Physica 115B (1982) 1–4
North-Holland Publishing Company

TRANSMISSION OF ELECTROMAGNETIC RADIATION THROUGH AN ELECTRON INVERSION LAYER OF FINITE THICKNESS IN A METAL-OXIDE-SEMICONDUCTOR (MOS) STRUCTURE

R.F. O'CONNELL and G. WALLACE
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

Received 14 May 1982

We investigate the transmission coefficient for propagation of electromagnetic radiation, of frequency ω, through an electron inversion layer, of finite thickness d, in the metal-oxide-semiconductor (MOS) structure. The corresponding result in the case where the inversion layer is treated as infinitely thin (two-dimensional electron gas) has been given by Chiu et al. Subject to the assumptions $\omega d/c \ll 1$ and constant density, it is possible to establish a point of contact between the finite d result and the Chiu et al. result. This comparison demonstrates that it is a good approximation to treat the inversion layer as a two-dimensional system and also provides a recipe for the calculation of the surface conductivity $\sigma^{(2)}$, viz. $\sigma^{(2)} = \sigma d$, where σ is the three-dimensional conductivity.

Cyclotron resonance is a widely used technique in the study of inversion layers at the semiconductor surface of a metal-oxide-semiconductor (MOS) system. The MOS structure is of much current interest both for technological and for basic scientific reasons.

Chiu et al. [1] have calculated the transmission amplitude and transmittance for an inversion layer of zero thickness. This result has been used widely by various authors [2–4] to interpret experimental data. However, it is known [5] that the electron inversion layer extends into the semiconductor to the order of 100 Å. Thus we are motivated to study inversion layers of finite thickness. In particular, we would like to address the following questions: (a) How good an approximation is it to treat the inversion layer as a two-dimensional system? (b) What recipe should one use for the calculation of the surface conductivity $\sigma^{(2)}$ appearing in the two-dimensional model?

The exact problem is complicated by the fact that the inversion layer has a varying density profile [5, 6]. The problem of electromagnetic transmission in inhomogeneous plasmas has been the subject of many investigations but only in very special cases is the problem easily solvable [7]. In fact, the most common type of approach to problems of this nature is to use a WKB approach. However, the latter method is based on the assumption that "... the properties of the medium vary very slowly ..." over a distance of the order of the wavelength of the radiation (geometric optics approximation) [8]. Unfortunately, this is the opposite of the situation we have in the case of the inversion layer, where the dielectric tensor varies rapidly across the layer, the latter being very much smaller than the typical wavelengths of interest. We conclude that a solution of the exact problem is accessible only by means of a complicated numerical approach.

On the other hand, if we consider the idealized case of an inversion layer of constant density, an exact analysis is possible, as we shall see, and, in addition, considerable light is thrown on the questions stated above. In particular, it will enable us to establish a point of contact between the finite d result and the Chiu et al. result. In essence, what the constant density model does is to average out the dielectric properties across the inversion layer but it should be emphasized that the two-dimensional model is also subject to a somewhat similar limitation because – by virtue of its very nature – it cannot incorporate a density profile. We turn now to the details of the calculation.

We consider the propagation of linearly polarized electromagnetic radiation of frequency ω through the oxide–inversion-layer–semiconductor system, in the presence of an applied external dc magnetic field $\boldsymbol{B} = B\hat{z}$, $B > 0$. The electric fields in the three regions of fig. 1 are:

$$z<0: \quad \boldsymbol{E}_{i\pm} = E_{i\pm}\,e^{i(k_O z-\omega t)}\hat{\varepsilon}_\pm\,, \quad \boldsymbol{E}_{r\pm} = E_{r\pm}\,e^{-i(k_O z+\omega t)}\hat{\varepsilon}_\pm\,, \tag{1a}$$

$$0<z<d: \quad \boldsymbol{E}_{u\pm} = E_{u\pm}\,e^{i(k_{I\pm} z-\omega t)}\hat{\varepsilon}_\pm\,, \quad \boldsymbol{E}_{v\pm} = E_{v\pm}\,e^{-i(k_{I\pm} z+\omega t)}\hat{\varepsilon}_\pm\,, \tag{1b}$$

$$z>d: \quad \boldsymbol{E}_{t\pm} = E_{t\pm}\,e^{i(k_S z-\omega t)}\hat{\varepsilon}_\pm\,, \tag{1c}$$

where

$$\hat{\varepsilon}_\pm \equiv (\hat{x}\pm i\hat{y})/\sqrt{2} \tag{2}$$

and the "±" notation refers to the left and right circularly polarized components of the corresponding wave. The total waves are

$$\boldsymbol{E}_i = \boldsymbol{E}_{i+} + \boldsymbol{E}_{i-}\,, \quad \boldsymbol{E}_r = \boldsymbol{E}_{r+} + \boldsymbol{E}_{r-}\,, \ldots\,, \quad \boldsymbol{E}_t = \boldsymbol{E}_{t+} + \boldsymbol{E}_{t-}\,, \tag{3}$$

where k is the wave number and the subscripts O, I and S refer to the oxide, inversion layer and semiconductor, respectively, and are related to the refractive index and absorption coefficient of the corresponding medium by

$$k_O = (\omega/c)n_O\,, \tag{4a}$$

$$k_{I\pm} = (\omega/c)(n_{I\pm} + i\kappa_{I\pm})\,, \tag{4b}$$

$$k_S = (\omega/c)n_S\,, \tag{4c}$$

where c is the speed of light.

The fields given in eq. (1) satisfy the wave equation

$$\frac{\partial^2 \boldsymbol{E}}{\partial z^2} = -\frac{\omega^2}{c^2}\,\varepsilon \boldsymbol{E} - i\,\frac{4\pi\omega}{c^2}\,\boldsymbol{J}\,, \tag{5}$$

where $\boldsymbol{J}$ is the three-dimensional current density induced in the inversion layer by the electric field $\boldsymbol{E}$.

The boundary conditions at $z = 0$ and $z = d$ are (a) continuity of the tangential component of the ac electric field and (b) equality of the discontinuity of the ac magnetic field and $(4\pi/c)\boldsymbol{j}$, where $\boldsymbol{j}$ is the two-dimensional surface current density. In the model of Chiu et al. [1], the inversion layer was treated as *infinitesimally small*, so that $\boldsymbol{J} = \boldsymbol{j}\delta(z)$. However, in the case of an inversion layer of *finite* thickness, as Jackson points out [9], "...there cannot actually be a surface layer of current...". This is true also when the layer is "...macroscopically small, but microscopically large..." [9]. Thus, for our analysis we take $\boldsymbol{j} = 0$ and so the boundary conditions give the following results (suppressing the "±" subscripts):

$$z=0: \quad E_i + E_r = E_u + E_v\,, \tag{6a}$$

$$k_O(E_i - E_r) - k_I(E_u - E_v) = 0\,, \tag{6b}$$

$$z=d: \quad E_u\,e^{ik_I d} + E_v\,e^{-ik_I d} = E_t\,e^{ik_S d}\,, \tag{6c}$$

$$k_I(E_u\,e^{ik_I d} - E_v\,e^{-ik_I d}) - k_S E_t\,e^{ik_S d} = 0\,. \tag{6d}$$

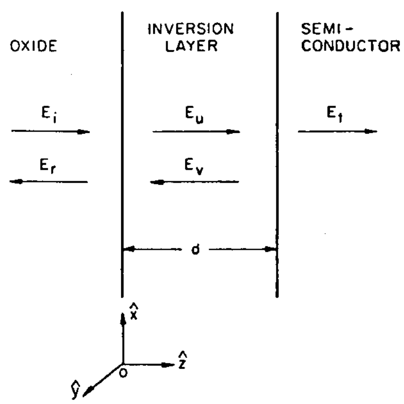

Fig. 1. Geometry for the propagation of electromagnetic radiation through an inversion layer of finite thickness d in a MOS structure.

After some algebra, we find that the solution of eq. (6) for the transmission coefficient $t \equiv E_t/E_i$, in circularly polarized coordinates, is

$$t_\pm = 4k_O k_\pm\,e^{-ik_S d}\,\{(k_O - k_\pm)(k_\pm - k_S)\,e^{ik_\pm d} + (k_O + k_\pm)(k_\pm + k_S)\,e^{-ik_\pm d}\}^{-1}\,, \tag{7}$$

where we have written $k_\pm$ for $k_{I\pm}$ (and we will do likewise for $n_{I\pm}$ and $\kappa_{I\pm}$ below).

Using eq. (4), eq. (7) may be written as

$$t_\pm = 4n_O(n_\pm + i\kappa_\pm)\, e^{-i\beta_S d} \times \{(n_O - n_\pm - i\kappa_\pm)(n_\pm + i\kappa_\pm - n_S)\, e^{-\alpha_\pm d/2}\, e^{i\beta_\pm d} + (n_O + n_\pm + i\kappa_\pm)(n_\pm + i\kappa_\pm + n_S)\, e^{\alpha_\pm d/2}\, e^{-i\beta_\pm d}\}^{-1}, \tag{8}$$

where $\alpha_\pm/2 \equiv \omega\kappa_\pm/c$, $\beta_\pm \equiv \omega n_\pm/c$ and $\beta_S \equiv \omega n_S/c$. Note, in particular, that $t_\pm$ depends explicitly on the optical constants $n_\pm$, $\kappa_\pm$ of the inversion layer. In the absence of a magnetic field, eq. (8) reduces to a result obtained by Stratton [10].

Since the thickness d of the inversion layer is typically $\approx 10^{-6}$ cm, then, for wavelengths λ in the region of, say, 3×10^{-2} cm (far infra-red), $d \ll \lambda$ or equivalently $(\omega d/c) \ll 1$. Thus, to first order in d, we have

$$t_\pm = \frac{2n_O}{n_O + n_S}\left\{1 + i\,\frac{(n_\pm + i\kappa_\pm)^2 - n_S^2}{n_O + n_S}\left(\frac{\omega d}{c}\right)\right\}. \tag{9}$$

Next, we note that the dielectric constant of the inversion layer is given by

$$\varepsilon_\pm = \varepsilon_\ell + i\,\frac{4\pi}{\omega}\,\sigma_\pm\,, \tag{10}$$

where $\sigma_\pm$ is the three-dimensional conductivity, and ε_ℓ is the dielectric constant of the bulk silicon. But since

$$\varepsilon_\pm = (n_\pm + i\kappa_\pm)^2 \tag{11}$$

and

$$\varepsilon_\ell = n_S^2\,, \tag{12}$$

it follows that

$$t_\pm = \frac{2n_O}{n_O + n_S}\left\{1 - \left[\frac{1}{c}(4\pi\sigma_\pm d)\Big/(n_O + n_S)\right]\right\}. \tag{13}$$

to first order in d.

We now wish to make contact with the Chiu et al. [1] result obtained in the case $d = 0$ viz.

$$t_\pm = \frac{2n_O}{n_O + n_S + (4\pi/c)\sigma_\pm^{(2)}}, \tag{14}$$

where $\sigma_\pm^{(2)}$ is the two-dimensional surface conductivity. We find that if we set

$$\sigma_\pm^{(2)} = \sigma_\pm d \tag{15}$$

in eq. (14) and expand to lowest order in d that we obtain a result for $t_\pm$ identical to that given in eq. (13).

To conclude, we have obtained a general result for the case of the transmittance of linearly polarized light through an inversion layer of finite thickness and uniform density. In addition, by demanding consistency between our results for $\omega d/c \ll 1$ and those of Chiu et al. [1] for $d = 0$, we have given a recipe for the calculation of the surface conductivity appearing in the Chiu et al. results.

It should be emphasized that our results hold *regardless of what explicit form is chosen for* $\sigma_\pm$. On the other hand, they were derived for the idealized case of an inversion layer of uniform carrier density, whereas in fact the carrier density is highly non-uniform [5]. This non-uniformity leads to a very complicated dielectric constant and is particularly important "... whenever the frequencies of interest are of the order of the frequency of the single-particle transitions to the first excited subband ..." [6]. However, it can be argued that the use of an *average* density is a good approximation in *intraband* situations where we are dealing with photons of low frequency such that no subband excitations occur (as is the situation in, for example, Faraday rotation [11] experiments). Furthermore, it should be stressed that the use of the two-dimensional approximation does not obviate the question of how to deal with non-uniformity – in essence it "sweeps it under the rug" and in the end one is faced with the question of evaluating the two-dimensional conductivity $\sigma_\pm^{(2)}$. The latter, of course, is an idealization, not having the physical content of the three-dimensional conductivity $\sigma_\pm$. In fact what is done in practice is, in essence, to write $\sigma^{(2)}$ in terms of $\sigma_\pm$ and hence it is subject to all the limitations inherent in the choice of $\sigma_\pm$. One result of our present investigation is that, within the framework of these limitations, we have given the correct recipe viz. eq. (15) and, in addition, we have reinforced confidence in the assumption of treating the inversion layer in MOS as a two-dimensional system. Also, we note that our recipe is in contrast to another result sometimes quoted in the literature viz. $\sigma_{\pm}^{(2)} = N^{-1/3}\sigma_{\pm}$, where N is the number of particles per unit volume. However, it is interesting to note that, for the inversion layer in typical MOS systems, both results are numerically close to one another. For example, a typical value for the surface electron concentration, N_s say, is $10^{12}\,cm^{-2}$. The $\sigma_{\pm}^{(2)}$ chosen by Chiu et al. [1] is proportional to this number. However, the value of $\sigma_{\pm}^{(2)}$ as given by eq. (15) is proportional to $10^{18} \times d\,cm^{-2}$. But, since d is $\approx 10^{-6}\,cm$, we see that this is again $\approx 10^{12}\,cm^{-2}$.

Finally, we would like to emphasize that eq. (8) is a very general result in the sense that it can be applied to light transmission, in the presence of a magnetic field, through any 3-media system in which the center medium is of uniform density and conducting.

Acknowledgments

This research was partially supported by the Department of Energy, Division of Materials Science, under contract No. DE-AS05-79ER10459.

Note added in proof

An important point which emerges from the above analysis is that the two-dimensional model of Chiu et al. [1] automatically includes multiple reflection effects in the inversion layer (E_v in fig. 1). Such effects play an important role in the analysis of measurements of ellipticity and Faraday rotation [12].

References

[1] K.W. Chiu, T.K. Lee and J.J. Quinn, Surface Science 58 (1976) 182.
[2] T.A. Kennedy, R.J. Wagner, B.D. McCombe and D.C. Tsui, Phys. Rev. Lett. 35 (1975) 1031.
[3] T.A. Kennedy, R.J. Wagner, B.D. McCombe and J.J. Quinn, Solid State Commun. 18 (1976) 275.
[4] R.J. Wagner, T.A. Kennedy, B.D. McCombe and D.C. Tsui, Phys. Rev. B22 (1980) 945.
[5] F. Stern and W.E. Howard, Phys. Rev. 163 (1967) 901.
[6] A. Equiluz and A.A. Maradrudin, Ann. Phys. (N.Y.) 113 (1978) 29 (in particular, see p. 66).
[7] See, for example, V.L. Ginzburg, The Propagation of Electromagnetic Waves in Plasmas (Pergamon Press, New York, 1970) 2nd ed., ch. IV.
[8] Ref. 7, p. 226.
[9] J.D. Jackson, Classical Electrodynamics, 2nd ed. (J. Wiley, New York, 1975) pp. 23 and 336.
[10] J.A. Stratton, Electromagnetic Theory (McGraw-Hill, New York, 1941) p. 512.
[11] As an example, we refer to the discussion in the "Note added in proof" section of R.F. O'Connell and G.L. Wallace, Solid State Commun. 38 (1981) 429.
[12] R.F. O'Connell and G. Wallace, Phys. Rev. B26 (1982) 2231.