Superfluidity of cold fermionic atomic gases

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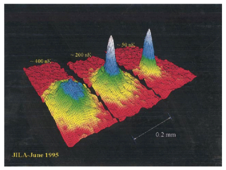
Overview

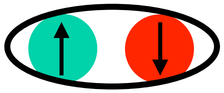
- Many-body physics of cold atomic gases

Bose-Einstein condensation



Anderson et al Science 95
Davis et al PRL 95

- Superfluidity of atomic fermions
  - Pairing of fermions



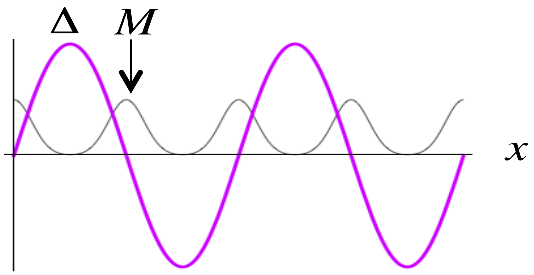
↔ Pairing amplitude Δ

  - Experimental knob: Interactions

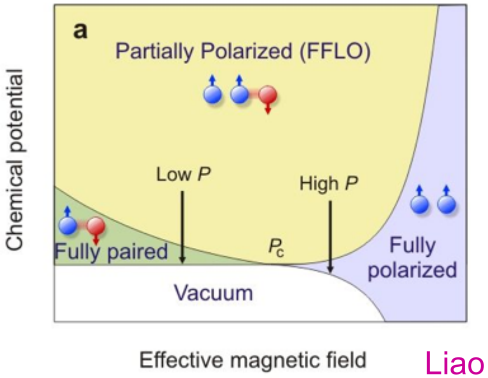
BEC-BCS crossover $\longrightarrow N_\uparrow = N_\downarrow$ favored

- **Recent Work**: Apply “population imbalance”
  - Polarized: More ↑ than ↓
    - ➢ Magnetization: $M = (N_\uparrow - N_\downarrow)/V$
    - ➢ “Zeeman” magnetic field: $h = \mu_\uparrow - \mu_\downarrow$
  - FFLO phase

Cartoon picture:



- 3D gas: Only thin window of stability
- 1D gas: Possibly broad range!
  - No direct evidence of FFLO
- What about FFLO in 2D?



Liao et al Nature 2010

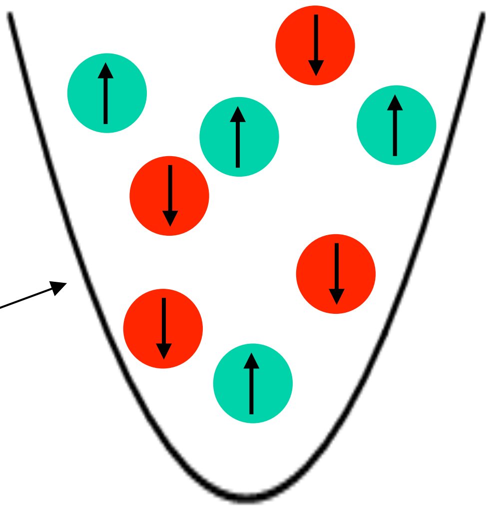
Fermionic pairing of cold atoms

Regal et al PRL 04; Zwierlein et al ibid; Kinast et al ibid, Bartenstein et al ibid, Bourdel et al ibid; Partridge et al ibid 05…

- Fermionic superfluidity: atomic fermions ^{40}K, ^{6}Li

Ultracold: ~10-100 nK   Dilute: ~ 10^{10}-10^{13} cm^{-3}

- Harmonic trap

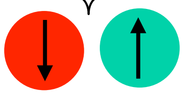


n.b.: actual
exp’ts have
~10^5 -10^7 particles

$$V(\mathbf{r}) = \frac{1}{2} m\omega^2 r^2$$

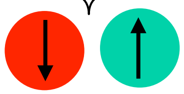
- “Spin” -- different atomic hyperfine-Zeeman levels

e.g. ^{40}K: F=9/2, m_f = $\underbrace{-9/2,-7/2}$



total atom
spin

^{6}Li: F=1/2, m_f = $\underbrace{1/2,-1/2}$



Next: Hyperfine-Zeeman states

Atomic fermions in a magnetic field

- Total atom spin: $\vec{F} = \vec{I} + \vec{S}$

Nuclear spin

Electron spin

Lithium-6:

$I = 1$

$A = 152.1\mathrm{MHz}$

- Hyperfine-Zeeman Hamiltonian

$$H = A\vec{I}\cdot\vec{S} + g\mu_B\vec{B}\cdot\vec{S}$$

Hyperfine Coupling

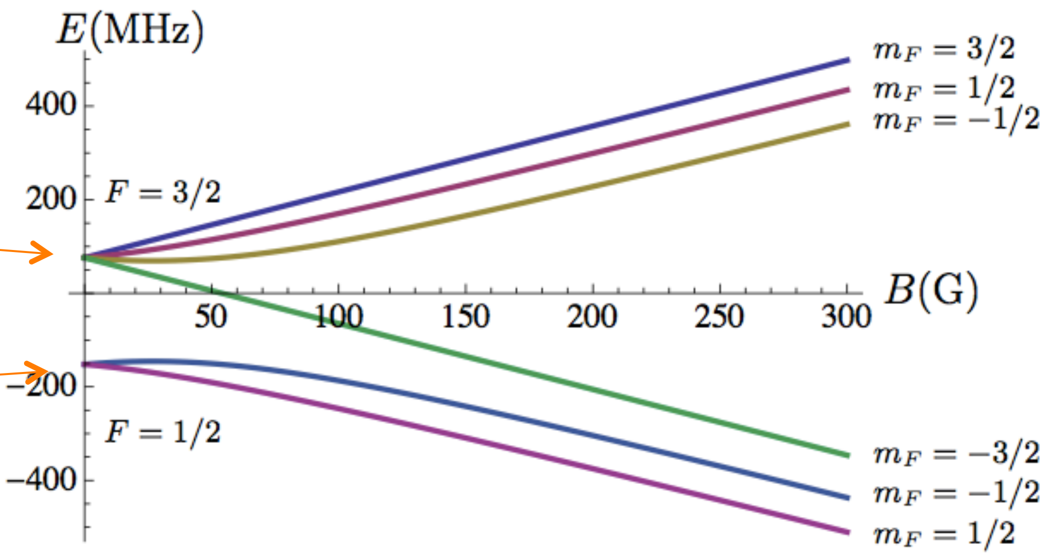
External magnetic field

- Breit-Rabi formula

Low B: Hyperfine dominant

$\vec{I}\cdot\vec{S} = \frac{1}{2}$

$\vec{I}\cdot\vec{S} = -1$



Large B: Zeeman dominant

$S_z \approx \frac{1}{2}$

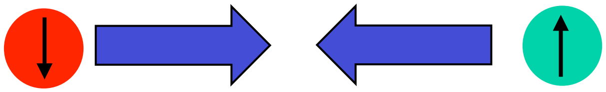
$S_z \approx -\frac{1}{2}$

Our “spin-1/2” system

Next: Interactions

Feshbach Resonance

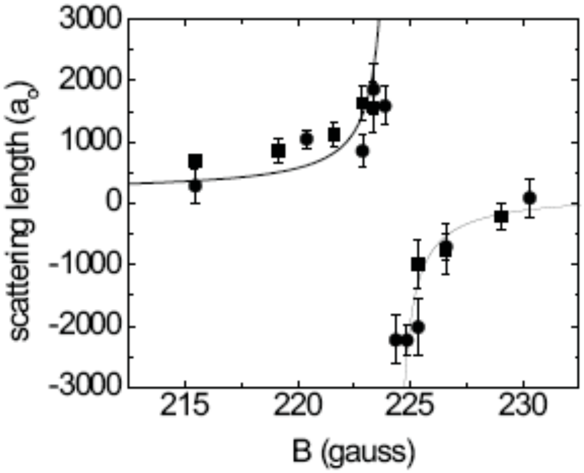
- Interactions controllable via magnetic field B



- Scattering depends on electronic spin
- Electron spin controlled by *B*

- Feshbach resonance: *B* field values where scattering enhanced

s-wave
scattering length



Regal & Jin PRL 90, 230404 (2003)

s-wave
scattering length

$$a_s \propto -\frac{1}{B - B_0}$$

B_0 "resonance position"

Formation of two-particle
bound state

- Dimensionless interaction parameter: $-\frac{1}{k_F a_s} \propto B - B_0$

Can tune interactions into
the strong-coupling regime

Next: BEC-BCS crossover

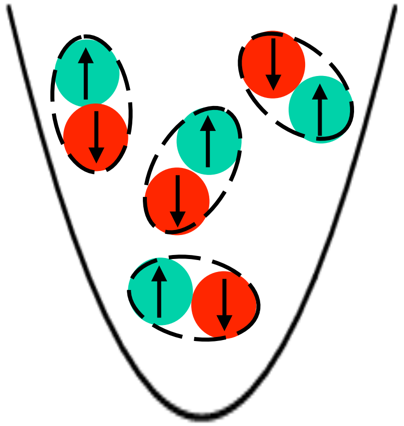
Balanced gas: BEC-BCS crossover

Leggett 1980
Nozieres & Schmitt Rink 1985
Sa De Melo et al 1993

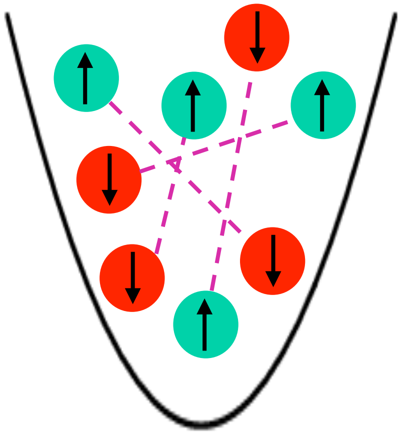
- Balanced: Equal number of ↓, ↑

$N_\uparrow = N_\downarrow$

Pairing at $T \to 0$



BEC limit: strong attraction



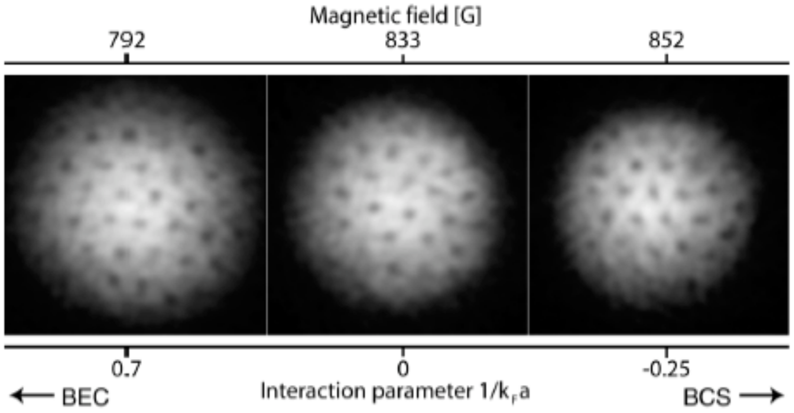
BCS limit: weak attraction

- Numerous experiments: Regal et al PRL 2004, Zwierlein et al PRL 2004, Chin et al Science 2004, Partridge et al PRL 2005, Kinast et al PRL 2004….

- Singlet superfluidity for all coupling values

Zwierlein et al Nature 2005

Vortices in a Bose-Einstein condensate



Vortices in a neutral BCS superconductor

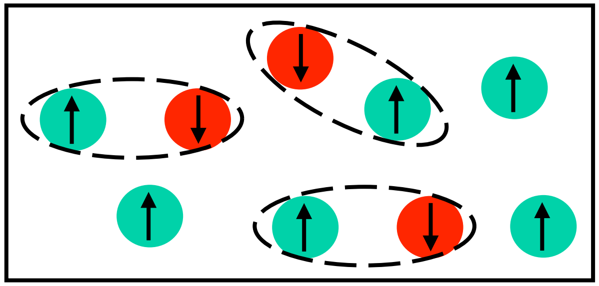
Next: Polarization

Applied spin polarization

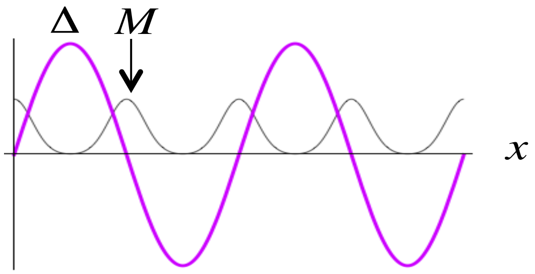
- Recent work*: Explore changing relative number of ↓ , ↑
  - Experimental "knob": Frustrates pairing!
    - Polarization: $P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$
    - "Zeeman" field/chemical potential diff.

$$h = \mu_\uparrow - \mu_\downarrow$$

*Exp't: Zwierlein et al Science 06, Partridge et al Science 06, Shin et al PRL 06, Partridge et al PRL 07, Schunck et al Science 07, Shin et al Nature 08, Shin et al PRL 08, Schirotzek et al PRL 2009,Nascimbene et al 2009, Liao et al 2009…



- Conventional pairing suppressed!
  - –Exotic Phases? (FFLO) Fulde & Ferrell PR 1964; Larkin & Ovchinnikov JETP 1965
    - Spatially inhomogeneous pairing



$$\Delta(x) \propto \cos[Qx]$$

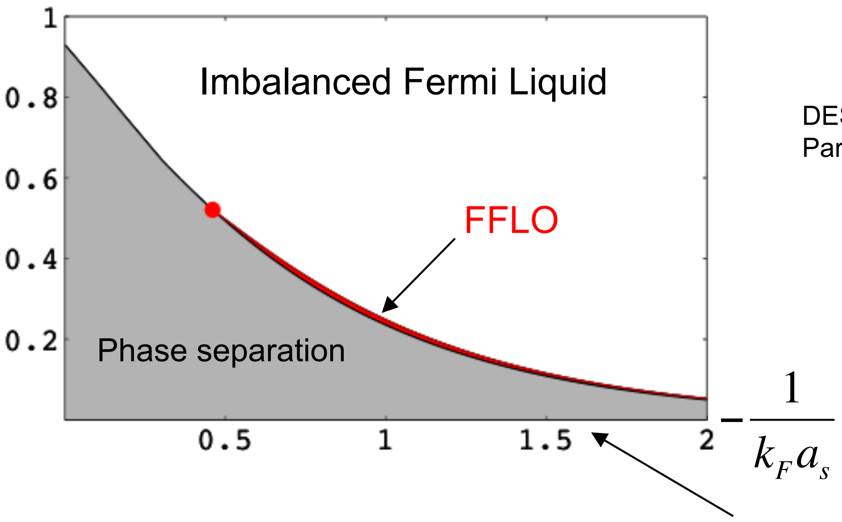
Pairing wavevector: $\mathbf{Q} \approx p_{F\uparrow} - p_{F\downarrow}$

Next: However…

3D Gas: Only a thin range of FFLO

$$P = \frac{M}{n} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

Ground state phase diagram



Fully paired BCS

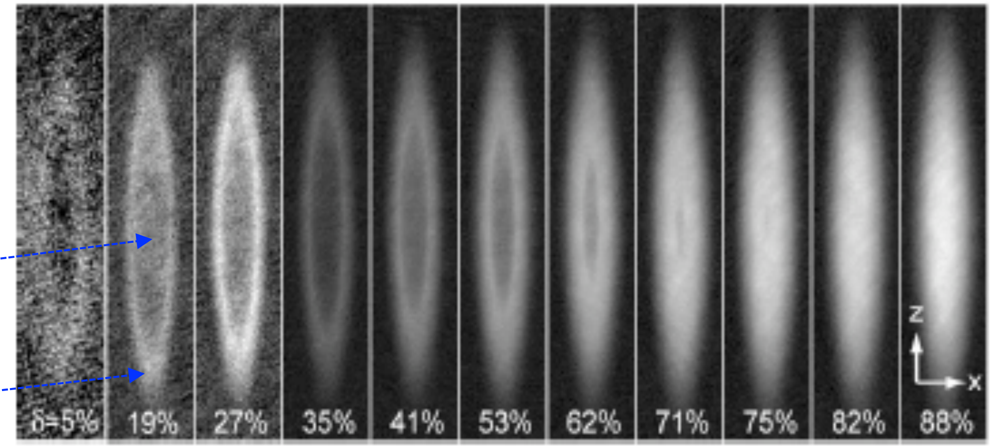
DES, L. Radzihovsky PRL 2006, Ann. Phys. 2007,
Parish et al Nat. Phys. 07

- Experiments: phase separation

- ➢ Magnetization

$$M(r) = n_\uparrow(r) - n_\downarrow(r)$$

- ➢ Fully paired core (BCS)
- ➢ Magnetized edge (Imbalanced FL)

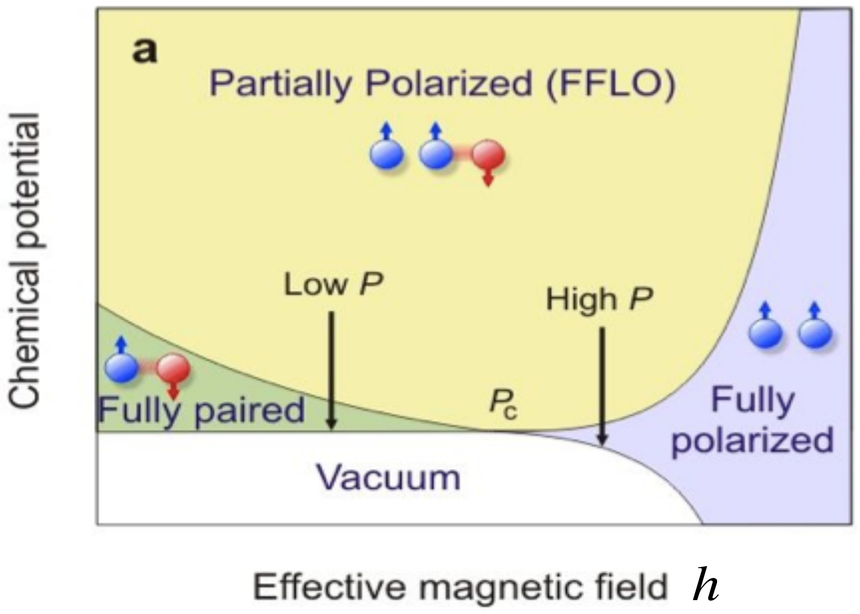


Shin et al PRL 2006

Next: 1D

One-dimensional case: Wide FFLO regime

Bethe Ansatz:
Orso PRL 2007
Hu et al PRL 2007

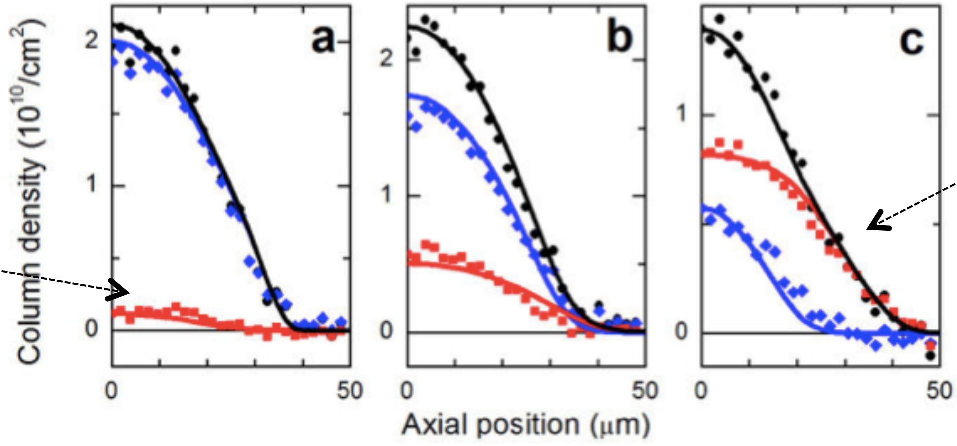


Experiment:
Liao et al Nature 2010

^{6}Li in "tube" geometry

- Density profiles consistent with phase diagram
  - LDA: Trap acts like a spatially-varying chemical potential

Red curves:
Magnetization



Low P: Magnetized in center, fully paired on edge

High P: Magnetized in center, fully polarized on edge

But how to observe the "FFLO"?

No clear evidence of FFLO!

- In 1D: Experiments consistent with theoretical phase diagram

Liao et al Nature 467, 567 (2010)

  - ➢ But they did not observe “FFLO” correlations

$\Delta(z) \propto \cos[\mathbf{Q}z]$ → Pairing correlations peaked at wavevector

$$\mathbf{Q} \approx p_{\mathrm{F}\uparrow} - p_{\mathrm{F}\downarrow}$$

- In 3D: FFLO stable only for thin window

- What about 2D?

  - ➢ 2D Fermi gas experiments: Zhang, et al PRL 108, 235302 (2012)
Sommer, at al PRL 108 045302 (2012)

- Deep optical lattice potential:

Independent 2D
“pancakes”

Orso & Shlyapnikov PRL 2005: Scattering amplitude exhibits 2D limit

Next: Model

Model of attractive quasi-2D fermions

- Effective Hamiltonian:

$$H = \sum_{\sigma} \int d^2r\, \psi^{\dagger}_{\sigma}(r)\left(\frac{p^2}{2m} - \mu_{\sigma}\right)\psi_{\sigma}(r) \; + \lambda \int d^2r\, \psi^{\dagger}_{\uparrow}(r)\psi^{\dagger}_{\downarrow}(r)\psi_{\downarrow}(r)\psi_{\uparrow}(r)$$

- Interactions: Parameterized by $\lambda < 0$
  - Depend on bulk 3D scattering length & trap

- 2-body problem: Bound-state for *any* attractive interactions

Scattering amplitude: $f(E) = \dfrac{4\pi}{\log\left(E_b/E\right) + i\pi}$

Negative energy pole
for any weak coupling

Binding energy: $E_b = 2D\exp\left[\dfrac{2}{\lambda g}\right]$

UV cutoff

2D density of states $g = \dfrac{m}{2\pi\hbar^2}$

Next: BEC-BCS crossover

BEC-BCS crossover ?

- 2-body problem: Bound-state for *any* attractive interactions

$$E_b = 2D\exp\left[\frac{2}{\lambda g}\right]$$

UV cutoff

2D density of states $g = \frac{m}{2\pi\hbar^2}$

- 3D: Critical coupling strength λ_c

Strong Attraction: $|\lambda| > \lambda_c$ ⇒ Vacuum bound state — "BEC" limit of preformed pairs

Weak Attraction: $|\lambda| < \lambda_c$ ⇒ No bound state — "BCS" limit of many-body pairing

- Finite density 2D gas: Always BEC?
  - ➢ Compare 2D scattering length $\underbrace{a_{2D}}_{E_b = \frac{\hbar^2}{m a_{2D}^2}}$ to Fermi wavevector $k_F \propto \sqrt{n}$
  - ➢ "BCS" regime: $k_F a_{2D} > 1$
  - ➢ "BEC" regime: $k_F a_{2D} < 1$

$$\left.\begin{matrix} k_F a_{2D} > 1 \\ k_F a_{2D} < 1 \end{matrix}\right\} \Rightarrow$$ Effective coupling parameter $\log\left(k_F a_{2D}\right)$

Next: BEC-BCS crossover

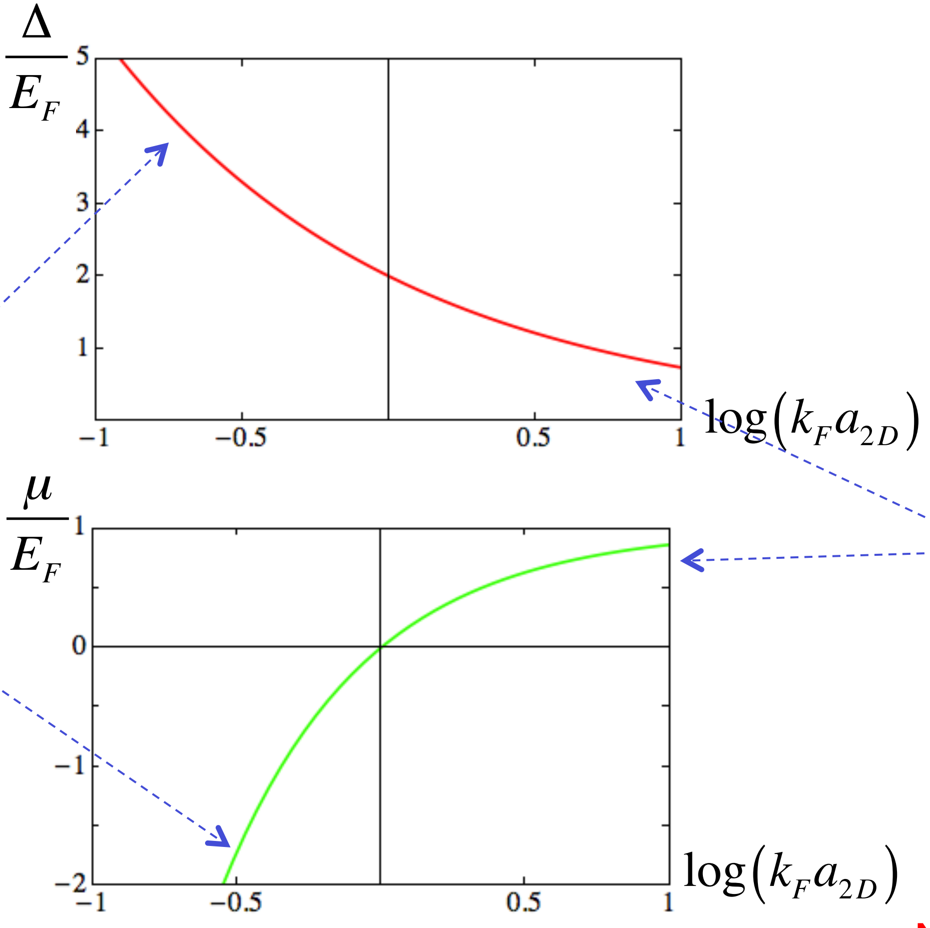
Mean-field theory $(T = 0)$

Randeria, et al PRL 1989

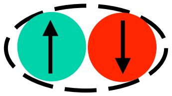
- Assume pairing amplitude Δ ➡ Mean-field free energy $F(\Delta,\mu)$

Gap equation: $\frac{\partial F}{\partial \Delta} = 0$

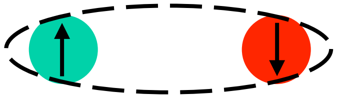
Number equation: $-\frac{\partial F}{\partial \mu} = N$



"BEC" regime:
Strong pairing, negative chemical potential



"BCS" regime:
Weak pairing, chemical potential close to Fermi energy



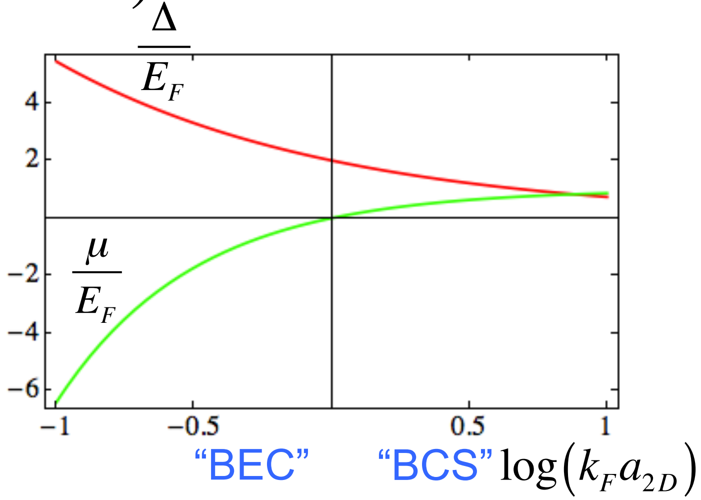
Next: Unusual feature…

Mean-field theory $(T=0)$

- Mean-field gap & chemical potential:

The curves we plotted:

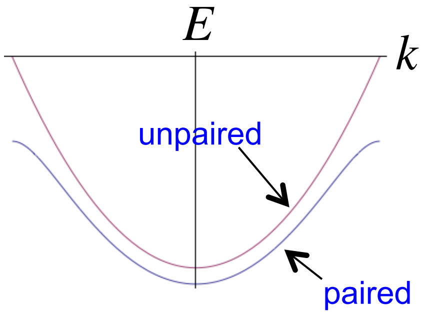
$$\frac{\Delta}{E_F}=\sqrt{2\frac{E_b}{E_F}} \qquad \frac{\mu}{E_F}=1-\frac{E_b}{2E_F}$$



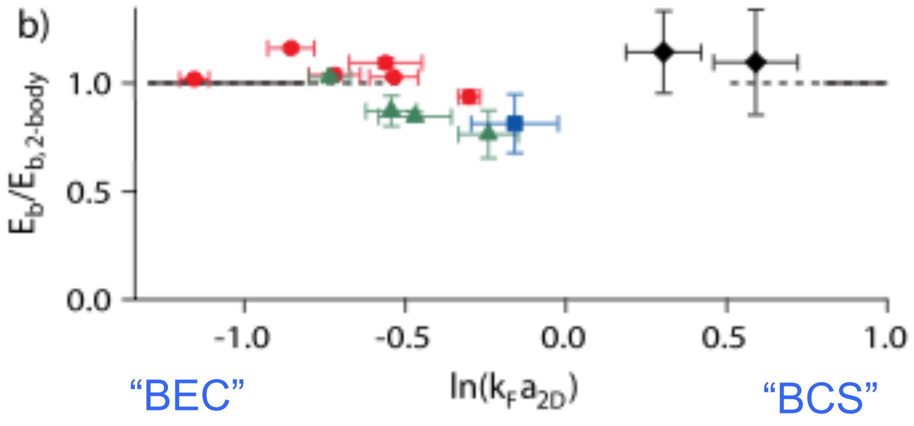
- Excitation gap equals binding energy!

$$E_{gap}=\sqrt{\Delta^2+\mu^2}-\mu \leftarrow \text{Measured in RF spectroscopy}$$

$$=E_b \leftarrow \text{Two-body binding energy}$$



- Recent experiments: Sommer, at al PRL 108 045302 (2012)



Is pairing in 2D trivial somehow?

Next: Imbalance?

Impose population imbalance

- Same effective Hamiltonian: $\mu_\uparrow > \mu_\downarrow$ ⇨ Impose density imbalance

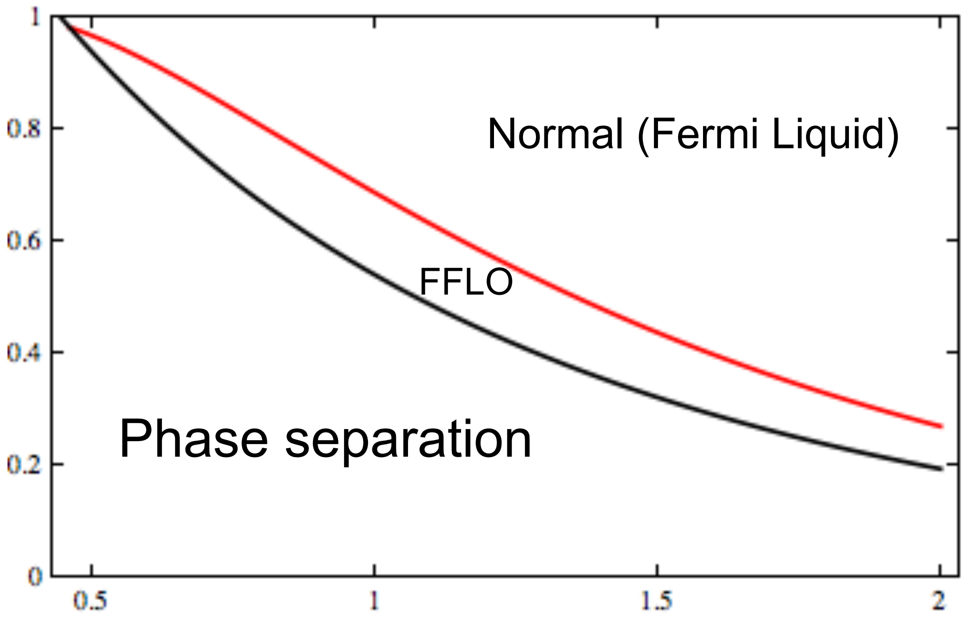
$$H = \sum_\sigma \int d^2 r\, \psi_\sigma^\dagger(r) \left(\frac{p^2}{2m} - \mu_\sigma \right) \psi_\sigma(r) \; + \lambda \int d^2 r\, \psi_\uparrow^\dagger(r) \psi_\downarrow^\dagger(r) \psi_\downarrow(r) \psi_\uparrow(r)$$

- Main result: Phase diagram

Red curve: Continuous

Black curve: 1st order

$$P = \frac{M}{n} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



$\log\left(k_F a_{2D}\right)$

Note: Experiments studied range -1 to 1

See also: Conduit et al PRA 2008

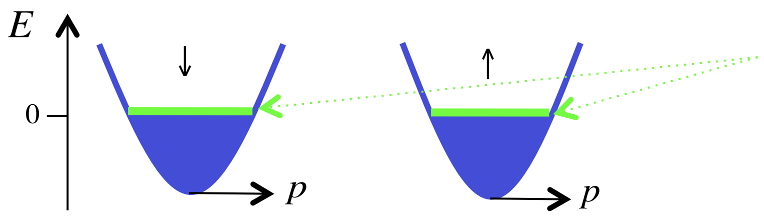
Next: How to get this?

First neglect FFLO phase

- Fate of BCS state with increasing $h=(\mu_\uparrow-\mu_\downarrow)/2$

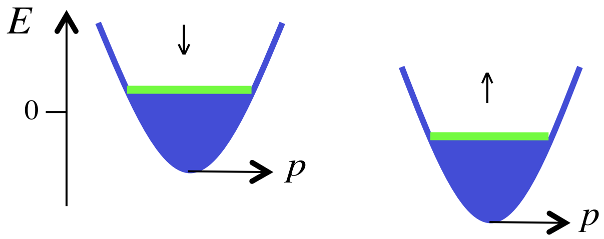
$h=0:$

$\boxed{\mu_\uparrow=\mu_\downarrow}$



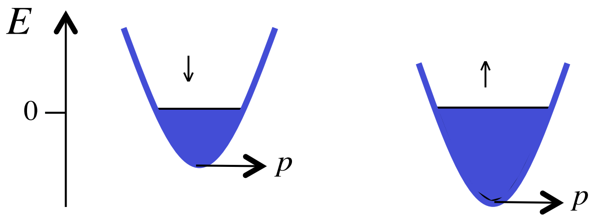
- Cooper pairing at Fermi surface
- BCS ground state

Small h:



- System stays in BCS ground state
- Note: $N_\uparrow=N_\downarrow \quad (M=0)$
- Pay “Zeeman” energy, gain pairing energy

Large h:



- System becomes unpaired
- Gain “Zeeman” energy, pay pairing energy $M>0$

- 1st order transition at $h_c=\frac{1}{2}E_b\sqrt{1+4\mu/E_b}$

3D: Clogston 1962

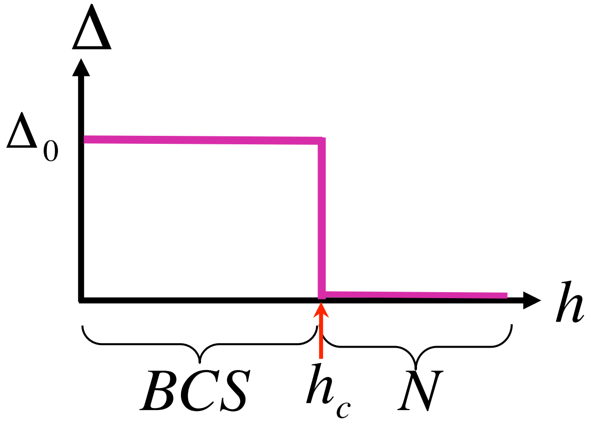
Next: Fixed *M* picture

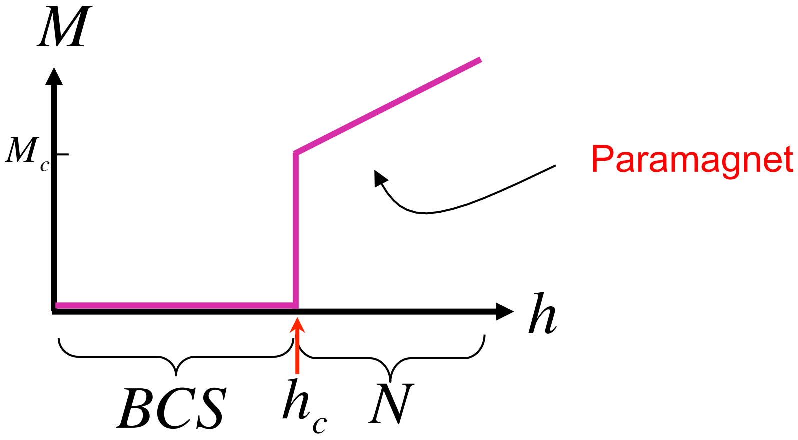
First-order BCS-to-N transition

- 2D Clogston transition: $h_c = \frac{1}{2} E_b \sqrt{1+4\mu/E_b}$

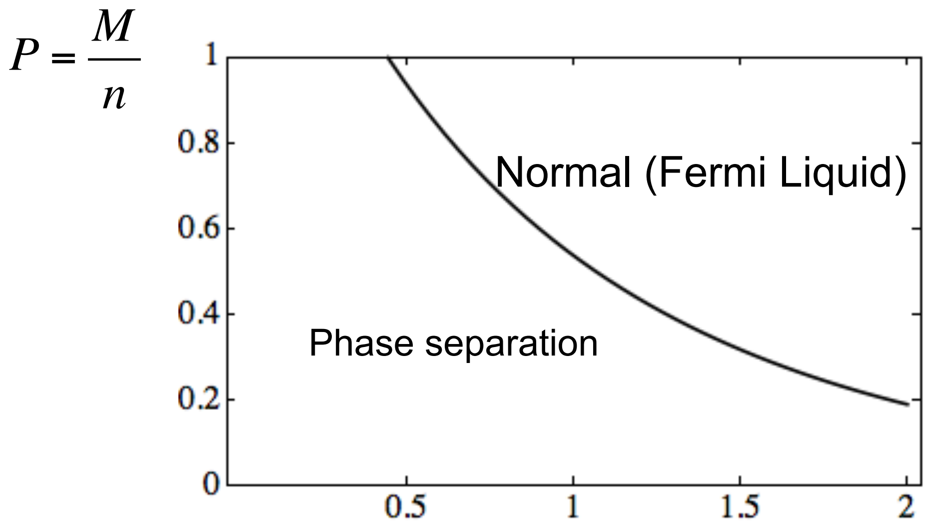
$E_b = \frac{\hbar^2}{ma_{2D}^2}$ =2-body binding energy

- Magnetization (M) and pairing (Δ) jump





- Fixed-M ensemble: Phase separation



$P_c \sim E_b \longleftrightarrow$ Increases towards BEC limit

Note: Experiments studied range -1 to 1

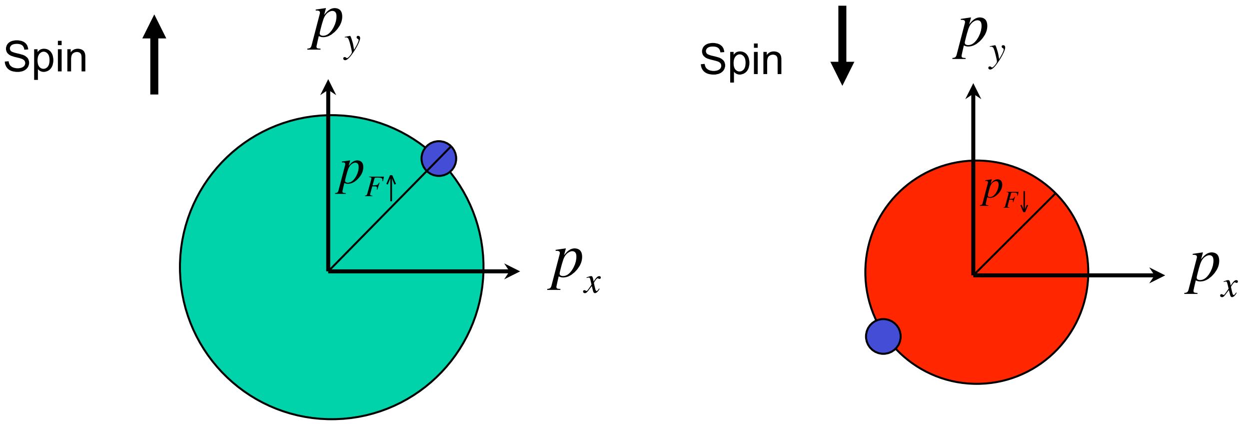
$\log(k_F a_{2D})$

Next: FFLO phase?

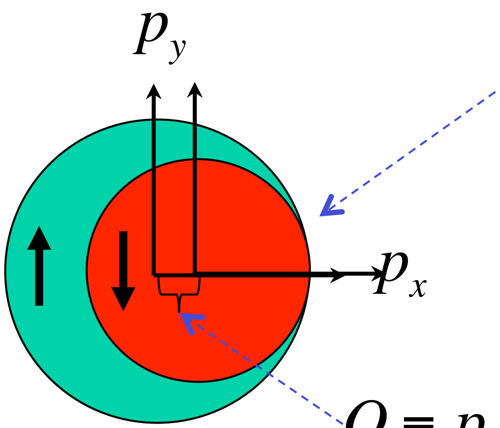
FFLO state

Fulde&Ferrell PR 1964;
Larkin&Ovchinnikov JETP 1965

- Excess spin $\uparrow$: Larger Fermi surface $p_{F\uparrow} > p_{F\downarrow}$



- Pairing of low-energy states near Fermi surface $Q = p_{F\uparrow} - p_{F\downarrow}$
  - ➢ Cooper pairs have finite momentum!
- “Shifted” Fermi seas



Susceptible to pairing!

Finite-momentum pairing

$$\Delta(x) \propto \cos[Qx]$$

$$Q \equiv p_{F\uparrow} - p_{F\downarrow}$$

Next: Many-body t-matrix

FFLO state

- Instability of imbalanced normal phase

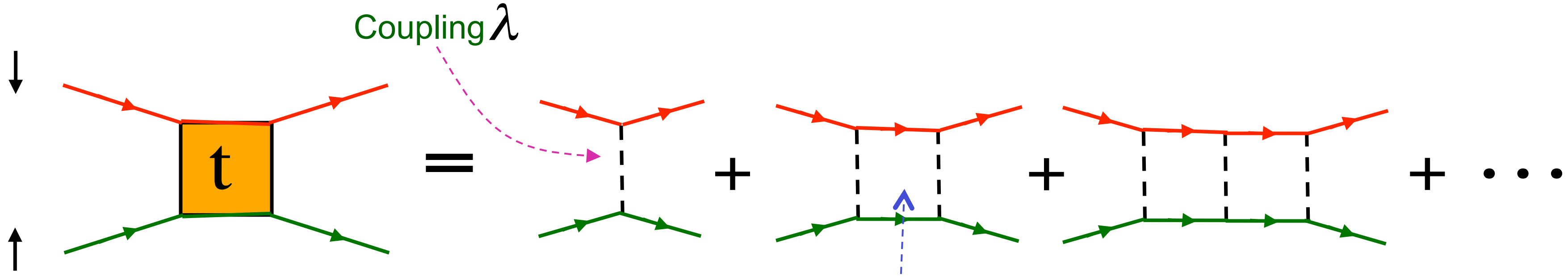
Zero frequency

Pair-pair correlations: $\langle \Delta(\mathbf{r})\Delta^\dagger(\mathbf{r}')\rangle \propto \sum_{\mathbf{q}} t(\mathbf{q},0)\exp[i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')]$

$\Delta(\mathbf{r}) = \psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r})$

t-matrix

- t-matrix: Repeated scattering of ↑ , ↓



- Geometric (ladder) series: $t(q,\omega) = \dfrac{\lambda}{1+\lambda\Pi(q,\omega)}$

- If $\dfrac{1}{t(q,0)} < 0$ for all momenta ⇨ Normal phase is stable

- If $\dfrac{1}{t(q,0)} = 0$ for some q ⇨ Pairing instability!

Next: Exact result

Inverse t-matrix for 2D imbalanced gases

- T=0 limit…

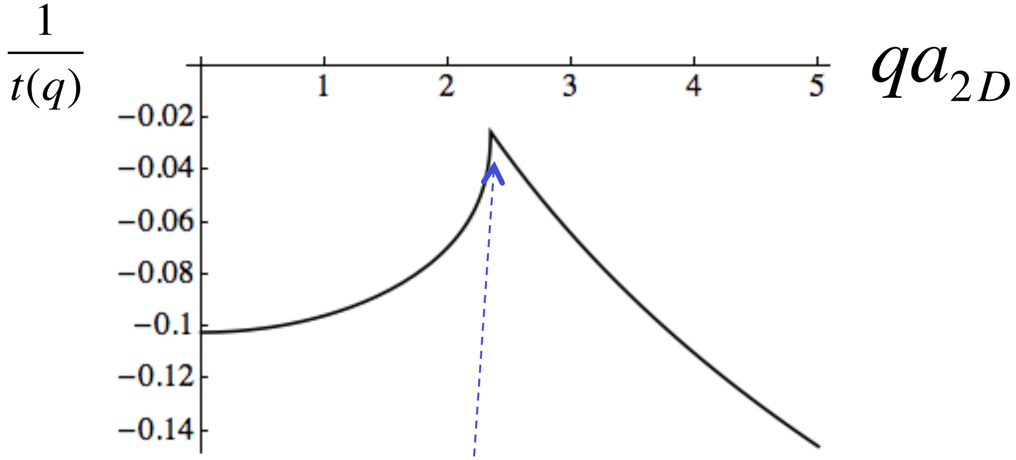
$$\frac{1}{t(q,0)} = \frac{1}{\lambda} + \sum_{\mathbf{p}} \frac{1 - n_F\left(\varepsilon_{\mathbf{p}} - \mu_\downarrow\right) - n_F\left(\varepsilon_{\mathbf{p+q}} - \mu_\uparrow\right)}{\varepsilon_{\mathbf{p}} - \mu_\downarrow + \varepsilon_{\mathbf{p+q}} - \mu_\uparrow} = \frac{1}{2\pi}\mathrm{Re}\log\left[\frac{\sqrt{2E_b(\mu - q^2/8)}}{h + \sqrt{h^2 - \frac{1}{2}q^2(\mu - q^2/8)}}\right]$$

  - Difficult method: Contour integration
  - Easy method: Jensen formula from complex analysis

- Let's plot it…

$\mu = 2E_b$

$h = 2E_b$



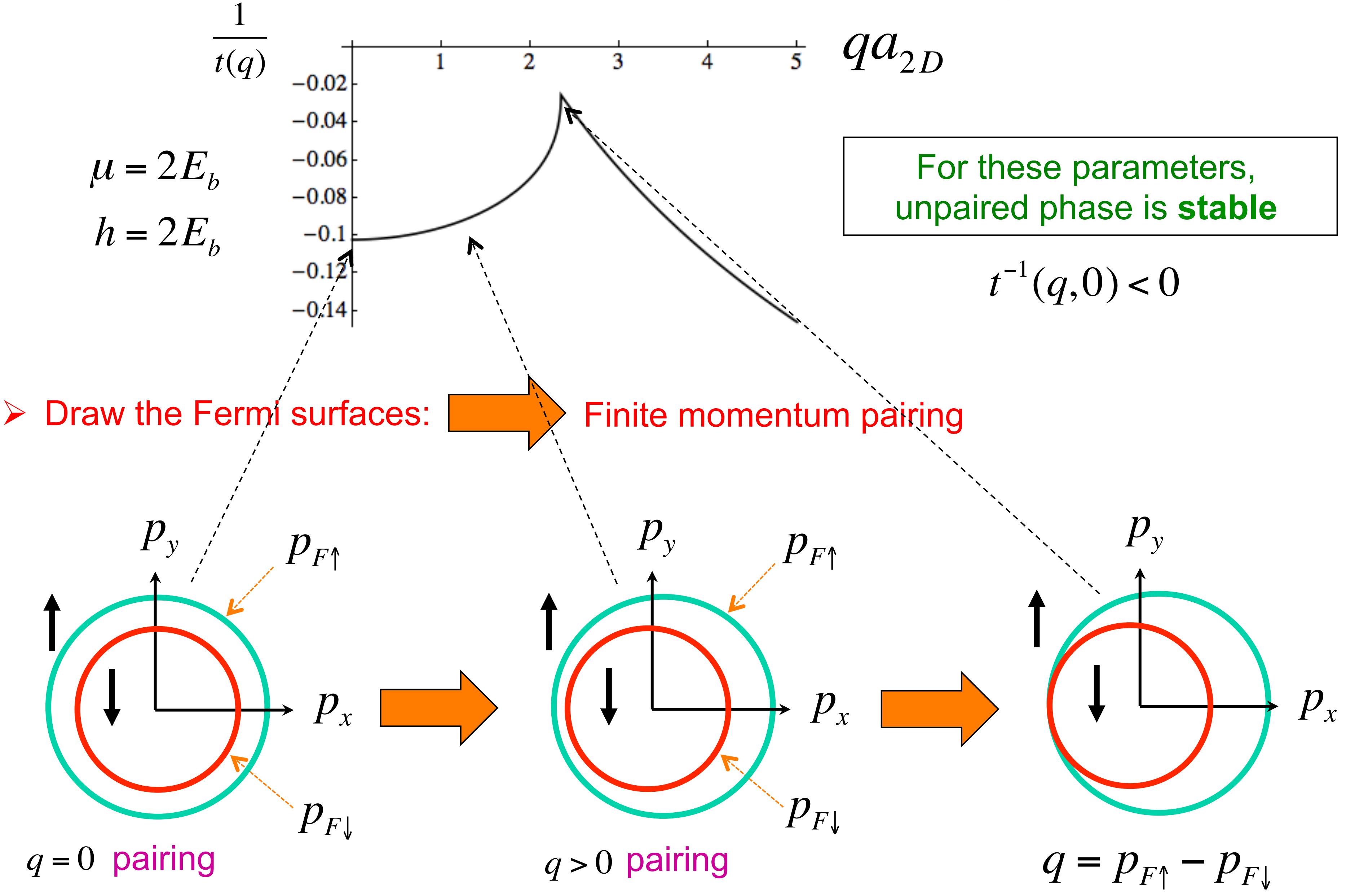
- Singularity as a function of FFLO wavevector q!
  - At $q = p_{F\uparrow} - p_{F\downarrow}$

Fermi wavevectors

$$\begin{cases} p_{F\downarrow} = \sqrt{2m(\mu - h)} \\ p_{F\uparrow} = \sqrt{2m(\mu + h)} \end{cases}$$

Next: Draw the Fermi surfaces

Inverse t-matrix for 2D imbalanced gases



$\mu = 2E_b$

$h = 2E_b$

For these parameters, unpaired phase is **stable**

$t^{-1}(q,0) < 0$

- Draw the Fermi surfaces: ➡ Finite momentum pairing

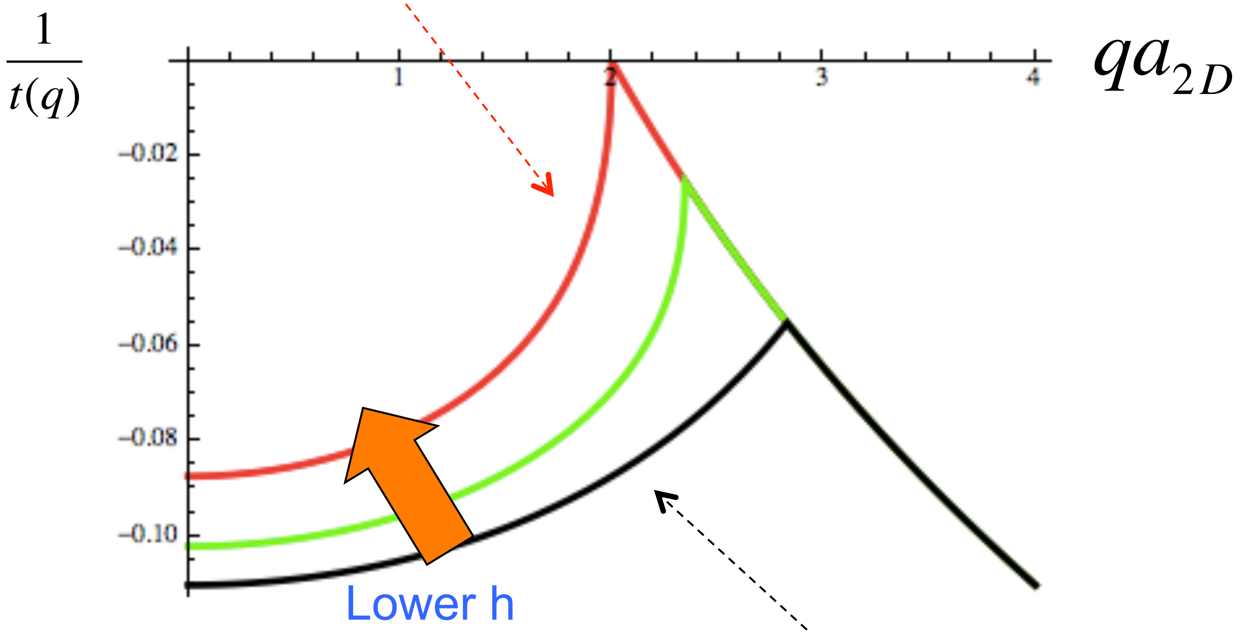
$q = 0$ pairing

$q > 0$ pairing

$q = p_{F\uparrow} - p_{F\downarrow}$

FFLO instability with decreasing imbalance

- Critical Zeeman: FFLO instability $h = \sqrt{3}E_b$ and $\mu = 2E_b$



- Large Zeeman: Stable normal state $h = 2E_b$ and $\mu = 2E_b$

- Further decreasing imbalance: Continuous phase transition!

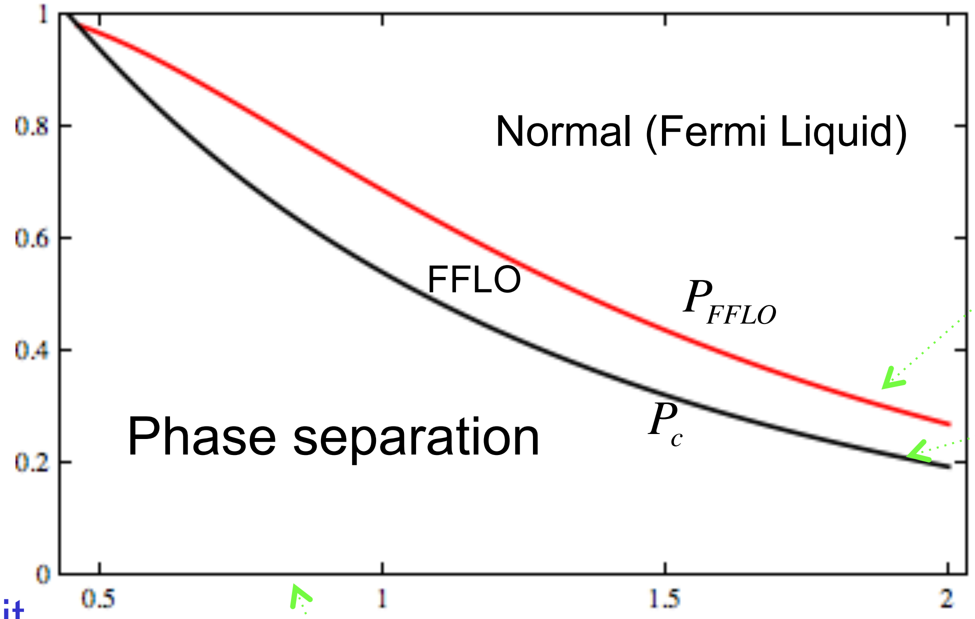
$$h_{FFLO} = E_b\sqrt{2\mu / E_b - 1}$$

- Lower Imbalance: Ordered phase

Next: Phase diagram

Phase diagram

$$P = \frac{M}{n} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



Red curve: Continuous

Black curve: 1st order

$\log(k_F a_{2D})$

See also: Conduit et al PRA 2008

BCS phase: *P=0*

Note: Experiments studied range -1 to 1

Analytical results:

$$\begin{cases} P_{FFLO} = \dfrac{E_b}{E_F}\sqrt{2\dfrac{E_F}{E_b} - 1} \\ P_c \approx \dfrac{1}{2}\dfrac{E_b}{E_F}\sqrt{4\dfrac{E_F}{E_b} + 1} \end{cases}$$

(neglecting FFLO condensation)

Next: Ginzburg-Landau breakdown

Effective Ginzburg-Landau Theory?

- Usual picture: Analytic Ginzburg-Landau expansion

➢ Effective action: $$S(\{\Delta_q\},\mu,h) = \sum_q t^{-1}(q)\left|\Delta_q\right|^2 + \underbrace{\ldots}_{}$$

All possible FFLO instabilities

Quartic & higher order terms $\sim \left|\Delta_q\right|^4$

- Balanced Gas: $S(\Delta,\mu,0) \sim |\Delta|^2 \ln|\Delta|^2$

Nonanalytic!

No T=0 GL theory

- 3D FFLO: GL theory OK

- We find: Highly singular effective action in 2D!

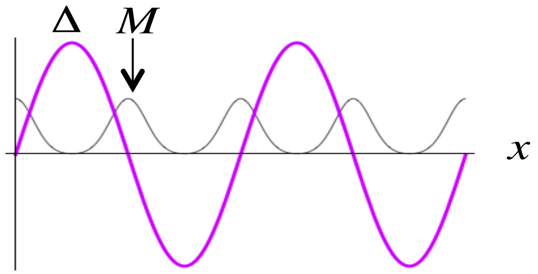
$$S(\{\Delta_q\},\mu,h) = \sum_q T^{-1}(q,\Delta_q)\left|\Delta_q\right|^2$$

Has step function nonanalyticities vs. Δ_q

Concluding remarks

- FFLO state still not observed!
  - ➢ Extremely narrow window of stability in 3D

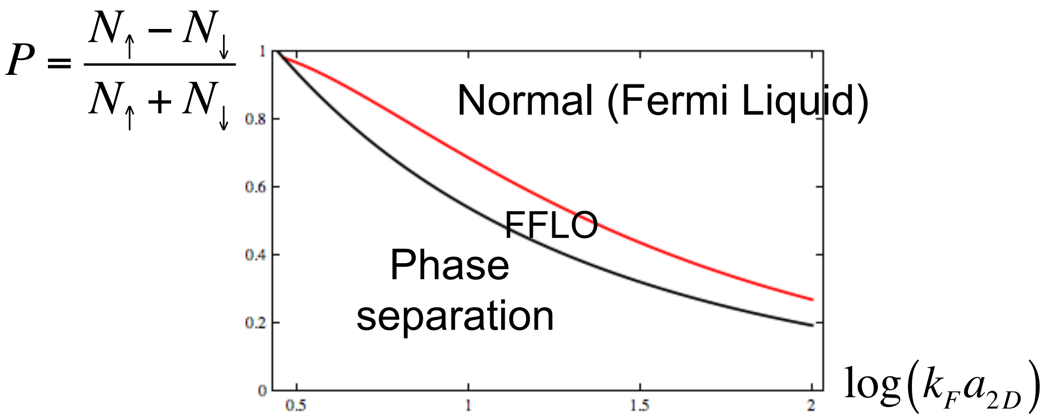
Cartoon picture:



- 2D Fermi gases: Unusual BEC-BCS crossover

Sommer, at al PRL 108 045302 (2012)

- Imbalanced 2D gas: FFLO state in BCS regime
  - ➢ Much wider than 3D



$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

- Effective FFLO action: Highly nonsingular!
  - ➢ Controls emergence of pairing in ordered phase
  - ➢ Effective theory for FFLO fluctuations

Radzihovsky PRA 2011