

## INTRABAND AND INTERBAND NULL FARADAY ROTATION

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In an earlier paper we have pointed out the advantage of null Faraday rotation measurements in the case of intraband phenomena, using a classical model for the free electron plasma. Further results are given here and then we present an extension to interband transitions by considering a model of classically bound electrons. Particular emphasis is placed on the application to MOS systems.

### 1. Introduction

The surface charge layer of a MOS system has been extensively studied in recent years. In particular, it is well-known that an electric field at a semiconductor–insulator interface quantizes the electron motion normal to the interface, with the result that the electronic states form two-dimensional energy bands called electric subbands. As in many other types of materials, there are two parameters of basic theoretical interest, viz. the effective mass  $m^*$  and the collision frequency  $\nu$  ( $\nu = \tau^{-1}$  where  $\tau$  is the relaxation time). It has been found that  $m^*$  is a function of many parameters such as carrier frequency [1]. In addition, it has been calculated that  $\nu$  is not only temperature dependent but also frequency dependent [2]. Thus, it is desirable to obtain as much information as possible about  $m^*$  and  $\tau$  using, if possible, a variety of techniques. One such method is the use of Faraday rotation for electromagnetic wave propagation in a magnetoplasma. However, for the most part, this method has been used mainly to determine  $m^*$  since the determination of  $\nu$  is much more cumbersome. Even the analysis in the case of  $m^*$  is complicated by such factors as (a) variable plasma density along the length of the inversion and accumulation layers and (b) multiple reflection (which can be obviated to some degree with the use of wedges but with a subsequent loss in sensitivity). Our purpose here is to point out that a *clean* determination, not only of  $m^*$ , but also of  $\nu$ , can be achieved by using Faraday rotation techniques in a novel way. What we propose is that the external parameters be adjusted to achieve a *null* Faraday rotation. The theoretical condition necessary to achieve a null rotation is simply a relation between the basic frequencies. In the case of a quasi-free electron gas the frequencies in question are collision, photon, cyclotron, and plasma frequencies, denoted by  $\nu$ ,  $\omega$ ,  $\omega_c$ , and  $\omega_p$ , respectively. In the case of classically bound electrons we also have  $\omega_0$ , the natural frequency of classically bound electrons.

We consider the propagation of a linearly polarized wave, which may be decomposed into right and left circularly polarized components. The corresponding dielectric constants  $\epsilon_{\pm}$  are written as follows:

$$\epsilon_{\pm} \equiv \epsilon'_{\pm} + i\epsilon''_{\pm}. \quad (1)$$

It may then be shown that the condition for null Faraday rotation is [3]

$$\{(\epsilon'_+)^2 - (\epsilon''_-)^2\}^2 = 4(\epsilon'_+ - \epsilon'_-)\{\epsilon'_+(\epsilon''_-)^2 - \epsilon'_-(\epsilon''_+)^2\}. \quad (2)$$

We will now consider two different specific choices for  $\epsilon$ .

## 2. Quasi-free electron gas

This has been studied extensively in the case of semiconductors [4,5] and is also applicable [6, 7], with suitable modifications, to the two-dimensional plasma forming the surface charge layer of a MOS system (although there is a possibility that two interacting charge systems, with different effective masses, might have to be considered [8]). Specifically,

$$\varepsilon_{\pm} = \varepsilon_l \left\{ 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c + i\nu)} \right\}, \quad (3)$$

where  $\varepsilon_l$ , which is the dielectric constant of the lattice, is real, homogeneous, and isotropic. In addition,

$$\omega_p^2 = 4\pi n e^2 / m^* \varepsilon_l, \quad (4)$$

where  $m^*$  is the effective mass and we take [6, 8, 9]  $n = N^{3/2}$  where  $N$  is the surface concentration.

When eq. (3) is substituted into eq. (2) we obtain [3], after much algebra, the condition for null Faraday rotation in the form of a quintic equation for  $x \equiv (\omega/\Omega)^2$ :

$$f(x) \equiv 4x^5 + 8 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] x^4 - 8 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] x^2 - 4x - \left( \frac{\omega_p}{\Omega} \right)^2 \left\{ 3x^4 + 8 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] x^3 + 6x^2 - 1 \right\} = 0, \quad (5)$$

where

$$\Omega \equiv (\omega_c^2 + \nu^2)^{1/2}. \quad (6)$$

We have carried out a detailed analysis of this equation, with the conclusion that it has two positive roots but that only the larger of the two roots gives  $\theta = 0$ , where  $\theta$  is the Faraday rotation angle. In addition, we find that:

- (i) the desired root always occurs for  $\omega > \Omega$ ;
- (ii)  $\theta = 0$  if

$$\omega = \Omega(1 + \omega_p^2/4\Omega^2), \quad \omega_p \ll 2\Omega; \quad (7)$$

- (iii)  $\theta = 0$  if

$$\omega = 0.866\omega_p \gg \Omega. \quad (8)$$

For arbitrary values of  $\omega_p$  we simply use graphical means to obtain the value of  $\omega/\Omega$  for which  $f = 0$ . In fig. 1 we present typical plots of  $f(x)$  versus  $x$ . In table I we present a list of values of  $\omega/\Omega$ , for which null Faraday rotation is achieved, for various values of  $\omega_p$ ,  $\nu$ , and  $\omega_c$ . It is apparent that many useful investigations can thus be carried out. For example, one could determine how  $\nu$  depends on the temperature or gate voltage by simply finding where the zero of  $f$  occurs for different values of these quantities and then making use of eq. (6) to obtain  $\nu$ .

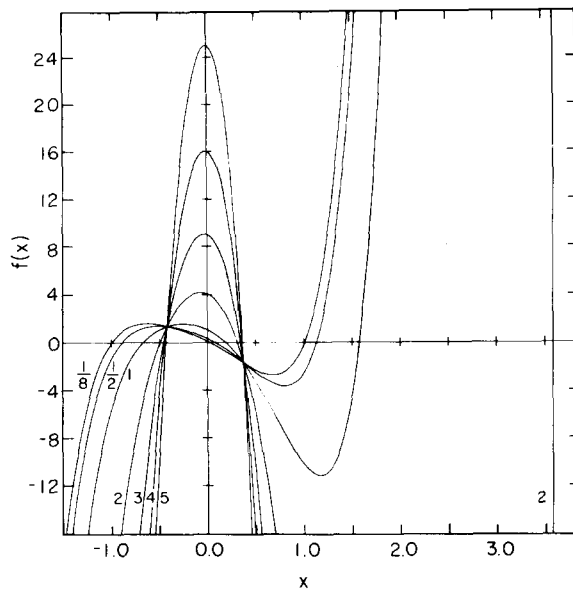


Fig. 1. Plot of  $f(x)$  versus  $x$  for  $\omega_c/\Omega = 3/5$ ,  $\nu/\Omega = 4/5$  and for  $\omega_p/\Omega$  values as indicated on the curves.

Table I  
Values of  $\omega/\Omega$  for which null Faraday rotation is obtained, for various values of  $\omega_c$ ,  $\nu$  and  $\omega_p$

$\omega_c/\Omega$	$\nu/\Omega$	$\omega_p/\Omega$	$\omega/\Omega$	$\omega_c/\Omega$	$\nu/\Omega$	$\omega_p/\Omega$	$\omega/\Omega$
3/5	4/5	$10^{-2}$	1.000025	$2/\sqrt{5}$	$1/\sqrt{5}$	$10^{-2}$	1.000025
		$10^{-1}$	1.002503			$10^{-1}$	1.002503
		1	1.257436			1	1.235972
		10	8.672438			10	8.638203
		$10^2$	86.60362			$10^2$	86.60023
4/5	3/5	$10^{-2}$	1.000025	$1/\sqrt{2}$	$1/\sqrt{2}$	$10^{-2}$	1.000025
		$10^{-1}$	1.002503			$10^{-1}$	1.002503
		1	1.248634			1	1.254067
		10	8.650919			10	8.661792
		$10^2$	86.60146			$10^2$	86.60254
$1/\sqrt{5}$	$2/\sqrt{5}$	$10^{-2}$	1.000025			$10^{-2}$	1.000025
		$10^{-1}$	1.002503			$10^{-1}$	1.002503
		1	1.260005			1	1.260005
		10	8.684338			10	8.684338
		$10^2$	86.60485			$10^2$	86.60485

### 3. Classically bound electrons [5]

The appropriate starting-point is

$$\epsilon_{\pm} = \epsilon_l \left\{ 1 - \frac{\omega_p^2}{(\omega \pm \omega_c + i\nu) - \omega_0^2} \right\}. \tag{9}$$

Using the same techniques as above, we obtain the condition for null Faraday rotation in the form of a ninth order equation for  $x \equiv (\omega^2 - \omega_0^2)/\Omega^2$ :

$$\begin{aligned}
g(x) = & 4x^9 + \left\{ 8 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] - 3 \left( \frac{\omega_p}{\Omega} \right)^2 \right\} x^8 - 8 \left[ \left( \frac{\omega_p}{\Omega} \right)^2 - \left( \frac{\omega_0}{\Omega} \right)^2 \right] \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] x^7 \\
& - 2 \left\{ 4 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] + 3 \left( \frac{\omega_p}{\Omega} \right)^2 + 4 \left( \frac{\omega_0}{\Omega} \right)^2 \left( \frac{\omega_p}{\Omega} \right)^2 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] \right\} x^6 \\
& - 4 \left\{ 1 + 3 \left( \frac{\omega_0}{\Omega} \right)^2 \left[ 2 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] + \left( \frac{\omega_p}{\Omega} \right)^2 \right] \right\} x^5 \\
& + \left\{ \left( \frac{\omega_p}{\Omega} \right)^2 - 2 \left( \frac{\omega_0}{\Omega} \right)^2 \left[ 3 \left( \frac{\omega_0}{\Omega} \right)^2 \left( \frac{\omega_p}{\Omega} \right)^2 + 12 \left( \frac{\omega_0}{\Omega} \right)^2 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] + 8 \right] \right\} x^4 \\
& + 4 \left( \frac{\omega_0}{\Omega} \right)^2 \left\{ \left( \frac{\omega_p}{\Omega} \right)^2 - 6 \left( \frac{\omega_0}{\Omega} \right)^2 - 2 \left( \frac{\omega_0}{\Omega} \right)^4 \left[ 2 \left( \frac{\nu}{\Omega} \right)^2 - 1 \right] \right\} x^3 \\
& - 2 \left( \frac{\omega_0}{\Omega} \right)^4 \left[ 8 \left( \frac{\omega_0}{\Omega} \right)^2 - 3 \left( \frac{\omega_p}{\Omega} \right)^2 \right] x^2 - 4 \left( \frac{\omega_0}{\Omega} \right)^6 \left[ \left( \frac{\omega_0}{\Omega} \right)^2 - \left( \frac{\omega_p}{\Omega} \right)^2 \right] x + \left( \frac{\omega_0}{\Omega} \right)^8 \left( \frac{\omega_p}{\Omega} \right)^2 \\
= & 0.
\end{aligned} \tag{10}$$

By inspection of eq. (10) we have

$$g(\pm\infty) = \pm\infty, \tag{11}$$

$$g(0) = \left( \frac{\omega_0}{\Omega} \right)^8 \left( \frac{\omega_p}{\Omega} \right)^2 > 0, \tag{12}$$

$$g\left(-\left(\frac{\omega_0}{\Omega}\right)^2\right) = -\left(\frac{\omega_0}{\Omega}\right)^{16} \left[ 4 \left( \frac{\omega_0}{\Omega} \right)^2 + 3 \left( \frac{\omega_p}{\Omega} \right)^2 \right] < 0. \tag{13}$$

Eqs. (12) and (13) prove the existence of a root  $x_1$  (or  $\omega_1$ ) of eq. (10) such that

$$-\left(\frac{\omega_0}{\Omega}\right)^2 < x_1 < 0 \quad (\text{or } 0 < \omega_1 < \omega_0). \tag{14}$$

Evaluation of the Faraday rotation at  $\omega = \omega_1$  gives  $\theta = 0$ .

Alternatively, eq. (9) may be written as

$$\varepsilon_{\pm} = \varepsilon_i \left\{ 1 - \frac{\tilde{\omega}_p^2}{\tilde{\omega}(\tilde{\omega} \pm \omega_c + i\nu)} \right\}, \tag{15}$$

where

$$\tilde{\omega} \equiv (1 - \omega_0^2/\omega^2)\omega, \tag{16}$$

and

$$\tilde{\omega}_p^2 \equiv (1 - \omega_0^2/\omega^2)\omega_p^2. \tag{17}$$

Note that eq. (15) is identical to eq. (3) with  $\omega$  and  $\omega_p^2$  replaced by  $\tilde{\omega}$  and  $\tilde{\omega}_p^2$ , respectively.

Thus we may write eq. (10) in the equivalent form

$$\begin{aligned} \tilde{f}(\tilde{x}) \equiv & 4\tilde{x}^5 + \left\{ 8\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right] - 3\left(\frac{\tilde{\omega}_p}{\Omega}\right)^2 \right\} \tilde{x}^4 - 8\left(\frac{\tilde{\omega}_p}{\Omega}\right)^2 \left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right] \tilde{x}^3 \\ & - 2\left\{ 4\left[2\left(\frac{\nu}{\Omega}\right)^2 - 1\right] + 3\left(\frac{\tilde{\omega}_p}{\Omega}\right)^2 \right\} \tilde{x}^2 - 4\tilde{x} + \left(\frac{\tilde{\omega}_p}{\Omega}\right)^2 = 0, \end{aligned} \quad (18)$$

where  $\tilde{x} \equiv (\tilde{x}/\Omega)^2$ . The solutions of eq. (18), or equivalently eq. (10), may be obtained using the solutions of eq. (5). Exact solutions may be obtained graphically, as already indicated. Approximate analytic results may be obtained, using eqs. (7) and (8), as we will now demonstrate.

Recall that in eq. (5) the independent variable is  $x \equiv (\omega/\Omega)^2$ . Neglecting the  $\omega_p$  terms the solution of eq. (5) is  $(\omega/\Omega)^2 = \pm 1$ , of which the negative root is rejected as unphysical, leaving  $\omega/\Omega = \pm 1$ . The negative root is rejected as unphysical, but, as will be seen, this is no longer so when  $\omega_0 \neq 0$  in which case the negative root leads to a physical solution.

By analogy with eq. (7), we see that the lowest order solution of eq. (18) may be written as

$$\frac{\tilde{\omega}}{\Omega} = \pm 1, \quad \left| \left(\frac{\tilde{\omega}_p}{\Omega}\right)^2 \right| \ll 1. \quad (19)$$

This equation gives two quadratic equations for  $\omega/\Omega$  and thus four solutions, of which two must be rejected as unphysical, since they give a negative  $\omega$ . The remaining two roots for positive  $\omega$  may be written as

$$\frac{\omega}{\Omega} = \frac{1}{2} \left\{ \left[ 1 + 4\left(\frac{\omega_0}{\Omega}\right)^2 \right]^{1/2} \pm 1 \right\}, \quad (20)$$

where  $|\omega^2 - \omega_0^2| \omega_p^2 \ll 4\omega^2 \Omega^2$  or  $(\omega_p^2 + 2\Omega^2)^2 \ll 4\Omega^2(\Omega^2 + 4\omega_0^2)$ , upon substitution for  $\omega$  from eq. (20). Note that eq. (20) is also valid in the region  $\omega_0/\Omega \gg 1$  and reduces to

$$\omega = \omega_0 \pm \frac{1}{2}\Omega, \quad \omega_0 \gg \Omega. \quad (21)$$

The solution of eq. (10), or eq. (18), corresponding to eq. (8) may be written as

$$\frac{\tilde{\omega}}{\Omega} = \frac{\sqrt{3}}{2} \frac{\tilde{\omega}_p}{\Omega}, \quad \left| \left(\frac{\tilde{\omega}_p}{\Omega}\right)^2 \right| \gg 1. \quad (22)$$

Hence

$$\omega = \left\{ \omega_0^2 + \frac{3}{4} \omega_p^2 \right\}^{1/2} \quad \text{if} \quad \frac{3}{4} \omega_p^2 \left( \frac{3}{4} \omega_p^2 - \Omega^2 \right) \gg \omega_0^2 \Omega^2. \quad (23)$$

Table II is a list of values of  $\omega/\Omega$  for which null Faraday rotation is achieved for various values of  $\omega_c$ ,  $\nu$ ,  $\omega_p$  and  $\omega_0$ .

In summary, we have shown that the introduction of the natural frequency of classically bound electrons has the effect of admitting other solutions for the photon frequency for which null Faraday rotation is achieved.

Table II  
Values of  $\omega/\Omega$  for which null Faraday rotation is obtained,  
for various values of  $\omega_c$ ,  $\nu$ ,  $\omega_p$  and  $\omega_0$

$\omega_c/\Omega$	$\nu/\Omega$	$\omega_p/\Omega$	$\omega_0/\Omega$	$\omega/\Omega$	
3/5	4/5	10	2	1.71554	8.90077
		2	10	9.55488	10.56943
4/5	3/5	10	2	1.69886	8.87869
		2	10	9.55445	10.56879

Finally, we turn to a discussion of the potential usefulness of our results, particularly with respect to Faraday rotation measurements on MOS systems, presently being carried out by our colleague, Dr. H. Piller, along with various collaborators at the NRL. In contrast to the usual type of measurements on such systems as semiconductors, here we are dealing with a charged plasma with a varying density. Thus, we have the complication of a varying plasma frequency across the length of the plasma. At best, the density profile through the plasma must be calculated theoretically [10]. Hence the calculation of a *nonzero* value for the Faraday rotation  $\theta$  is more involved and uncertain. However, if we select our parameters so that eq. (7) holds we see that the condition for *null* Faraday rotation is only *weakly dependent on*  $\omega_p$ . Furthermore, we have verified numerically that, for  $\omega_p$  values close to the value for which null Faraday rotation is obtained ( $\omega_p^{(0)}$  say),  $\theta$  is very small and proportional to  $\omega_p^{(0)} - \omega_p$ . Thus, the optimum accuracy is achieved by selecting  $\omega_p^{(0)}$  to be a suitably weighted average of the plasma frequencies and also by taking  $\omega_p^{(0)}/\Omega$  to be as small as possible. Further details will be given in the thesis of one of us (GLW) [11]. Also, we note that null measurements should provide greater sensitivity.

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