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QUASI-LANDAU RESONANCES : ANALYTIC TREATMENT OF THE HYDROGENIC SPECTRUM IN THE TWO-DIMENSIONAL MODEL AND RELATION TO OTHER STRONG-FIELD PROBLEMS

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Résumé: Le modèle WKB à 2 dimensions est à l'origine de plusieurs approches numériques du spectre quasi-Landau de l'atome d'hydrogène. On montre ici que les résultats peuvent être obtenus sous forme analytique en termes d'intégrales elliptiques. Celles-ci peuvent être calculées aisément ce qui permet d'obtenir les prédictions numériques en bon accord avec l'expérience. On montre aussi que plusieurs autres problèmes en champs intenses sont des cas particuliers d'un potentiel plus général pour lequel les niveaux d'énergie et leur espacement peuvent être exprimés en termes d'intégrales elliptiques.

Abstract: The two-dimensional WKB model has been the basis for several investigations of the quasi-Landau hydrogenic spectrum. Whereas other authors have used numerical integration, we show that the results can be obtained analytically in terms of elliptic integrals. The later are easily generated by even programmable pocket calculators, from which numerical results - which are in good agreement with experiments - are easily obtained. A further advantage of using elliptic integrals is that several strong-field problems can be shown to be special cases of a general potential whose energy and spacing is expressible in terms of them.

Discussion. In 1969 Garton and Tomkins |1| published a paper the result of which was to unleash an avalanche of research in the general area of what is now referred to as quasi-Landau resonances. They studied the absorption spectrum of barium in a magnetic field of 24 kG, for n values as high as 75, and found broad resonances spaced by approximately $1.5\,\hbar\omega$ near $E = 0$ ($\omega = eB/Mc$ is the cyclotron frequency where M is the electron mass). These quasi-Landau resonances were observed only in the σ spectrum ($\Delta m = \pm 1$) and not in the π spectrum ($\Delta m = 0$). Thus, since the σ lines come from states which are essentially localized in the x-y plane (as distinct from the π lines which come from a state with a node in the x-y plane), it is natural to neglect the motion in the z direction and treat the problem as two-dimensional. This was the procedure used by both Edmonds |2| and Starace |3|, who reduced the problem to motion in a potential $V(\rho)$ where $\rho^2 = x^2 + y^2$. Thus it was then natural to solve the problem using WKB techniques. Whereas previous investigators have solved the WKB integrals using numerical techniques, we have recently shown |4| that results can be obtained analytically in terms of elliptic integrals, not only for the energy spacings but also for the energy levels for zero and non-zero values of m. The results so obtained are in very good agreement with, for example, the experimental results of Delande and Gay |5| and with the numerically generated results of McDowell |6|.

In the present brief communication our emphasis will be on why we feel it is advantageous to work in terms of elliptic integrals, of which there are only three kinds. First of all, they are easily generated by even programmable pocket calculators, from which numerical resulcs are easily obtained. As a result it very much facilitates comparison with work of other authors. In addition, we see explicitely that the quasi-Landau problem fits into a general framework of strong-field problems. This was emphasized by one of us several years ago |7| and elaborated on at more length in a review paper presented at the present Colloque |8|. To be specific, we mention first of all the calculation of the energy spectrum of electrons outside a free surface of liquid helium. These electrons are trapped in an image potential which is essentially one-dimensional Coulombic and for diagnostic purposes an electric field is also generally superimposed. This combined field problem is another example of a strong-field problem whose solution can be expressed in terms of elliptic integrals |7,8| as in the case of the quasi-Landau resonances |4|. A further example is the Stark problem, which was solved to very good accuracy using WKB techniques |9| and the results obtained - including the case of autoionizing states - were all expressed in terms of elliptic integrals. Finally, we mention the calculation of the spectrum of Charmonium |10|. In essence, the potentials for all the problems are special cases of a general potential whose energy spectrum and spacing is expressible in terms of elliptic integrals. Elsewhere, we will present in more detail a general framework which can be used for the consideration of a large class of strong-field problems.

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