

A TRINARY MODEL FOR SS 433

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ABSTRACT

We present a trinary model for SS 433 that consists of the Martin and Rees close central binary ($5 m_{\odot}$ black hole and $0.4 m_{\odot}$ degenerate dwarf) and an additional star with a 13 day orbital period. We generalize the Martin-Rees model by including quadrupole, eccentricity, and two-body effects. We then analyze the effect of the central binary on the 13 day star and vice versa. We predict that the total angular momentum axis of the Martin-Rees binary will not remain at 78° to the line of sight but will precess about the total angular momentum axis of the trinary with a period of the order of 300 years.

Subject headings: black holes — stars: binaries — stars: emission-line — stars: individual

I. INTRODUCTION

The object SS 433 has excited much interest because of its peculiar optical properties. Its spectrum shows stationary Balmer and He I emission lines each with two strong satellite lines which move cyclically from zero to large redshifts and blueshifts over a period of 164 days. In addition to its spectral peculiarities, SS 433 is also a radio and X-ray source centrally located in the presumed supernova remnant W50. The peculiar spectral features of the star SS 433 have been explained as being due to two oppositely directed gas jets (with speeds of $0.27c$) which are precessing with a 164 day period about an axis 78° to the line of sight. The jets themselves are at an angle varying from 17° to 22° from this axis (Fabian and Rees 1979; Abell and Margon 1979; see review by Margon 1979).

Martin and Rees (1979) have proposed that SS 433 is a spinning black hole of about $5 m_{\odot}$ accreting matter from a degenerate dwarf of about $0.4 m_{\odot}$. Because the black hole's spin angular momentum vector is tilted with respect to the black hole-dwarf orbital angular momentum vector, each vector precesses due to general relativistic effects. The accreted material is assumed to be ejected in two jets along the black hole's precessing spin axis. This model thus explains the major features of SS 433's spectrum.

Crampton, Cowley, and Hutchings (1980) have discovered that the "stationary" spectral emission lines show a 13 day periodic wavelength variation with an amplitude equivalent to a 73 km s^{-1} velocity variation. One interpretation of the 13 day variation is that SS 433 is a triple star with the Martin and Rees pair as a *close* central binary orbited at a *much larger* distance by another "13 day" star which generates the "stationary" spectrum. A triple SS 433 is quite probable. The statistics of multiple stars indicate that 25% to 33% of stars which are binary are also members of triple star systems (Batten 1973, p. 61). The alternate interpretation of only one central object and the "13 day" star has the problem that the star must be a giant to fill its Roche lobe and feed material to the central object. Since this pair's total angular momentum vector would have to be 78° from the line of sight, a deep photometric eclipse should be but is *not* observed (Margon 1979). No such problem exists if the Martin and Rees (1979) close binary model is assumed.

Although many other models have been proposed for SS 433 (e.g., De Young and Burbidge 1979), this article will be limited to further development and generalization of the dynamics of the Martin-Rees model. Our analysis will be based on the spin and orbital precession results obtained for the two-body problem in general relativity by Barker and O'Connell (1975*a, b*, 1978). We will also evaluate the possible dynamical consequences of the Martin-Rees model being part of a triple star system, considering the effect of the 13 day star on the close binary and vice versa.

II. THE BINARY SYSTEM

For the black hole member of the Martin-Rees binary, let m_2 , $S^{(2)}$, $n^{(2)}$, and $\Delta I^{(2)}$ denote the mass, spin, unit vector in the $S^{(2)}$ direction, and moment of inertia about polar axis minus moment of inertia about equatorial axis. Let us use a similar notation with 2 replaced by 1 for the degenerate dwarf companion. Also let A , L , n , $\bar{\omega}$, P , a , and e be the Runge-Lenz vector, orbital angular momentum, unit vector in the L direction, average orbital angular velocity, orbital period, semimajor axis, and eccentricity of the binary's orbit. We shall also set $M = m_1 + m_2$ and $\mu = m_1 m_2 / M$.

Martin and Rees (1979) assume (i) the large-mass approximation ($m_2 \gg m_1$), (ii) circular orbits only ($e = 0$), (iii) $L \gg S^{(2)} \gg S^{(1)}$ ($S^{(1)}$ can be neglected). We shall generalize the Martin and Rees model by eliminating the unnecessary

restriction (i), and partially eliminating the restriction (ii). We shall include e in all results except the Roche lobe equivalent radius and the numerical final results (since e is not known). We shall retain (iii) and, thus, neglect $S^{(1)}$. The results and implications when (iii) is relaxed (i.e., $S^{(1)}$ is *not* neglected) will be presented in another publication.

The two-body secular results (Barker and O'Connell 1975*a, b*, 1978) for the precession of the spin (of body 2) and the precession of the orbit are given, respectively, by (a dot denotes d/dt and av denotes average over an orbital period)

$$\dot{S}_{av}^{(2)} = \Omega_{av}^{(2)} \times S^{(2)}, \quad (1)$$

$$\Omega_{av}^{(2)} = \Omega_{DS\ av}^{(2)} + \Omega_{Q2\ av}^{(2)}, \quad (2)$$

where $\Omega_{DS\ av}^{(2)}$ is the de Sitter (geodetic) general-relativistic contribution and $\Omega_{Q2\ av}^{(2)}$ is the contribution due to the quadrupole moment of body 2 (acted on by the tidal field of body 1), and

$$\dot{L}_{av} = \Omega^* \times L, \quad \dot{A}_{av} = \Omega^* \times A, \quad (3)$$

$$\Omega^* = \Omega^{*(E)} + \Omega^{*(2)} + \Omega^{*(Q2)}, \quad (4)$$

where $\Omega^{*(E)}$ is the Einstein (spin-independent) contribution, $\Omega^{*(2)}$ is the contribution due to the spin of body 2, and $\Omega^{*(Q2)}$ is the contribution due to the quadrupole moment of body 2. We also have

$$\Omega_{DS\ av}^{(2)} = A_{DS}^{(2)} n, \quad (5)$$

$$\Omega_{Q2\ av}^{(2)} = A_{Q2}^{(2)} [n^{(2)} - 3(n \cdot n^{(2)})n], \quad (6)$$

$$\Omega^{*(E)} = A^{*(E)} n, \quad (7)$$

$$\Omega^{*(2)} = A^{*(2)} [n^{(2)} - 3(n \cdot n^{(2)})n], \quad (8)$$

$$\Omega^{*(Q2)} = A^{*(Q2)} \{2(n \cdot n^{(2)})n^{(2)} + [1 - 5(n \cdot n^{(2)})^2]n\}, \quad (9)$$

with

$$A^{*(E)} = \frac{3GLM/\mu}{c^2 a^3 (1 - e^2)^{3/2}}, \quad (10)$$

$$A^{*(2)} = \left(\frac{S^{(2)}}{L}\right) A_{DS}^{(2)}, \quad A_{DS}^{(2)} = \frac{GL(4 + 3m_1/m_2)}{2c^2 a^3 (1 - e^2)^{3/2}}, \quad (11)$$

$$A^{*(Q2)} = -\frac{3}{2} \left(\frac{S^{(2)}}{L}\right) A_{Q2}^{(2)}, \quad A_{Q2}^{(2)} = \frac{Gm_1 \Delta I^{(2)}}{2S^{(2)} a^3 (1 - e^2)^{3/2}}. \quad (12)$$

The form of the above A 's can be altered by using

$$\frac{L/\mu}{a^2(1 - e^2)^{1/2}} = \left(\frac{GM}{a^3}\right)^{1/2} = \frac{2\pi}{P} = \bar{\omega}. \quad (13)$$

Since we are neglecting $S^{(1)}$, the total angular momentum is $J = L + S^{(2)}$. We shall define $n^{(J)}$ as a unit vector in the J direction. It is clear from equation (1) that $\dot{S}_{av}^{(2)}$ will be unaltered if we replace $\Omega_{av}^{(2)}$ by $\Omega_R^{(2)}$, where these two quantities differ only by a component in the $n^{(2)}$ direction. It is also clear from equation (3) that \dot{L}_{av} (but *not* \dot{A}_{av}) will be unchanged if we replace Ω^* by Ω_R^* where these two quantities differ only by a component in the n direction. It is easy to see that $\Omega_R^{(2)}$ and $\Omega_R^{*(2)}$ can be put in the form

$$\Omega_R^{(2)} = A_R n^{(J)}, \quad \Omega_R^* = A_R n^{(J)}, \quad (14)$$

where

$$A_R = (J/L)[A_{DS}^{(2)} - 3A_{Q2}^{(2)}(n \cdot n^{(2)})]. \quad (15)$$

Because both L and $S^{(2)}$ precess about J at the same rate, $(n \cdot n^{(2)})$ will not change with time and, therefore, the precession rate will not change with time.

Clearly, the period of precession, \mathcal{P} , is given by $2\pi/A_R$ and can be written as

$$\mathcal{P} = \frac{\pi c^2 a^3 (1 - e^2)^{3/2}}{GJ[1 + \frac{3}{4}k(m_1/m_2)]}, \quad (16)$$

where

$$k = 1 - m_2 c^2 \Delta I^{(2)} (n \cdot n^{(2)}) / (LS^{(2)}). \quad (17)$$

For a black hole $\Delta I^{(2)}$ can be expressed as (Hernandez 1967)

$$\Delta I_{BH}^{(2)} = (S^{(2)}/c)^2 / m_2; \quad (18)$$

thus,

$$k_{\text{BH}} = 1 - (\mathbf{n} \cdot \mathbf{n}^{(2)})S^{(2)}/L. \quad (19)$$

Using equations (13) and (16), we obtain

$$a = \left[\left(\frac{\mathcal{P}}{\pi c^2} \right)^{2/5} (Gm_{\odot})^{3/5} \left(\frac{m_1^{2/5} m_2^{1/5}}{m_{\odot}^{3/5}} \right) \left(\frac{J}{L} \right)^{2/5} \left[\frac{(1 + \frac{3}{4} k m_1/m_2)^{2/5}}{(1 + m_1/m_2)^{1/5} (1 - e^2)^{2/5}} \right] \right]. \quad (20)$$

For $e = 0$, the Roche lobe equivalent radius that the dwarf fills is (Pringle and Webbink 1975)

$$R_{L1} = 0.46 m_1^{1/3} m_2^{-1/3} (1 + m_1/m_2)^{-1/3} a. \quad (21)$$

Our equations (16), (20), and (21) are the generalizations of Martin and Rees's equations (1), (2), and (3). In order to obtain numerical results, we shall set $m_1 = 0.4 m_{\odot}$, $m_2 = 5 m_{\odot}$, $e = 0$, $(J/L) = 1$, $k = 1$, and $\mathcal{P} = 164$ days. We then obtain

$$a = 86,700 \text{ km}, \quad R_{L1} = 16,700 \text{ km}. \quad (22)$$

Using equation (13), it follows that the orbital period is

$$P = 3.16 \text{ min} = 2.19 \times 10^{-3} \text{ d}. \quad (23)$$

instead of the 4 min result given by Martin and Rees.

III. THE TRINARY SYSTEM

Let m_{t2} be the mass of the Martin-Rees close binary and m_{t1} be the mass of the much more distant (since the orbital period is much greater) 13 day star. Thus, this trinary is a "binary" and we can now use the notation and results of § II with an additional subscript t for the trinary. With this notation, $m_{t2} = M = 5.4 m_{\odot}$ and $S_t^{(2)} = J$. Since $L_t \gg S_t^{(2)} \gg S_t^{(1)}$, we shall neglect $S_t^{(1)}$ so that $J_t = L_t + S_t^{(2)}$. We also know the orbital period and eccentricity of the trinary (Crampton, Cowley, and Hutchings 1980) to be $P_t = 13.08$ days and $e_t = 0.13$. In this section we shall use $e = 0$ for the Martin-Rees close binary.

a) Precession of J and L_t about J_t

The precessional period of the trinary \mathcal{P}_t can be expressed in terms of the general relativistic and quadrupole contributions as

$$\mathcal{P}_t^{-1} = \mathcal{P}_{t\text{GR}}^{-1} + \mathcal{P}_{tQ}^{-1}, \quad (24)$$

where

$$\mathcal{P}_{t\text{GR}} = 2\pi / [(J_t/L_t) A_{tDS}^{(2)}], \quad (25)$$

$$\mathcal{P}_{tQ} = -2\pi / [3(J_t/L_t) A_{tQ2}^{(2)} (\mathbf{n}_t \cdot \mathbf{n}_t^{(2)})]. \quad (26)$$

We shall use a time averaged result for $\Delta I_t^{(2)}$ by assuming that each of the masses m_1 and m_2 is distributed uniformly over its (circular) binary orbit. We then obtain

$$\Delta I_t^{(2)} = \frac{1}{2} \mu a^2. \quad (27)$$

Using the approximations $(J_t/L_t) = 1$ and $(J/L) = 1$, we obtain (see Appendix)

$$\mathcal{P}_{t\text{GR}} = (258,000 \text{ yr}) M_t^{1/3} m_{\odot}^{2/3} (m_{t1} + \mu_t/3)^{-1}, \quad (28)$$

$$\mathcal{P}_{tQ} = -(277 \text{ yr}) M_t m_{t1}^{-1} (\mathbf{n}_t \cdot \mathbf{n}_t^{(2)})^{-1}. \quad (29)$$

Thus, for the trinary precession result, the quadrupole effect is the most important. We conclude that J will not remain fixed at 78° to the line of sight but will precess about J_t with a period of the order of 300 years.

b) Precession of the Orbital Elements

We can write Ω_t^* in the form (Barker and O'Connell 1978)

$$\Omega_t^* = \dot{\Omega}_t' n_0 + \dot{\omega}_t' n_t + \dot{i}_t' n_0 \times n_t / |n_0 \times n_t|, \quad (30)$$

where n_0 is a unit vector perpendicular to the plane of the sky and directed away from Earth (the form of all our equations remains the same if one uses the other convention of directed toward the Earth), and Ω_t' , ω_t' , and i_t' denote the longitude of the ascending node, the argument of the periastron, and the inclination of the orbit in the plane of the sky reference system. We shall put

$$\dot{\omega}_t' = \dot{\omega}_{tE}' + \dot{\omega}_{tQ2}' + \dot{\omega}_{t2}', \quad (31)$$

$$\dot{i}_t' = \dot{i}_{tE}' + \dot{i}_{tQ2}' + \dot{i}_{t2}', \quad (32)$$

where

$$\dot{\omega}_{tE}' = A_t^{*(E)}, \quad \dot{i}_{tE}' = 0, \quad (33)$$

$$\dot{\omega}_{tQ2}' = A_t^{*(Q2)} \{ 2(\mathbf{n}_t \cdot \mathbf{n}_t^{(2)}) [\mathbf{n}_t^{(2)} \cdot \mathbf{n}_t - (\mathbf{n}_t^{(2)} \cdot \mathbf{n}_0) \cos i_t'] / \sin^2 i_t' + [1 - 5(\mathbf{n}_t \cdot \mathbf{n}_t^{(2)})^2] \}, \quad (34)$$

$$\dot{i}_{tQ2}' = A_t^{*(Q2)} \{ 2(\mathbf{n}_t \cdot \mathbf{n}_t^{(2)}) [\mathbf{n}_t^{(2)} \cdot (\mathbf{n}_0 \times \mathbf{n}_t)] / \sin i_t' \}, \quad (35)$$

and (see Appendix)

$$A_t^{*(E)} = (2.79 \times 10^{-3} \text{ deg yr}^{-1}) M_t^{2/3} m_\odot^{-2/3}, \quad (36)$$

$$A_t^{*(Q2)} = -(7.61 \times 10^{-3} \text{ deg yr}^{-1}) m_\odot^{2/3} M_t^{-2/3}. \quad (37)$$

The results for $\dot{\omega}_{t2}'$ and \dot{i}_{t2}' (being proportional to $A_t^{*(2)}$) are negligible compared to the other effects. We have not given the result for $\dot{\Omega}_t'$ since the longitude of the ascending node is not observed for spectroscopic binaries. Clearly i_t' and ω_t' vary at a rate of the order of $5 \times 10^{-3} \text{ deg yr}^{-1}$. However, since some of the dot products in equations (34) and (35) are not constant, one cannot determine the long term variation in the orbital elements from equations (31)–(37) without complicated integration.

c) Long-Term Change in the Orbital Elements

We can also write $\dot{\Omega}_t^*$ in the form

$$\dot{\Omega}_t^* = \dot{\Omega}_t'' \mathbf{n}_t^{(J)} + \dot{\omega}_t'' \mathbf{n}_t + \dot{i}_t'' (\mathbf{n}_t^{(J)} \times \mathbf{n}_t) / |\mathbf{n}_t^{(J)} \times \mathbf{n}_t|, \quad (38)$$

where the double primes indicate we are using the plane perpendicular to $\mathbf{n}_t^{(J)}$ reference system (the double prime system is introduced only in order to obtain long term results for the single prime quantities i_t' , Ω_t' , and ω_t'). It is convenient to introduce the longitude of the periastron, $\varpi_t'' = \Omega_t'' + \omega_t''$, because it, unlike ω_t'' , is measured from a fixed reference point. Let us set

$$\dot{\omega}_t'' = \dot{\omega}_{tE}'' + \dot{\omega}_{tQ2}'' + \dot{\omega}_{t2}'', \quad (39)$$

$$\dot{i}_t'' = \dot{i}_{tE}'' + \dot{i}_{tQ2}'' + \dot{i}_{t2}'', \quad (40)$$

where

$$\dot{\omega}_{tE}'' = A_t^{*(E)}, \quad \dot{i}_{tE}'' = 0, \quad (41)$$

$$\dot{\omega}_{tQ2}'' \approx A_t^{*(Q2)} [1 - 3(\mathbf{n}_t \cdot \mathbf{n}_t^{(2)})^2], \quad \dot{i}_{tQ2}'' = 0, \quad (42)$$

and $\dot{\omega}_{t2}''$ and \dot{i}_{t2}'' are negligible. In adding $\dot{\Omega}_{tQ2}''$ to $\dot{\omega}_{tQ2}''$, we made the approximation that $(J_t - L_t)/S_t^{(2)} \approx \mathbf{n}_t \cdot \mathbf{n}_t^{(2)}$. Since i_t'' is the angle between L_t and J_t , we have (see Appendix)

$$\begin{aligned} i_t'' &\approx (S_t^{(2)}/L_t) [1 - (\mathbf{n}_t \cdot \mathbf{n}_t^{(2)})^2]^{1/2} \\ &= (0.673 \text{ deg}) (M_t^{1/3} m_\odot^{2/3} m_{t1}^{-1}) [1 - (\mathbf{n}_t \cdot \mathbf{n}_t^{(2)})^2]^{1/2}, \end{aligned} \quad (43)$$

and, also when L_t goes once around J_t , we have

$$i_{t \max}'' - i_{t \min}'' = 2i_t'', \quad (44)$$

$$\Omega_{t \max}'' - \Omega_{t \min}'' \approx 2i_t'' [1 - (\mathbf{n}_0 \cdot \mathbf{n}_t^{(J)})^2]^{-1/2}. \quad (45)$$

The angle that the projection of L_t on the plane of the sky sweeps out (as L_t goes around J_t) is approximately $2i_t'' [1 - (\mathbf{n}_0 \cdot \mathbf{n}_t^{(J)})^2]^{-1/2}$, and because the line of nodes is perpendicular to this projection of L_t , equation (45) follows. Over a long period of time

$$\Delta\omega_t' \approx \Delta\varpi_t'' \quad (46)$$

with a maximum error of $\Omega_{t \max}'' - \Omega_{t \min}''$ which is of order of about 1° . Since $\dot{\omega}_t''$ does not vary with time, we have $\Delta\varpi_t'' = \dot{\omega}_t'' \Delta t$, where Δt does not have to be small. Thus, over a long period of time it is easier to obtain $\Delta\omega_t'$ from equation (46) than from integration of equation (31). Using equation (46), we find that ω_t' will change by 360° over a period of

$$\mathcal{P}_{t \text{ periastron}} = 2\pi/\dot{\omega}_t'' \quad (47)$$

which is of the order of 70,000 years (see eqs. [39], [41], [42], [36], and [37]).

General expressions for the numerical factors in equations (28), (29), (36), (37), and (43) are given in the Appendix in case it is necessary to recalculate the factors for revised parameter values.

IV. DISCUSSION AND CONCLUSION

In § II, we have generalized the Martin-Rees model by not making the large mass approximation ($m_2 \gg m_1$) and by not assuming $e = 0$. If the masses assumed by Martin and Rees ($5 m_\odot$ for the black hole and $0.4 m_\odot$ for the degenerate dwarf) are approximately correct, the error made by using the large mass approximation is small. However, if the masses turn out to be more comparable, the error can be significant. If the orbit turns out to be eccentric rather than circular, the error can be large. Certainly orbits are circularized with time in close binaries (Batten 1973). However, it is likely that one star in the Martin-Rees binary has lost mass in a supernova explosion, leaving the resulting orbit much more eccentric than before. One cannot at present say whether the circularizing mechanisms will have had sufficient time to act to a degree to justify Martin and Rees's assumption.

In § II, we have also generalized the Martin-Rees model to include quadrupole effects. While these effects were not important for the binary, they were included so that the results would be available to apply to the trinary where they are most important.

In § III, we have analyzed the effect of the central binary on the 13 day star and vice versa. The precession of the orbital angular momentum of the trinary and the spin angular momentum of the binary about the total angular momentum of the trinary is given in § IIIa. Although not all the parameters of the trinary are known, the quadrupole effect (eq. [29]) is clearly much greater, with a shorter period, than the general relativistic effect (eq. [28]). The spin angular momentum axis of the Martin-Rees binary will thus *not* remain at 78° to the line of sight, but will precess about the total angular momentum of the trinary with a period of the order of 300 yr.

According to equations (31)–(37) both the argument of the periastron ω_t' and the inclination of the orbit i_t' for the trinary in the plane of the sky reference system vary at a rate of the order of $5 \times 10^{-3} \text{ deg yr}^{-1}$. The change in ω_t' is caused by *both* quadrupole and general-relativistic effects while the change in i_t' is caused *only* by the quadrupole effect. Over a cycle of the order of 70,000 years (see eqs. [47]) ω_t' will change by 360° . However, i_t' will vary only by roughly 1° over its cycle of the order of 300 years (see eqs. [44], [43], and [29]).

Crampton, Cowley, and Hutchings (1980) give their uncertainty for ω_t' as $\pm 3^\circ$, so the order of $5 \times 10^{-3} \text{ deg yr}^{-1}$ change in ω_t' is certainly not obvious in the present data available. However, if observations continue so the uncertainty is reduced, the cumulative change in ω_t' might be detectable. Also with continued observations the variation of i_t' might be detected through a change in the mass function.

Incidentally, Faulkner, Gilliland, and Kemper (1979) have observed an additional 5 km s^{-1} periodic term in the “13 day” star besides its 73 km s^{-1} variation. This term has about the right period ($2.77 \times 10^{-3} \text{ day}$) to be caused by the short term effect of the Martin-Rees central binary of period $2.19 \times 10^{-3} \text{ days}$ (see eq. [23]). However, a rough calculation shows that the acceleration that the additional periodic term represents is about seven orders of magnitude greater than the calculated gravitational effect of the Martin-Rees binary on the 13 day star. This discrepancy supports Faulkner *et al.*'s remark that the 5 km s^{-1} variation may be instrumental.

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APPENDIX

The numerical factors given in equations (28), (29), (36), (37), and (43) have been evaluated, respectively, by using

$$\frac{4\pi c^2(1 - e_t^2)}{3(Gm_\odot)^{2/3}\bar{\omega}_t^{5/3}} = 258,000 \text{ yr}, \quad (\text{A28})$$

$$\frac{8\pi(1 - e_t^2)^{3/2}(GM)^{1/2}}{3\bar{\omega}_t^2 a^{3/2}} = 277 \text{ yr}, \quad (\text{A29})$$

$$\frac{3(Gm_\odot)^{2/3}\bar{\omega}_t^{5/3}}{c^2(1 - e_t^2)} = 2.79 \times 10^{-3} \text{ deg yr}^{-1}, \quad (\text{A36})$$

$$\frac{3a^2(\mu/M)\bar{\omega}_t^{7/3}}{8(Gm_\odot)^{2/3}(1 - e_t^2)^2} = 7.61 \times 10^{-3} \text{ deg yr}^{-1}, \quad (\text{A37})$$

$$\frac{a^{1/2}(\mu/M)(M/m_\odot)^{1/2}\bar{\omega}_t^{1/3}}{(Gm_\odot)^{1/6}(1 - e_t^2)^{1/2}} = 0.673 \text{ deg}. \quad (\text{A43})$$

In equations (29), (37), and (43) we have assumed that the Martin-Rees close binary has $e = 0$. Equations (28) and (36) do not depend on the value of e .

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