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ABSORPTION OF RADIATION PROPAGATING OBLIQUELY IN A MAGNETOPLASMA

A. KHANDKER AND R. F. O'CONNELL
Department of Physics and Astronomy, Louisiana State University
AND
G. W. FORD
Department of Physics, University of Michigan
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ABSTRACT

We consider the propagation of electromagnetic radiation in a magnetoplasma, with the propagation vector making an arbitrary angle θ with respect to the magnetic field, and we obtain a general expression for the power absorbed. In the particular case of propagation of circularly polarized radiation, we find that, for all angles, left circularly polarized radiation is absorbed more than right circularly polarized radiation. We discuss the implications of our results for the deduction of the magnitude of the magnetic field in magnetic white dwarfs.

Subject headings: plasmas — polarization — radiative transfer — stars: magnetic — stars: white dwarfs

Consider a monochromatic plane electromagnetic wave propagating in a plasma, with an electric vector given by

$$\boldsymbol{E}(t) = \mathrm{Re}\, \boldsymbol{E}_\omega e^{-i\omega t}. \tag{1}$$

Without loss of generality, we assume that the wave is propagating along the positive z-direction and making an angle θ with respect to the direction of an external magnetic field B. In the case where $\theta = 0$ (Faraday orientation) and $\theta = \pi/2$ (Voigt orientation), magnetic dichroism (i.e., the difference in the power absorbed for different polarizations) has been extensively studied in many branches of physics and astrophysics (see, for example, Ford and O'Connell 1980, and references therein). Many aspects of the case of arbitrary θ have also been considered by Palik and Furdyna (1970), but nowhere have we found a general discussion of magnetic dichroism in this case.

The starting point of our calculation is a general expression for the time-averaged power dissipated per unit volume (Landau and Lifshitz 1960):

$$P = \frac{\omega}{8\pi}\, \mathrm{Im}\,(\boldsymbol{E}^*_\omega \cdot \boldsymbol{D}_\omega), \tag{2}$$

where, in component form,

$$(\boldsymbol{D}_\omega)_l = \varepsilon_{lm}(\boldsymbol{E}_\omega)_m, \tag{3}$$

and ε_{lm} denotes the components of the plasma dielectric tensor. We now set

$$\boldsymbol{E}_\omega = E_0 \hat{e}, \tag{4}$$

where E_0 refers to the amplitude of the radiation and $\hat{e}$ is a unit vector in its direction of polarization. Hence, we obtain

$$P = -\frac{i\omega}{16\pi}|E_0|^2 e^*_l e_m (\varepsilon_{lm} - \varepsilon^*_{ml}). \tag{5}$$

Now, a general expression for the dielectric tensor is given by

$$\varepsilon_{lm} = \delta_{lm} - \frac{\omega_p^2}{\omega}\left[\frac{1}{\omega + i\nu} b_l b_m + \frac{\omega + i\nu}{(\omega + i\nu)^2 - \omega_c^2}(\delta_{lm} - b_l b_m) - \frac{i\omega_c}{(\omega + i\nu)^2 - \omega_c^2}\varepsilon_{nlm} b_n\right], \tag{6}$$

where δ_{lm} is the Kronecker delta, ε_{nlm} is the Levi-Civita symbol, b_l is the l component of the unit vector $\hat{b}$ in the direction of the magnetic field, and where ν, ω_c, and ω_p denote the collision, cyclotron, and plasma frequencies respectively. Also

$$\omega_c = eB/mc, \tag{7}$$

and

$$\omega_p = \left(4\pi n e^2/m\right)^{1/2}, \tag{8}$$

where $-e$, m, and n are the charge, mass, and number density of the electrons. Equation (6) is derived by combining equations (2.5), (2.16), and (2.36) of Palik and Furdyna (1970), noting that in our particular case the lattice dielectric constant ε_l (κ_l in the notation of Palik and Furdyna) is unity.

From equation (6), it follows—after some algebra—that

$$\varepsilon_{lm} - \varepsilon^*_{ml} = 2i\nu\frac{\omega_p^2}{\omega}\left[\frac{1}{\omega^2+\nu^2}b_l b_m + \frac{\left(\omega^2+\omega_c^2+\nu^2\right)\left(\delta_{lm}-b_l b_m\right)-2i\omega\omega_c\varepsilon_{nlm}b_n}{\left(\omega^2-\omega_c^2\right)^2+2\left(\omega^2+\omega_c^2\right)\nu^2+\nu^4}\right]. \tag{9}$$

Substituting equation (9) in equation (5), we finally obtain the general result

$$P = \frac{\omega_p^2\nu}{8\pi}|E_0|^2\left[\frac{|\hat{b}\cdot\hat{e}|^2}{\omega^2+\nu^2} + \frac{\left(\omega^2+\omega_c^2+\nu^2\right)\left(1-|\hat{b}\cdot\hat{e}|^2\right)+2i\omega\omega_c\hat{e}\cdot\left(\hat{e}^*\times\hat{b}\right)}{\left(\omega^2-\omega_c^2\right)^2+2\left(\omega^2+\omega_c^2\right)\nu^2+\nu^4}\right]. \tag{10}$$

This is the key result of our paper, which we will now apply to the particular case of propagation of circularly polarized radiation, whose propagation vector is along the z-direction, making an angle θ with $\hat{b}$. Thus we write

$$\hat{e} = \hat{e}_{l,r} = \frac{1}{2^{1/2}}\left(\hat{x} \pm i\hat{y}\right). \tag{11}$$

It follows that

$$|\hat{b}\cdot\hat{e}_{l,r}|^2 = \tfrac{1}{2}\left(b_x^2+b_y^2\right) = \tfrac{1}{2}\sin^2\theta, \tag{12}$$

and

$$i\hat{e}_{l,r}\cdot\left(\hat{e}^*_{l,r}\times\hat{b}\right) = \pm b_z = \pm\cos\theta. \tag{13}$$

Substituting equations (12) and (13) in equation (10), we obtain

$$P_{l,r} = \frac{\omega_p^2\nu}{8\pi}|E_0|^2\left[\frac{\sin^2\theta}{2\left(\omega^2+\nu^2\right)} + \frac{\left(\omega^2+\omega_c^2+\nu^2\right)\left(1-\frac{1}{2}\sin^2\theta\right)\pm 2\omega\omega_c\cos\theta}{\left(\omega^2-\omega_c^2\right)^2+2\left(\omega^2+\omega_c^2\right)\nu^2+\nu^4}\right], \tag{14}$$

where $P_{r,l}$ refers to the power absorbed per unit volume from beams of right and left polarization, respectively, and where E_0 refers to the amplitude of either radiation. A convenient comparison of the respective rates is obtained by defining

$$\Delta \equiv \left(P_l - P_r\right)/\left(P_l + P_r\right). \tag{15}$$

Hence we find, using equation (14), that

$$\Delta = \frac{2\omega\omega_2\cos\theta}{\left(\omega^2+\omega_c^2+\nu^2\right)\left(1-\frac{1}{2}\sin^2\theta\right)+\left[D\sin^2\theta/2\left(\omega^2+\nu^2\right)\right]}, \tag{16}$$

where

$$D \equiv \nu^4 + 2(\omega^2 + \omega_c^2)\nu^2 + (\omega^2 - \omega_c^2)^2. \quad (17)$$

In the case where $\theta = 0$, equation (16) reduces to a well-known result (Ford and O'Connell 1980; Kemp 1970, 1977), viz.,

$$\Delta(\omega) = \frac{2\omega\omega_c}{\omega^2 + \omega_c^2 + \nu^2}. \quad (18)$$

It should be emphasized that our present considerations are restricted to the so-called weak–electric-field regime, where the collision frequency is determined primarily by the temperature, as distinct from the electric field (Ford and O'Connell 1980). However, we should also remark that finite temperature effects are neglected in the expression for ε_{lm}, which corresponds to the use of the Drude model.

In the general case, we note that the dependence on θ is nontrivial and could not have been obtained heuristically from the result in the case of $\theta = 0$. However, in the special case that $\omega \gg \omega_c$, ν, we obtain

$$\Delta = \frac{2\omega_c}{\omega} \cos\theta. \quad (19)$$

In other words, this result is the same as the $\theta = 0$ result with B replaced by the component of B along the direction of propagation of the wave.

We turn now to some implications of our results. In particular, Kemp (1970, 1977) has pioneered the use of equation (18) to deduce the magnitude of the magnetic field in so-called magnetic white dwarfs, obtaining values for B in the range 10^6–10^8 gauss. This work has sparked tremendous interest because it is the first direct evidence for the existence of magnetic fields whose magnitude exceeds that of laboratory-produced fields. In addition, it has implications for pulsar theories where even greater values of B ($\sim 10^{12}$ gauss) are speculated to occur but for which the evidence is less direct. However, these observations are interpreted on the assumption that the angle θ between the magnetic field direction and the line of sight is zero. This assumption was made because of the difficulty in determining θ values. However, what it means is that the deduced values of B are, in general, lower than the actual values—which in fact should be deduced from our equation (16) once a value for θ has been determined. We should also remark that ion motion is neglected in both our analysis and that of Kemp. However, such effects are negligible in many situations and, in particular, in the analysis of magnetic white dwarfs.

The above results are also of interest for condensed matter studies.

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REFERENCES

Ford, G. W., and O'Connell, R. F. 1980, *Phys. Rev. A*, **22**, 295.
Landau, L. D., and Lifshitz, E. M. 1960, *Electrodynamics of Continuous Media* (Oxford: Pergamon), pp. 253–254.
Palik, E. D., and Furdyna, J. K. 1970, *Rept. Progr. Phys.*, **33**, 1193.
Kemp, J. C. 1970, *Ap. J.*, **162**, 169.
______. 1977, *Ap. J.*, **213**, 794.

G. W. Ford: Department of Physics, University of Michigan, Ann Arbor, MI 48109

A. Khandker and R. F. O'Connell: Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803