallout depends on weather conditions. The eruption of May 18 happened to occur during a period when weather conditions were similar to the conditions assumed for the 1959 study. On the other hand, the eruption of May 25 occurred during a rainy period. Not as many ash particles were thrown aloft during the May 25 eruption as during the May 18 eruption but the number was still more than usually expected from a surface detonation of a nuclear weapon. On May 25 rain mixed with the ash to form mud, which fell locally and not uniformly. However the fallout pattern of this ash was similar to that predicted by Storebø for "rainout" of debris from nuclear explosives, again making the event worthy of study as a possible predictor of effects which could follow the detonation of a nuclear weapon. (See VOLCANOLOGY.)~~

~~The series of events which result in acid rain are at the other extreme of the fallout picture for, instead of consisting of particulate matter of finite size, much of the material released from chimneys, smokestacks and motor-vehicle exhausts is composed of individual molecules or groups of molecules of the oxides of sulfur and nitrogen. In the atmosphere these substances go through chemical reactions to form sulfates and nitrates, which are acidic and which dissolve in water and fall as precipitation at distances of 100 to 1000 kilometers downwind from their original source. The mechanical result is a type of fallout not unlike that from an air-burst of a nuclear weapon, during which essentially all matter is vaporized. Although the particulate matter cannot be physically observed, as in the case of the fall of ash from Mount St. Helens, the chemical effects of the acid rain can be observed, for example, in the increased acidity of the lakes of the Adirondack Mountains, where the water of some lakes has become too acidic to support fish life. Similar nuclear-weapon fallout would be observed simply as an increase in radioactivity in the water.~~

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~~Cross-references: ATOMIC ENERGY, FISSION, FUSION, ISOTOPES, NUCLEAR REACTIONS, RADIOACTIVITY, VOLCANOLOGY.~~

FARADAY EFFECTS*

— R. F. O'CONNELL

In 1845, Michael Faraday discovered the first magnetooptical effect[1] when he observed the rotation of the plane of polarization of light as a result of its passage through lead borate glass in a direction parallel to an applied magnetic field B. This is known as the *Faraday effect* or *Faraday rotation* and was important historically because it provided the first concrete evidence for a connection between magnetism and light. Since then many other magnetooptical effects have been investigated—notably cyclotron resonance, magnetic dichroism, and the Voigt and Hall effects—their common linkage being their dependence on various components of the dielectric or conductivity tensor, as well as on the magnetic permeability tensor in the case of magnetic materials. Faraday rotation has now been shown to be a general property of matter and has been observed in a variety of solids (especially semiconductors), liquids, and gases, over a wide range of frequencies.[2] It is often a very useful technique[3,4] for the determination of various quantities such as effective mass m, collision frequency ν, and mobility μ. In fact, not only does it complement cyclotron resonance determinations of m but it is especially useful in determining electron and hole effective masses in solids in cases where cyclotron resonance is unobservable.[5] The latter circumstance occurs when $\nu \gg \omega$, where ω is the angular frequency of the radiation, and corresponds to large damping (since $\tau = \nu^{-1}$, where τ is the damping or relaxation time). Also, since cyclotron resonance occurs when $\omega = \omega_c$, where $\omega_c = (eB/mc)$ is the cyclotron frequency with e being the magnitude of the charge, it is clear that for a typical m value of $10^{-1}\ m_0$ (where m_0 is the free electron mass) and a maximum B value of 100 kG we have resonance at $\omega = 1.77 \times 10^{13}\ s^{-1}$, corresponding to a wavelength $\lambda = 326\ \mu m$, i.e., for practical purposes sharp cyclotron resonances typically do not occur for wavelengths smaller than infrared, and in many cases actually no smaller

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than microwave. In other words, Faraday rotation may be measured over a far wider range of frequencies and in a far wider range of materials than cyclotron resonance. However, it has the disadvantage of measuring an average effective mass in cases where the effective mass is anisotropic. It is also used to deduce interstellar magnetic field values as well as providing support for the conclusion that there is little or no antimatter in the galaxy. We use cgs units in this article.

A linearly polarized wave can be decomposed into two waves of opposite *circular polarization*, called *right* and *left* (RCP and LCP), which propagate independently. In general, optical rotation occurs in a medium when its refractive indices for right- and left-circularly polarized radiation, n_+ and n_-, are unequal so that the phase velocities c/n_+ and c/n_- are also unequal. In the case of *natural* optical activity, this arises from asymmetry among the atomic layers whereas *Faraday* rotation arises from the anisotropy produced by the magnetic field. The former (which will not concern us here) disappears on reflection back through a sample whereas the latter is doubled. In the case of an absorbing medium, the difference in absorption coefficients for the two components (referred to as *dichroism*) causes the emerging beam to be elliptically polarized.

The Faraday rotation θ is defined to be one-half the phase angle change between the RCP and LCP waves and corresponds to the amount of rotation of the major axis of the transmitted polarization ellipse.[3] For radiation of frequency ω propagating a distance d through the medium along the direction of the magnetic field, the rotation is given by

$$\theta = \frac{\omega d}{2c}(n_+ - n_-) = \frac{\pi d}{\lambda}(n_+ - n_-), \qquad (1)$$

where λ is the wavelength in vacuum. A theoretical evaluation of $n_\pm$ in the case of nonmagnetic materials (magnetic materials will be discussed below) starts with the relation

$$\epsilon_\pm = (n_\pm + ik_\pm)^2, \qquad (2)$$

where k, the imaginary part of the complex refractive index, arises from absorption and $\epsilon_\pm$ is related to the components of the dielectric tensor ϵ_{ij} $(i, j = x, y, z)$ by

$$\epsilon_\pm = \epsilon_{xx} \pm i\epsilon_{xy}. \qquad (3)$$

Thus the problem of calculating θ is reduced to a calculation of ϵ_{ij}. In general, rotation arises due to interaction of the radiation with either free or bound charge carriers. There are five basic frequencies to be considered: the wave frequency ω, the plasma frequency $\omega_p = (4\pi Ne^2/m)^{1/2}$, where N is the number of charge carries per unit volume, the collision frequency ν, the cyclotron frequency ω_c, and ω_0, which refers to either the natural frequency of the bound charges (classical model) or the frequency separation of spectral lines (quantum mechanical model). In a solid there are contributions to θ from both free and bound electrons and also the nuclei but, as a general rule, at optical and infrared frequencies, the dominant contribution to θ is from the free electrons. For low photon frequencies only transitions within the same band (intraband) are of importance in a semiconductor but as we approach the optical region the band to band (interband) effects must be included. A *classical* calculation of ϵ_{ij} is based on the Boltzmann equation or, more frequently although less rigorously, on the Drude model, which assumes that all charge carriers act independently. The Drude model result is often referred to as the *cold-plasma limit* since it corresponds to the result obtained by use of the Boltzmann equation in the limit of extreme degeneracy (zero temperature).

Consider now the case of electrons moving freely ($\omega_0 = 0$) in a crystal-lattice background—while being cognizant of the fact that, except for a change in the sign of θ, similar results hold for positive carriers such as, for example, holes in the valence band of semiconductors. Then the Drude model leads to[3]

$$\epsilon_\pm = \epsilon_l - [\omega_p{}^2/\omega(\omega \pm \omega_c + i\nu)], \qquad (4)$$

where $\epsilon_l = n^2$ is the (real) dielectric constant of the lattice. It follows that, if ω is much larger than the other three frequencies, the Drude model leads to

$$\theta = d\omega_c\omega_p{}^2/2cn\omega^2. \qquad (5)$$

Thus θ is proportional to the magnetic field B and also to m^{-2}, making clear how a measurement of θ can determine the effective mass m. This formula is also used to determine galactic magnetic fields from observations, over a range of frequencies, of the Faraday rotation associated with polarized radio waves from such objects as pulsars.[6] Estimates of primordial magnetic field values have also been made from observations of the θ of a distant extragalactic radio source.[7] Furthermore, a measurement of the Faraday rotation of radio waves emitted by artificial satellites and transmitted through the ionosphere can be used to measure the electron density along the path.[8] Turning to the question of how much antimatter there is in the galaxy, we note that positrons and electrons cause rotations in opposite directions. But polarized light traversing the interstellar medium does suffer Faraday rotation, demonstrating that there are not comparable numbers of electrons and positrons. In fact, when these results (which in essence give the difference in the number of electrons and positrons) are combined with dispersion measures (which depend on the sum of the number of electrons and positrons), it is found that the number of positrons is negligibly small.[9]

In the case where ω_c is much greater than the other three frequencies (high-field, low-frequency approximation),

$$\theta = -d\omega_p{}^2/2cn\omega_c, \qquad (6)$$

i.e., θ is now negative, and it is proportional to B^{-1} and independent of ν again and also ω. Also if $\omega_p \ll \omega$ then a zero in θ occurs[10] at a photon frequency $\omega = (\omega_c{}^2 + \nu^2)^{1/2}$, which can lead to a determination of ν. If $\nu \gg \omega$, $\omega_c \gg (\omega_p/n)$ then we get the *low-frequency Faraday rotation*

$$\begin{aligned}\theta &= -d\omega_c\omega_p{}^2/2cn\nu^2 \\ &= -2\pi d\sigma_0\mu B/c^2 n,\end{aligned} \qquad (7)$$

where $\mu = e\tau/m$ is the carrier mobility and $\sigma_0 = ne^2\tau/m$ is the static conductivity. Since the latter expression for θ does not contain m or ν explicitly, one can use known values of σ_0 and the other parameters to deduce the mobility of charge carriers from a measurement of θ.

In strong magnetic fields, account must be taken of the fact that electron energies in a magnetic field are confined to discrete *Landau levels* and, in the case of interband transitions, this gives rise to oscillatory effects in the Faraday rotation, while there is evidence for a contribution also from exciton transitions. In general, there are other complications[3] which are sometimes of importance. For example, the collision frequency ν can be frequency- and magnetic-field dependent. Also, in polar semiconductors there is a contribution from optical lattice vibrations to the dielectric tensor.

In the case of thin samples, multiple reflections can play an important role with an attendant increase in the complexity of the analysis.[11,12] An example of where such multiple reflections play a role[13] is the *two-dimensional electron space-charge layer* (the motion being quantized in one direction whose effective width is negligible compared to the wavelength of the transmitted radiation), which is formed in various modern microelectronics systems, such as at the semiconductor surface in a metal-oxide-semiconductor (MOS) system. There is also a contribution to θ due to boundary effects in the transmission of radiation through different material. In the case of metals, the usual method of observing θ in transmission is not convenient except for very thin films because metals are very good reflectors in the visible and infrared regions. As an alternative, θ is measured on reflection (the *polar reflection Faraday effect*) and gives information on the electron band structure of nonferromagnetic metals.

The advent of high-intensity laser radiation has motivated the inclusion of various nonlinear terms into the laws of optics. In particular, the Faraday effect is intensity dependent, especially in a strong magnetic field.[14] A closely related phenomenon is the *inverse Faraday effect*, i.e., the magnetization of the medium by intense polarized radiation,[15] which has been suggested as the basis of a nondemolition optical quantum counting measurement.[16]

Faraday rotation has also been used as a diagnostic tool to study and measure the large magnetic fields which are produced both in controlled thermonuclear fusion plasma and in laser-produced plasma,[17] as well as being one of the first phenomena to be studied by the megagauss magnetic fields which are being increasingly produced in many laboratories.[18]

All of the effects discussed so far depend on the dielectric tensor and arise from the interaction of the charge carriers with the electric field of the electromagnetic wave and, in addition, the spatial dependence of the electric field is generally neglected (electric dipole approximation). However, for magnetic materials the Faraday rotation depends on the magnetic permeability tensor. In ferromagnetic metals very large rotations occur which are proportional to the net magnetization and not to the external magnetic field.[19,20] Thus the rotation per cm in a magnetic field of 10^4 gauss is of the order of 2 degrees in quartz, 10^2 degrees in aluminum and 1.3×10^5 degrees in iron. The large value of the latter rotation arises from a spin-orbit interaction: the magnetic moment of an electron, due to its spin, interacts with the magnetic field which arises by virtue of its motion through the electric field created by the nuclei and all the other electrons in the absence of radiation. This phenomenon is often called the *ferromagnetic Faraday effect* and refers to the transmitted beam, whereas effects associated with reflection from a ferromagnetic material are called *polar Kerr magnetooptic effects*. In a certain sense the spin-orbit interaction can be looked on as the effect of a large internal magnetic field acting on the electrons. Also, the role of the external magnetic field is peripheral in that it serves only to magnetize the sample in a certain direction. It should also be mentioned that there is also a contribution from spin-orbit effects in nonferromagnetic material, albeit small compared to the situation for ferromagnetics.

Absorption in the metallic ferromagnets is very large except in the case of very thin films. On the other hand, ferrimagnetic substances are particularly good magnetooptical materials because they combine the low absorption of a good insulator with high permeability. For example, yttrium iron garnet (YIG) is transparent in the optical region and also gives rise to a large Faraday rotation which makes it an excellent material for the observation of magnetic domains. In fact, measurements of θ can be used to measure several macroscopic magnetic properties of thin rare earth garnet films which are used for magnetic bubble devices.[21] Ferrimagnetic materials are used extensively in microwave technology,[22] their importance stemming from the fact that they can be used to make Faraday isolators which permits a signal to be transmitted with low attenuation in one direction but causes the reflected signal to be highly attenuated. Thus, for example, this permits the decoupling of an oscillator from a measuring system. Similar devices have also been used in laser systems.[23] At higher frequencies it turns out that antiferromagnetic materials perform better.

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Cross-references: HALL EFFECT AND RELATED PHENOMENA, KERR EFFECT, LIGHT, MAGNETISM, POLARIZED LIGHT, PROPAGATION OF ELECTROMAGNETIC WAVES, SEMICONDUCTORS.

FEEDBACK

~~The concept of feedback lies at the heart of modern systems theory and control engineering. The term itself seems to have been used for the first time in a technical sense in 1920,[1] and refers to the return or feedback of system output signals to the inputs in order to improve or change the behavior of the system. A very simple example is the control of a heating system by using a bimetallic strip (Fig. 1). The system output is heat, the input the voltage across the heating element. When the desired temperature (the *set-point*) is reached the heat causes the bimetallic strip to deform sufficiently to switch off the heating element. As the temperature falls back below the set-point the strip returns to its original shape, contact is reestablished, and the heater switches on again. A second illustrative example may be found in the flyball governor (attributable to Sir James Watt) and used to control steam engines (Fig. 2). As more steam is fed to the engine the shaft~~