

INTERSUBBAND-CYCLOTRON COMBINED RESONANCE IN A SURFACE SPACE-CHARGE LAYER*

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Received 21 May 1980

In a magnetic field tilted with respect to the surface of Si, combined resonance transitions have been observed, resulting from a coupling of Landau levels and subband states. Because of some unexplained features in the observations, we analyze what one should expect on theoretical grounds. We conclude that unexplained discrepancies between theory and experiment still exist, which may indicate that collective effects are playing a role.

A strong electric field, ϵ , applied normal to a semiconductor surface gives rise to quantized electron motion in this direction, with the result that two-dimensional electric subbands are formed, with energies E_n . If $\epsilon = \epsilon_z$, and if a magnetic field H is also applied in the same direction then each subband is further quantized into discrete Landau levels so that the energy becomes $E_{n,N} = E_n + (N + 1/2)\hbar\omega_c$, where $\omega_c = (eH_z/cm_{\parallel})$ is the cyclotron frequency, and where $m_{\parallel} = 0.1905m_0$ is the effective mass in the direction parallel to the surface, m_0 denoting the free electron mass.

In a recent experiment, Beinvoogl and Koch [1] investigated electrons on Si(1,0,0), in the presence of a magnetic field tilted with respect to the sample surface (H_y and H_z components), and observed combined resonance transitions because of a coupling of Landau levels and subband states. Surprisingly, they found that the sum of the separations for the $\Delta N = 1$ and $\Delta N = -1$ transitions is $\{0.7 - 1.6\} \hbar\omega_c = 2.3\hbar\omega_c$ i.e. 15% higher than the expected result of $2\hbar\omega_c$.

Prior theoretical work on this problem by Ando [2] reached the conclusion that

$$E_{n,N} = E_n + \Delta E_n + E_N, \quad (1)$$

where

$$E_N = (N + \frac{1}{2})\hbar\omega_c, \quad (2)$$

$$\Delta E_n = \frac{e^2}{2m_{\parallel}c^2} H_y^2 [(z^2)_{nn} - (z_{nn})^2]. \quad (3)$$

Thus we get the characteristic energy changes of $(\hbar\omega_c)\Delta N$ corresponding to ΔN transitions. As shown by Ando [2] this conclusion is not affected by the inclusion of many-body and other effects. The latter affects the difference in the positions of the main ($\Delta N = 0$) and a combined ($\Delta N \neq 0$) resonance peak but does not affect the difference in the positions of two combined resonance peaks. A basic assumption made by Ando was to treat H_y as a perturbation so that its influence on the z -part of the wave function is neglected. Recently Ando [3] carried out a more detailed investigation without this restriction but it is clear (see fig. 9 of ref. 3) that discrepancies between experiment and theory still exist.

It is our purpose here to return to the original perturbation analysis to investigate more precisely the extent of its validity. As we shall see our conclusion is that it should be very good as far as an analysis of the Beinvoogl–Koch observations are concerned. To this end we will calculate ΔE_n explicitly by using the simplest realistic model for the z -potential, $V(z)$.

* This research was partially supported by the Department of Energy under contract no. DE-AS05-79ER10459.

Following Stern [4] we use the triangular-potential approximation i.e. $V(z) = eez$ for $z > 0$, with an infinite barrier for $z < 0$. The corresponding wavefunction is an Airy function, from which it readily follows that [4]

$$E_n \approx (\hbar^2/2m_{\perp})^{1/3} \left[\frac{3}{2} \pi e \epsilon \left(n + \frac{3}{4} \right) \right]^{2/3}, \quad (4)$$

and

$$z_{nn} = 2E_n/3e\epsilon; \quad \langle z^2 \rangle_{nn} = \frac{6}{5} (z_{nn})^2, \quad (5)$$

and where $m_{\perp} = 0.916m_0$ is the effective mass in the z -direction. Hence, from eqs. (3) and (5), we obtain

$$\Delta E_n = \frac{2}{45} \frac{e^2 H_y^2}{m_{\parallel} c^2} \left(\frac{E_n}{e\epsilon} \right)^2, \quad (6)$$

and thus

$$E_n + \Delta E_n = E_n \left\{ 1 + \frac{2}{45} \left(\frac{H_y}{\epsilon} \right)^2 \frac{E_n}{m_{\parallel} c^2} \right\}. \quad (7)$$

The magnitude of the H_y^2 term inside the braces, compared to unity, will be a measure of goodness of the perturbation approach. If it is $\ll 1$ then the perturbation analysis should be very good. Since $E_n \sim \epsilon^{2/3}$ we see that this H_y^2 term $\sim \epsilon^{-4/3}$ and thus it increases with increasing n and decreasing ϵ .

Now the observations were carried out in a sweep of the gate voltage for fixed infra-red energies $\hbar\omega = 10.45$ and 15.81 meV. The values selected for H_z were 5 and 3.5 T, so that the corresponding values of $\hbar\omega_c$ are 3.0 and 2.1 meV, respectively. The values of H_y ranged from 0 to 6 T. Typical values for E_n and ϵ (chosen to maximize the H_y^2 term inside the braces) are 10 meV and 10^5 V/cm (3.3×10^2 V/cm (stat.)), respectively. Also, $m_{\parallel} c^2 = 9.7 \times 10^4$ eV. Thus, choosing the maximum value of H_y used in the observations, $(H_y/\epsilon)^2 \approx (6 \times 10^4/3.3 \times 10^2)^2 \approx 3.3 \times 10^4$ and $(E_n/m_{\parallel} c^2) \approx 10^{-7}$. As a result, we conclude that ΔE is typically $\approx 10^{-4}$ times smaller than E_n and thus negligible. It is also less than 10^{-3} times the $\hbar\omega_c$ term. In other words, we are led to the basic conclusion that

ΔN transitions should give the familiar $(\hbar\omega_c) \Delta N$ energy changes.

Since the values we have used for H_y and H_z are comparable the question remains as to why H_z makes the dominant contribution to the energy. The reason is that H_z makes the dominant contribution to the energy in the x - y plane. On the other hand, H_y affects the motion in the z and x directions. In the z direction its effect is overwhelmed by the electric field effects. With regard to its effect on the x motion we note that the basic Hamiltonian contains a $P_x H_y$ term, (in addition to a H_y^2 term) where P_x is the momentum in the x -direction. However, this term does not contribute a linear H_y contribution to the energy (unlike the H_z term, whose contribution to the energy is linear in H_z) for the simple reason that P_x averages to zero in lowest order (i.e. absence of H_y). Thus the contribution of $P_x H_y$ to the energy is of order H_y^2 and thus higher order than might have been first surmised.

As already pointed out by many authors, the choice of a more realistic $V(z)$, to include polarization, excitonic and many-body effects, has important consequences. In particular the latter effects give rise to a shift in the energy of the pure subband resonance ($\Delta N = 0$). However, as mentioned above, such effects were shown to be irrelevant [2] to the discussion of the difference in the positions of two combined resonance peaks, which is the main theme of this communication. Our choice of $V(z)$ is also the dominant contribution to the real potential.

We conclude that perturbation theory should give very good results. This conclusion is also implicit in the work of others but no explicit calculations have been presented—as we have done above—showing the extent of the validity of the results obtained from perturbation theory.

We conclude that a definite discrepancy still exists between theoretical expectations and the observations of Beinvoogl and Koch. This could be due to a missing ingredient in the theoretical analysis or else a misinterpretation of the observations. One possibility, for example, is that collective efforts are playing a role. There has been very recent evidence [5] for the existence of a highly correlated or crystallized ground state—a *Wigner lattice* [6]—in Si inversion layers in the extreme quantum limit. This evidence came from infrared measurements of the cyclotron resonance in the two-dimensional electron gas, which revealed a

remarkable line narrowing and shift in the resonance frequency to higher values. As emphasized by Wilson, et al. [5], “. . . the strong line narrowing is not a feature of the one-electron theories of Ando . . .,” and of course the one-electron theories are the basis of all the existing theoretical investigations of transition energies. However, before jumping to what might be a premature conclusion it is important to ascertain precisely the various range of temperature, density, and magnetic field parameters for which the formation of a Wigner lattice is likely. Qualitatively speaking, the formation of a lattice is facilitated by having low inversion-layer electron concentrations n_s , low temperatures T , and high magnetic fields B . In the experiments of Wilson et al. [5], typical values used were $n_s \approx 10^{11}/\text{cm}^2$, $T = 1.2$ K, and B values of 6.15 T and 7.69 T. The values of n_s and B are not very different than those used by Beinvogl and Koch [1], but the latter authors used $T = 4.2$ K. Now Wilson et al. [5] point out that the cyclotron resonance broadens and

shifts to lower frequency (i.e. the Wigner lattice starts to disappear) across the temperature range of 5–20 K and that the temperature dependence is apparently independent of the value of electron density, electric field, or magnetic field. The T value of 4.2 K used by Beinvogl and Koch is thus seen to be on the borderline and perhaps the system itself is borderline between the extremes of an electron gas and a Wigner lattice. This question is presently under study.

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