MULTIPLE REFLECTIONS IN THE THEORY OF THE FARADAY EFFECT

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In the case of the transmission of linearly polarized light, of angular frequency ω, through a single medium, surrounded by vacuum on both sides, Donovan and Medcalf have given an exact treatment of multiple reflections in the theory of the Faraday effect. For the purpose of treating transmission through more complicated systems, Piller showed that, for $\omega_c \ll \omega$ (where ω_c is the cyclotron frequency), the problem is amenable to a simpler analysis. In the case of thin samples, we have been able to obtain simple analytic results expressing the multiple reflection rotation as a simple multiple of the single-pass result. In particular, we show that the two models are in disagreement for the case of weak absorption in thin samples. We also present a generalization of the Donovan–Medcalf results to the case of three distinct media.

Consider the electromagnetic propagation of linearly polarized radiation, of angular frequency ω and wavelength λ, through a medium of refractive index $N = n + i\kappa$, surrounded by vacuum on both sides. As with Donovan and Medcalf [1] and Piller [2,3] we consider normal incidence along the positive z direction, which is also the direction of an externally applied magnetic field B. Then, as is well known [4], the *single-pass* Faraday rotation of the direction of polarization is given by

$$\theta = (\omega d/2c)(n_+ - n_-)\,, \tag{1}$$

where d is the length of the medium along the direction of propagation, c is the velocity of light and where n_+ and n_- are the real parts of the refractive indices of the right and left circularly polarized components of the linearly polarized wave.

In the case of the transmission of linearly polarized light, of angular frequency ω, through a *single* medium, surrounded by vacuum on both sides, an exact treatment of multiple reflections in the theory of the Faraday effect has been given by Donovan and Medcalf [1]. Their result for the *multiple-pass* Faraday rotation, Θ say, is a complicated result (eqs. (13) and (14) of ref. [1]) involving the arctan of a non-trivial argument. For the purpose of being able to handle even more complicated transmission problems, Piller [2,3] presented a physically appealing model based on the assumption that the weak-field limit is applicable or, in other words, $\omega \gg \omega_c$, where ω_c is the cyclotron frequency. Thus it follows, in particular, that $N_+ + N_- = 2N$ to lowest order.

However, subsequent investigations [5–8] have indicated that agreement between the models is not possible until additional assumptions are imposed. It is our purpose here to carry out an analytic calculation for the purpose of clarifying explicitly what these assumptions are.

The result for the Faraday rotation in Piller's model, Θ_P say, is again written as the arctan of a complicated function (eq. (14) of ref. [3]), which is not easily comparable to the corresponding result for Θ. In the future we plan to carry out a comparison numerically. However, we have found that, in a certain regime, both results become very simplified so that direct comparison is easily achieved.

We have actually generalized the Donovan–Medcalf results to the case of *three* different media and, in addition, we have been able to decompose the final general result for Θ in a manner which we feel gives added physical insight into the various contributions to Θ. Specifically, we find that for propagation through media 1, 2 and 3 in that order (the details leading to this result will be published elsewhere [9])

$$\Theta = \theta + \theta_t + \theta_{mr}\,, \tag{2}$$

where θ is the single-pass rotation of eq. (1), θ_t is due purely to transmission at the boundaries, and θ_{mr} is due purely to the multiple reflections. Denoting the complex refractive indices as

$$N_{j\pm} \equiv n_{j\pm} + \mathrm{i}\kappa_{j\pm} \quad (j = 1,2,3)\,, \tag{3}$$

and propagation from medium i to medium j by the pair of indices (ij), the results for θ_t and θ_{mr} may be written in the form

$$\theta_t \equiv \tfrac{1}{2}(\gamma_{12+} - \gamma_{12-} + \gamma_{23+} - \gamma_{23-})\,, \tag{4}$$

$$\theta_{mr} \equiv \tfrac{1}{2}(\eta_+ - \eta_-)\,, \tag{5}$$

where

$$\tan\gamma_{ij\pm} = \frac{n_{j\pm}\kappa_{i\pm} - n_{i\pm}\kappa_{j\pm}}{n_{i\pm}(n_{i\pm} + n_{j\pm}) + \kappa_{i\pm}(\kappa_{i\pm} + \kappa_{j\pm})}\,, \tag{6}$$

$$\tan\eta_\pm = \frac{-|r_{23\pm}r_{21\pm}|\,\mathrm{e}^{-\alpha_\pm d}\sin(2\beta_\pm d + \xi_{21\pm} + \xi_{23\pm})}{1 - |r_{23\pm}r_{21\pm}|\,\mathrm{e}^{-\alpha_\pm d}\cos(2\beta_\pm d + \xi_{21\pm} + \xi_{23\pm})}\,, \tag{7}$$

$$\tan\xi_{ij\pm} = \frac{2(n_{i\pm}\kappa_{j\pm} - n_{j\pm}\kappa_{i\pm})}{n_{j\pm}^2 - n_{i\pm}^2 + \kappa_{j\pm}^2 - \kappa_{i\pm}^2}\,, \tag{8}$$

$$r_{ij\pm} = \frac{N_{j\pm} - N_{i\pm}}{N_{j\pm} + N_{i\pm}}\,, \tag{9}$$

are the Fresnel reflection coefficients, and where the complex phase shift $\delta_\pm$ due to medium 2, of thickness d, is written as

$$\delta_\pm \equiv (\omega d/c)(n_{2\pm} + \mathrm{i}\kappa_{2\pm}) \equiv (\beta_\pm + \tfrac{1}{2}\mathrm{i}\alpha_\pm)d\,. \tag{10}$$

In the particular case where $N_1 = N_3 = 1$ (and from henceforth we will write $N_2 \equiv N = n + \mathrm{i}\kappa$), it may be readily verified that these results reduce to those of Donovan and Medcalf and we turn now to a comparison of this case with the corresponding results of Piller. Also, the regime we wish to consider is such that (a) $\omega_c \ll \omega$ (as with Piller [2,3]), (b) $\kappa \ll n$ (weak absorption) and (c) $4\pi nd \ll \lambda$ (thin sample).

Under such assumptions it is seen from eqs. (4) and (6) that θ_t does not contribute to Θ and from eq. (8) that $\xi_{21\pm} = \xi_{23\pm} = 0$. Thus, the trigonometric functions appearing in eq. (7) may be replaced by their small-angle approximations and, with some additional algebra, we obtain the simple result

$$\Theta = n\theta\,. \tag{11}$$

In a similar manner,

$$\Theta_P = [(1 + n^2)/2n]\theta\,. \tag{12}$$

Thus we see explicitly the difference between the Θ_P result and the exact result Θ for a configuration corresponding to weak fields, weak absorption, and thin samples. It is clear that, for $n > 1$, $\Theta > \Theta_P$ and for large n the two models give results differing by close to a factor of 2.

In summary, we have presented a quantitative comparison between two theoretical models of multiple reflections in the theory of the Faraday effect for a situation which is amenable to a simple analytic treatment. This work will be extended in the future (perforce numerically) to check if these models predict similar numbers in other situations.

Finally, we would like to point out the relevance of our results to the observations of Faraday rotation on thin InSb films [10]. Since $n \approx 4$, eq. (11) – which is based on the exact results of Donovan and Medcalf – predicts an enhancement of the single-pass Faraday rotation by a factor of 4, if conditions (a) to (c) are fulfilled. As it turns out, in the experiment of White et al. [10], enhancement factors ≈ 100 have been observed. This is probably due to the fact that they are dealing with strong absorption so that condition (b) is not fulfilled. Whereas strong absorption per se actually mitigates multiple reflection, it can give rise to large rotation effects in the *transmission* of light from vacuum to medium, whereas in the case considered above the transmission coefficient θ_t was zero. This matter is presently under investigation.

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