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Faraday rotation in the Appel-Overhauser model for inversion-layer electrons in Si

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In order to explain puzzling observations of cyclotron resonance in electron inversion layers on Si, Appel and Overhauser have proposed a system composed of two different types of degenerate electrons. We examine Faraday rotation in this system in the limits of both weak and strong electron-electron scattering. In the former case (two noninteracting systems) we find that the Faraday rotation has a zero, at a frequency intermediate between the cyclotron frequencies of the individual systems.

The study of electron inversion layers on Si is of much current interest, both from a technological point of view and also because it provides an ideal "laboratory" for the study of many-body effects. Recent observations of cyclotron resonance have given rise to a dilemma, which is discussed at length by Appel and Overhauser.[1] In order to explain the observations, the latter authors proposed a model for the system viz. that it consists of two different types of degenerate electrons, characterized by their respective masses m_1 and m_2 and relaxation times τ_1 and τ_2.

Such a model is also of interest from the point of view of other systems, as for example electron-hole scattering.[2] Thus, we have been motivated to carry out a detailed investigation of the magneto-optical properties of such a system. Here we point out some striking results that arise in the case of Faraday rotation. A more detailed analysis will be given elsewhere.[3]

In the case of a single system of electrons, of mass m and relaxation time τ ($=\nu^{-1}$, where ν is the collision frequency), the Faraday rotation θ per unit length l of plasma is given by[4]

$$\frac{d\theta}{dl}=\frac{\omega_c}{2c}\frac{\omega_p^2}{\omega^2}\frac{1}{(\mathrm{Re}\epsilon_l)^{1/2}}\left[1-\frac{\omega_c^2}{\omega^2}\right]^{-1} \quad \text{if } |\omega-\omega_c| >> \nu,\omega_p \tag{1}$$

where c is the velocity of light, $\mathrm{Re}\epsilon_l$ is the real part of the lattice permittivity, and where ω, ω_p, and ω_c refer to the photon, plasma, and cyclotron frequencies, respectively ($\omega_c = eB/mc$ and $\omega_p^2 = 4\pi ne^2/m$, where B denotes the magnitude of the magnetic field and n the electron concentration). It is clear that θ can never be zero under the conditions governing Eq. (1). However, because of the factor $[1-(\omega_c^2/\omega^2)]$, it can be positive or negative depending on the relative magnitudes of ω and ω_c. Consider now the system of two different types of electrons discussed by Appel and Overhauser. An important parameter entering into their model is the electron-electron interaction (e-e) scattering time τ_e. Taking $\tau_1=\tau_2\equiv\tau$, Appel and Overhauser have shown that for strong e-e interactions one obtains a single cyclotron resonance but that two resonance peaks are obtained in the case of $\tau/\tau_e=0$ (characteristic of noninteracting electrons).

What is the corresponding situation in the case of Faraday rotation? In the case of strong e-e interaction ($\tau/\tau_e >> 1$), it is not difficult to see that the result for θ is the same as that given by Eq. (1) with m replaced by

$$(n_1m_1+n_2m_2)/(n_1+n_2) \ ,$$

where n_1 and n_2 are the concentration of electrons 1 and 2, respectively.

In the case of weak e-e interaction ($\tau/\tau_e << 1$) we are dealing with two systems of noninteracting electrons, and it follows that

$$\theta=\theta_1+\theta_2 \ , \tag{2}$$

using an obvious notation. Denoting the respective cyclotron frequencies by ω_1 and ω_2, it follows for photon frequencies restricted to the range

$$\omega_1>\omega>\omega_2 \tag{3}$$

that θ_1 is positive and θ_2 is negative. In fact, with the use of Eqs. (1) and (2), it may be verified that θ is actually zero for a photon frequency ω given by

$$\left[\frac{\omega}{\omega_1}\right]^2=\left\{1-\left[1-\left[\frac{m_1}{m_2}\right]^2\right]\Big/\left[1+\frac{n_1}{n_2}\left[\frac{m_2}{m_1}\right]^2\right]\right\} \tag{4}$$

Selecting, as in Ref. 1, $m_1 = 0.19m$ and $m_2 = 0.42m$ where m is the free-electron mass, it follows that

$$\left[\frac{\omega}{\omega_1}\right]^2=1-\frac{0.795}{1+0.205(n_1/n_2)} \ . \tag{5}$$

Now the values of n_1 and n_2 at the Si(100) surface are determined essentially by the energy difference between the two different sets of overlapping subbands, which in turn are determined by the temperature and the uniaxial stress. However, as noted in Ref. 1, the latter dependence is not known accurately enough to precisely predict the values of n_1 and n_2. However, estimates can be made and it is clear that if we vary ω within the range given by Eq. (3) that a zero in θ can be found provided we are in the weak-coupling regime. The advantages of making null Faraday rotation measurements are delineated elsewhere.[5] In Fig. 1 we present the results of an *exact* numerical calculation, where we have chosen parameters for which the above analytic treatment holds. The existence of the zero between ω_1 and ω_2 is striking and in agreement with the analytic result given in Eq. (4). The other zeros in the vicinity of ω_1 and ω_2 are also expected.[5]

The investigations of Ref. 1 indicate, at least for the experiments presently of interest, that the appropriate regime is either of strong or intermediate coupling. Thus, we are motivated to investigate the dependence of θ on τ_e and, in particular, as we go from $\tau_e = \infty$ to smaller values at what value does the zero in θ disappear. This should enable us to establish the strength of the coupling. Work along these lines is presently in progress.

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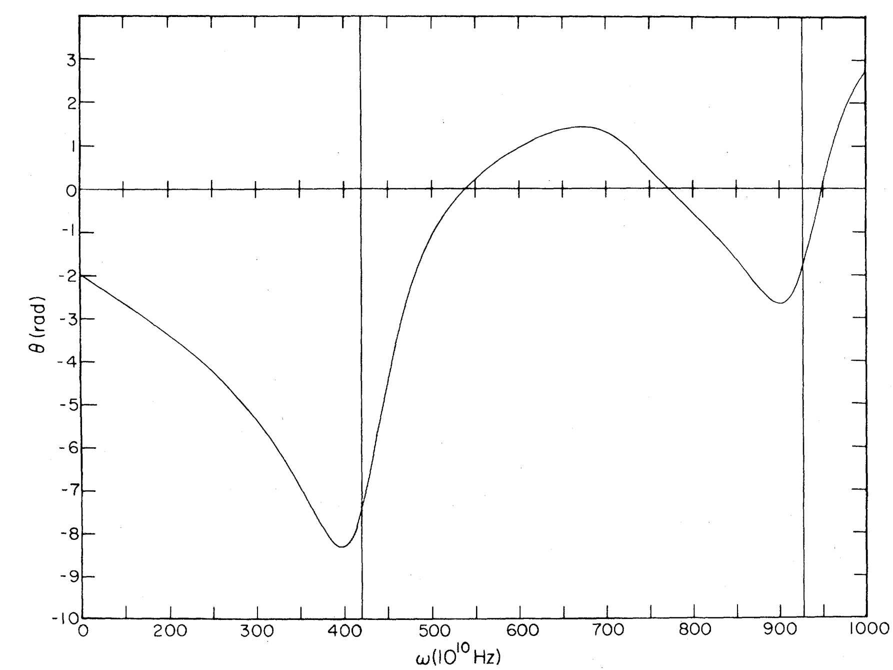

FIG. 1. Plot of the Faraday rotation θ versus angular photon frequency ω for the Appel-Overhauser model, using parameters $m_1 = 0.19m$, $m_2 = 0.42m$, $n_1 = 6.7\times10^{15}\text{cm}^{-3}$, $n_2 = 5.3\times10^{16}\text{cm}^{-3}$, $\tau_1=\tau_2 = 2.3\times10^{-12}$ s, $\tau_2 = 10^{40}$ s, $l = 10^{-2}$ cm, $B = 10^5$G and $\epsilon_l = 11.8$. The vertical lines indicate the corresponding values of $\omega_1 = 9.26\times10^{12}\ \text{s}^{-1}$ and $\omega_2 = 4.20\times10^{12}\ \text{s}^{-1}$.

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