Canonical quantum gravity

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- Lecture 1: Introduction: The early beginnings (1984-1992)
- Lecture 2: Formal developments (1992-4)
- **Lecture 3**: Physics (1994-present)

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Today’s lecture

- Dynamics and the Hamiltonian constraint.
- Applications to gamma-ray-bursts?
- Approximation ideas with large cosmological constant.

Dynamics: The Hamiltonian constraint

We discussed in the first lecture how one could write the Hamiltonian constraint and surprisingly find some solutions. The results were unregulated and formal. Part of the problem stemmed from the fact that we were looking at the Hamiltonian constraint in its double-densitized form,

$$\tilde{\tilde{H}} = \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j F^k_{ab}$$

Suppose one wants to promote this to an operator in the loop representation. We have at hand a manifold and loops. How could one build a density of weight two? The answer is: one can't, without the aid of external structures. Most regularizations of the "early years" did exactly that.

Couldn't we consider the single-densitized Hamiltonian? Then one could represent it as a Dirac delta, which is defined intrinsically.

The reason that stopped people from trying this is the complicated, non-polynomial form of the single-densitized constraint in terms of Ashtekar's new variables,

$$\tilde{H} = \frac{\epsilon^{ijk}\tilde{E}^a_i\tilde{E}^b_j}{\sqrt{\epsilon^{ijk}\tilde{E}^a_i\tilde{E}^b_j\tilde{E}^c_k\epsilon^{abc}}}F^k_{ab}$$

Remarkably, Thiemann (1996) discovered the identity,

$$\frac{\tilde{E}^a{}_i\tilde{E}^b{}_j e^{ijk}}{\sqrt{\tilde{E}^a{}_i\tilde{E}^b{}_j\tilde{E}^c{}_k e^{ijk} e_{abc}}} = 2e^{abc}\{A_c{}^k, V\}$$

Which allows us to write the single-densitized Hamiltonian as,

$$\tilde{H} = -2\mathrm{Tr}(F_{ab}\{A_c, V\})e^{abc}$$

This is not only remarkably simple, but Thiemann found something else as well...

Recall that when Ashtekar introduced the new variables, there was a free parameter in the canonical transformation (the Immirzi β parameter) and if one did not choose it equal to the imaginary unit the Hamiltonian constraint looked like,

$$\tilde{\tilde{S}} \equiv -\zeta \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j F_{abk} + \frac{2(\beta^2\zeta - 1)}{\beta^2} \tilde{E}^a_{[i} \tilde{E}^b_{j]} (A^i_a - \Gamma^i_a)(A^j_b - \Gamma^j_b) = 0$$

Thiemann found that the ugly looking second piece (divided by det g) can be written as,

$$\tilde{H}_2 = 4e^{abc}\mathrm{Tr}(\{A_a, K\}\{A_b, K\}\{A_c, V\})$$

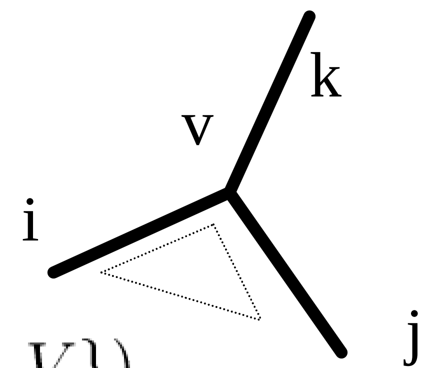
Where,

$$K = -\{\frac{V}{\kappa}, \int_\Sigma d^3x H^E(x)\}$$

That is, not only is the "Euclidean" piece of the Hamiltonian constraint (single densitized) easy. The full Hamiltonian constraint without using complex variables is reasonably simple too!

Thiemann proceeds to quantize this expression. He starts by discretizing it on a lattice, obtained by triangulating the spatial manifold in terms of tetrahedra,

$$\tilde{H} = -2\mathrm{Tr}(F_{ab}\{A_c, V\})e^{abc}$$



$$H_\Delta^E[N] := -\frac{2}{3}N_v \epsilon^{ijk}\mathrm{tr}(h_{\alpha_{ij}(\Delta)} h_{s_k(\Delta)}\{h^{-1}_{s_k(\Delta)}, V\})$$

When $\Delta\to 0$, $h_{\alpha_{ij}} \to 1 + F_{ij}s^i s^j \Delta^2$, $h_{s_k} = 1 + A_k s^k \Delta$

So the expression tends (pointwise) to the above one (times Δ^3) when the triangulation is shrunk. Moreover it is manifestly gauge invariant since loops are all closed in the end. Therefore,

$$H_T^E[N] = \sum_{\Delta \in T} H_\Delta^E[N]$$

Is a good approximation to the “smeared” classical Hamiltonian.

Since both the holonomy and the volume operator are well defined quantities in the space of cylindrical functions of spin networks, it is immediate to promote H to a quantum operator.

$$\hat{H}^E_\Delta[N] := -2\frac{N(v(\Delta))}{3i\ell_p^2}\epsilon^{ijk}\mathrm{tr}(h_{\alpha_{ij}(\Delta)}h_{s_k(\Delta)}[h^{-1}_{s_k(\Delta)},\hat{V}])$$

The operator is finite. It is well defined if we are acting on diffeomorphism invariant states. Otherwise we would have to specify where to add the “crossing” line of the loop α.

Due to this property, the operator is also anomaly-free. That is, the commutator of two Hamiltonians, vanishes, which agrees with the classical Poisson algebra on diffeomorphism invaraint functions.

$$[\hat{H}(N),\hat{H}(M)]=0 \qquad \{H(N),H(M)\}=\int d^3x(N\partial_a M - M\partial_a N)g^{ab}C_b$$

Notice the crucial role that diffeomorphism invariance played in the construction. If the functions were not diffeomorphims invariant, the added line would have to be shrunk to the vertex and possible divergences could appear.

The same construction can be applied to the Hamiltonian of general relativity coupled to matter: scalar fields, Yang-Mills fields, Fermions. In all cases the theory is finite, anomaly free and well defined. Gravity indeed appears to be acting as a “fundamental regulator” of theories of matter.

A similar construction, applied in 2+1 dimensions, yields a theory of quantum gravity that correctly contains the physical states found by Witten through his quantization.

What we therefore have is a well defined theory of quantum gravity.
Is this “THE” theory?

Although this could only be settled in detail when the semiclassical limit is worked out, there are certain worries.

The first one is the sort of action the Hamiltonian has. It only acts at vertices and it acts by “dressing up” the vertex with lines. It does not interconnect vertices nor change the valences of the lines (outside the “dressing”). This immediately suggests super-selection rules and quantities that are anomalously conserved. For instance, one can consider surfaces that enclose a vertex (diffeomorphically invariantly defined). The area of such surfaces would commute with the Hamiltonian.

This hints at the theory “failing to propagate”.

Another objection that was raised by Lewandowski and Marolf has to do with the fact that it is easy to construct a more general "habitat" of functions where Thiemann's Hamiltonian is well defined. Consider any function of a spin net with n vertices. Multiply it times a scalar function with n entries, evaluated at each vertex,

$$| s, f >= \int d^3 x_1 ... d^3 x_n \, f(x_1, ..., x_n)$$

These functions are invariant under diffeomorphism that leave the vertices of the spin network fixed. Otherwise diffeomorphisms are correctly implemented geometrically: $C(\vec{N}) \, | s, f >= | s, L_{\vec{N}} f >$

It is obvious that on these states one can implement Thiemann's Hamiltonian. It is also obvious that on these states the Hamiltonian will have an Abelian algebra too.

Is the theory inconsistent? For that we should evaluate the right hand side of the commutator, on these states.

$$\{H(N), H(M)\} = \int d^3x \omega_a(x) q^{ab}(x) \tilde{C}_b(x)$$

Which in turn requires computing the doubly-contravariant metric. Remember that we need to write it in terms of the fundamental variables. Using Thiemann's construction:

$$q^{ab}(x) = \frac{1}{4} \epsilon^{acd} \epsilon_{ijk} \epsilon^{bef} \epsilon_{ilm} \frac{e_c^j e_d^k}{\sqrt{det(q)}} \frac{e_e^l e_f^m}{\sqrt{det(q)}}$$

And it is not hard to see that this operator vanishes identically.

The theory is therefore consistent. But it appears (to some) that one is paying too high a price: the contravariant metric identically vanishes.

This is not uncommon in field theory. The contravariant metric is a highly non-linear combination of the fundamental variables and its relation to operators that are well defined and non-vanishing like the area and volume.

So is it a problem or not? At the moment the issue is debated. It is interesting that the same problem arises in 2+1 dimensions and one nevertheless recovers Witten's correct quantization.

Active research is ongoing in proposing modifications to Thiemann's Hamiltonian that may fix some of these issues. A recent proposal generalizes this Hamiltonian to the space of Vassiliev invariants, where one naturally finds the Chern-Simons state we discussed in the first lecture and a spin network generalization of the Jones poly.

R. Gambini, J. Griego, C|. Di Bartolo, JP (2000)

Second part: a possibly testable effect of quantum gravity?

Based on a proposal by Giovanni Amelino-Camelia and collaborators at CERN. This proposal have received a significant amount of attention:

-First prize Gravity Research Foundation 1999.

-Cover of Nature.

-Several experimental groups published papers concerning them.

Limits to Quantum Gravity Effects on Energy Dependence of the Speed of Light from Observations of TeV Flares in Active Galaxies

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We have used data from a TeV γ-ray flare associated with the active galaxy Markarian 421 to place bounds on the possible energy dependence of the speed of light in the context of an effective quantum gravitational energy scale. Recent theoretical work suggests that such an energy scale could be less than the Planck mass and perhaps as low as 10^{16} GeV. The limits derived here indicate this energy scale to be in excess of 6×10^{16} GeV for at least one approach to quantum gravity in the context of D-brane string theory. To the best of our knowledge, this constitutes the first convincing limit on such phenomena in this energy regime.

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The idea: The light that comes to us from gamma ray bursts has traveled a very long distance in terms of the number of wavelengths involved. If each wavelength is disturbed by a quantum gravity effect of order L_{planck} during the wave propagation then there is a chance we could observe the effects.

G. Amelino-Camelia, J. Ellis, N. Mavromatos, D. Nanopoulos, S. Sarkar, Nature 393, 763 (1998).

Effects stem from the fact that one does not expect Lorentz invariance to be a true symmetry of nature at the quantum gravity scale. Therefore one could conjecture that the dispersion relation of a photon propagating in vacuum will acquire corrective terms,

$$c^2 p^2 = E^2\left[1 + x\,\frac{E}{E_{\text{Planck}}} + O\left(\frac{E^2}{E_{\text{Planck}}^{\;2}}\right)\right]$$

A photon moving with the modified dispersion relation would suffer a time delay,

$$\Delta t \approx x \frac{E}{E_{\text{Planck}}} \frac{L}{c}$$

For a gamma ray like the ones current experiments observe, $E \approx 300 keV$ and assuming that the burst happens happens at $z = 1$ then $L/c \approx 3 \times 10^{17} s$.

If we demand $\Delta t < 0.01 s$

Then $\mathrm{E}_{\text{Planck}} > 10^{16}\,\mathrm{GeV}$

So if the measurement could be refined by three orders of magnitude, we would be probing the real Planck scale at 10^{19}GeV.

How do we know that $\Delta t < 0.01s$?

Evidence for sub-millisecond structure in a γ-ray burst

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GAMMA-RAY bursts (GRBs) vary in duration from hundreds of seconds down to several milliseconds. Early studies[1] suggested that bursts with durations of <100 ms form a distinct class, accounting for a few per cent of the total number of detected bursts, and there is some evidence[2] for a break in the distribution of GRB durations at ~600 ms, perhaps implying separate physical mechanisms for long and short bursts. Recently the estimated number of short GRBs has risen substantially. The shortest burst recorded so far is GRB820405, with duration ~12 ms (ref. 3), and the shortest spike within a burst, an unresolved feature with width <5 ms, was in GRB841215 (refs 4–7). GRB790305 had the shortest rise-time, 0.2 ms. We report here that GRB910711, with apparently the shortest duration (~8 ms) yet seen by the Burst and Transient Source Experiment (BATSE), has a time profile that shows significant submillisecond structure. The responses to this burst in the different BATSE detectors, from both direct and Earth-scattered γ-rays, show that the burst is both narrower and of higher energy than is indicated by a light-curve summed over all detectors. We detected a narrow spike of duration 200 μs in the light curve; variations on this timescale have not previously been observed in GRBs, and their explanation should be a stringent test of any GRB theory.

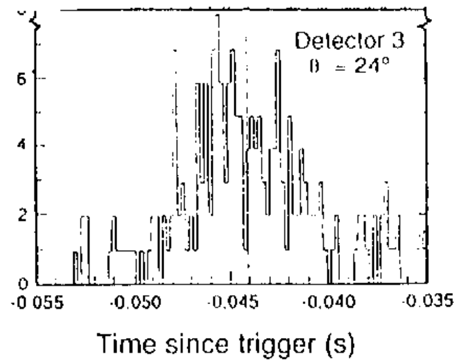

FIG. 1 Time profiles for the different triggered detectors in 200-μs bins for energy $E > 30$ keV. The vertical dashed line shows the position of the centroid for each. The detector numbers are indicated, along with θ, the angle between the detector axis and the Earth's centre. ($\theta = 180°$ corresponds to a detector pointed towards the zenith.) The spike is indicated by the arrow.

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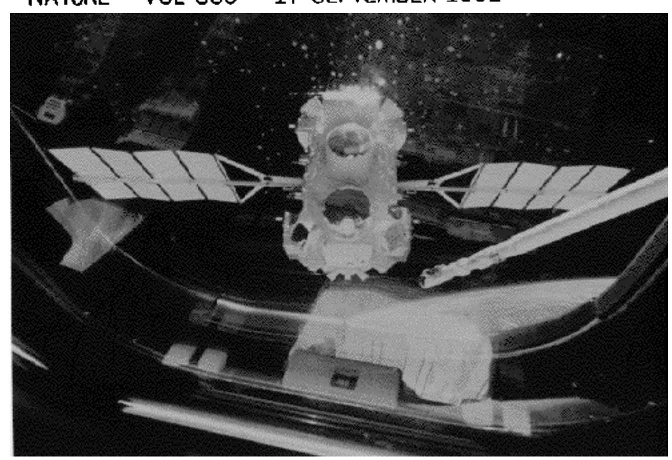


CGRO/
BATSE

The bursts arrive at the same time in all energy channels, 30keV-300keV

But how do we know that the dispersion relation looks like

$$c^2 p^2 = E^2 \left[1 + x \frac{E}{E_{\text{Planck}}} + O\left(\frac{E^2}{E_{\text{Planck}}^2} \right) \right]$$

In particular, what happens if ξ vanishes?

Then we are quite away from the Planck scale..

A quantum gravity model with non-vanishing ξ is provided by the (non-critical) Liouville strings. This was the model put forward by Amelino-Camelia et. al. in their original proposal.

This model does not appear to have a wide following among string theorists.

We took a look at the possibility that a nonvanishing ξ could appear in loop quantum gravity at a **kinematical level.**

R. Gambini, JP PRD59, 124021 (1999)

What one is interested in is in studying the propagation of light in a "semiclassical state". At the moment all of this is very vague since there is not available a rigorous semiclassical picture in this approach. One handwaving picture that has been put forward is the idea of the "weave" in which a loop state with many strands intersecting and knotting approximates a classical geometry in a certain sense (for instance, the area and volume operators have expectation values close to classical values and small dispersions).

$$< \Delta | \hat{\underset{\sim}{g}}_{ab} | \Delta >= \delta_{ab} + O(\frac{\ell_P}{\Delta}),$$

Where Δ is a large characteristic distance.

The idea would therefore be to examine the propagation of light on such a state. One should expect a vast phenomenology akin to propagation of light in solids.

As a first cut for a calculation we consider the term in the Hamiltonian constraint that couples the Maxwell theory to gravity.

$$H = \frac{e_a{}^i e_b{}^i}{\sqrt{\det g}} \left(E^a E^b + B^a B^b \right)$$

Thiemann (Class. Quan. Grav. 15, 1281 (1998)) has proposed a quantization of the Hamiltonian constraint of gravity (including couplings to matter) in the loop representation that is finite and consistent (anomaly-free). Within such framework, the electric term in the Hamiltonian above can **chosen** as,

$$\hat{H} = \int d^3x \int d^3y \, w_a(x) w_b(y) \hat{E}^a(x) \hat{E}^b(y) \, f_e(x-y)$$

Where the w's are well defined operators associated with the geometry that are non-vanishing at intersections of the weave. A similar expression can be written for the magnetic term.

We have also made the assumption, following Thiemann, that quantum gravity acts as a "fundamental regulator" and therefore delocalized the product of electric fields to two different points tied together by the regulator f

We now consider the electric and magnetic fields to be in a coherent states such that their dynamics can be thought of as classical, and evaluate the expectation value of the term in the Hamiltonian on a weave state for the gravitational field. Since the w's are only non-vanishing on the vertices of the weave, one obtains an expression like,

$$< \Delta | \hat{H}^E_{\rm Maxwell} | \Delta > = \frac{1}{2} \sum_{v_i, v_j} < \Delta | \hat{w}_a(v_i) \hat{w}_b(v_j) | \Delta > E^a(v_i) E^b(v_j)$$

Where the sum runs over all vertices.

We now consider a cell of size Δ and assume the electric and magnetic fields are very weakly varying through it. We take advantage of this fact to expand them around a "midpoint" P,

$$E^a(v_i) \sim E^a(P) + (v_i - P)_c \partial^c E^a(P) + \cdots,$$

The vector (v_i-P) is of length Δ, whereas the derivative is of the order of the inverse wavelength of the field, so the term is order Δ/λ.

When summed over the weave the first term just gives back the usual Hamiltonian for the Einstein-Maxwell theory, since $< \Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta >$ just gives back the classical metric to leading order in the weave approximation.

The next term requires evaluating,

$$< \Delta|\hat{w}_a(v_i)\hat{w}_b(v_j)|\Delta > (v_i - P)_c.$$

This quantity adds up to zero in leading order since we assume the weave is isotropic and we are summing a vector. Therefore the term is of order L_{planck}/Δ, since as we make the length larger the more points we include and the more isotropic the weave looks like.

If we are interested in the value of this term, since it has to be a rotationally invariant three index tensor, it has to be of the form,

$$x e_{abc} \frac{\ell_{Planck}}{\Delta}$$

With χ a proportionality factor of order one.

The addition of this term in the interaction Hamiltonian introduces a modification of Maxwell's equations in vacuum,

$$\partial_t \vec{E} = -\nabla \times \vec{B} + 2\chi\ell_P \Delta^2 \vec{B}$$

$$\partial_t \vec{B} = \nabla \times \vec{E} - 2\chi\ell_P \Delta^2 \vec{E}.$$

The terms are not Lorentz invariant, as expected. If one works out the modified wave equation,

$$\partial_t^2 \vec{E} - \Delta^2 \vec{E} - 4\chi\ell_P \Delta^2 (\nabla \times \vec{E})$$

And seeks solutions with a definite helicity

$$\vec{E}_\pm = \mathrm{Re}\left((\hat{e}_1 \pm i\hat{e}_2) e^{i(\Omega_\pm t - \vec{k}\cdot\vec{x})}\right).$$

The dispersion relation implies a birrefringence,

$$\Omega_\pm = \sqrt{k^2 \mp 4\chi\ell_P k^3} \sim |k|(1 \mp 2\chi\ell_P |k|).$$

That is, one helicity propagates faster than the other.

What broke the symmetry? The weave chosen. If we choose a parity-conserving weave then the constant χ vanishes identically and there is no effect.

How do we know if the weave is parity conserving? At the moment we do not. There is no definite dynamical mechanism that could break the invariance. So we do not have a prediction, we rather have a constraint on the quantum states allowed by the theory: they should not break parity at the level constrained by gamma ray burst observations.

A recent paper by Alfaro, Morales and Urrutia gr-qc/9909079 has studied similar calculations for propagation of neutrinos. They find remarkably large effects.

Words of caution:

Should one believe these calculations? A big question mark on them is the issue of the kinematical nature of the calculations.

If one is doing canonical quantum gravity presumably one wishes states that are annihilated by the constraints. The states we are considering are not. The states that are annihilated by the constraints tend to have distinctive properties. For instance, they are diffeomoprhism invariant, therefore one cannot compute operators like q_{ab} on them. That is, calculations like the ones presented simply do not work on states annihilated by the constraints.

Last topic: observables and approximation methods

The observations of the last slide suggest that we should try to make progress in finding observable quantities that allow us to study the true dynamics of quantum gravity.

It is well known that finding observables (quantities that commute with all the constraints) is very hard. In a sense it it tantamount to solving the Einstein equations!

It is also accepted that the problem should be tackled by an approximation scheme.

The problem is finding an approximation scheme that does not destroy the appealing features of the non-perturbative treatment.

Proposal: consider GR with cosmological constant

$$H(N) = H_{\Lambda=0}(N) + \Lambda \int d^3x N(x) \sqrt{\det q(x)}$$

Divide by Lambda (rescale the lapse),

$$H(N) = H^{(0)}(N) + \Lambda^{-1} H^{(1)}(N)$$

Where $H^{(0)}$ is just the volume term and $H^{(1)}$ is the Hamiltonian constraint of usual GR.

The idea is to study the theory perturbatively in inverse powers of Lambda in the limit when Lambda is large.

The zeroth order theory is analogous to the Husain-Kuchar model but with a cosmological constant.

Finding Observables:

At zeroth order, any quantity that commutes with det q and is diffeomorphism invariant is an observable.

Simple example: the volume of the slice.

More interesting quantities can be obtained by coupling the theory to matter in order to build quantitites that are left invariant by the action of the diffeomorphism. Usually when this is done, the quantities fail to have vanishing Poisson brackets with the Hamiltonian constraint.

e.g. Bergmann, Rovelli, Husain, Smolin

In our approach, the matter couplings of the Hamiltonian are suppressed by Lambda, so indeed this technique can be used to generate genuine observables of the zeroth order theory.

Assuming that the observable is expandable in power series,

$$O_\Lambda = O^{(0)} + \Lambda^{-1} O^{(1)} + \dots$$

For it to have vanishing PB| with the constraint, we get the following conditions,

$$\left\{O^{(0)}, H^{(0)}\right\} = 0$$

$$\left\{O^{(1)}, H^{(0)}\right\} + \left\{O^{(0)}, H^{(1)}\right\} = 0$$

Since the zeroth order Hamiltonian is only a function of E, one is left with a **linear** (functional) PDE for the first order correction O^1.

The story repeats at each order in perturbation theory: one always gets a linear PDE with an inhomogeneous term that grows in complexity.

It is therefore relatively easy to generate the successive corrections!

Quantum states:

$$\lambda = \Lambda^{-1}$$

Assume the states are expandable,

$$|\psi(\lambda) >= |\psi >^{(0)} + \lambda|\psi >^{(1)} + \lambda^2|\psi >^{(2)} + \dots$$

You want to find states that are annihilated by your constraint,

$$H(\lambda)|\psi(\lambda) >= 0$$

We will achieve this by first solving,

$$H(\lambda)|\psi(\lambda) >= \epsilon(\lambda))|\psi(\lambda) >$$

And then setting ε(L) to zero. And we will do this to a given order in perturbation theory. Notice that what we are doing is akin to solving a perturbed quantum mechanical system and finding perturbed states of “zero energy”. The detailed calculations look very much like usual degenerate perturbation theory.

As in the case of the observables, solving the perturbative quantum mechanical equations is easy (even, to a certain extent in full 4D quantum general relativity).

Approximation schemes like this could have been introduced 30 years ago. In fact, Bergmann and Newman in 1957 discuss the issue of finding observables and quantum states approximately.

What makes them of interest now is that in the case of quantum canonical general relativity we have the mathematical tools (thanks to the work of Ashtekar, Lewandowski, Thiemann and others) that make the calculations doable **in detail**. Before these developments, one could only hope for formal results.

Immediate questions:

The fact that the scheme seems to make things **so easy**, naturally raises certain level of skepticism:

• Why is this of any interest? The cosmological constant is 10^{-132} in Planck units. Aren't you solving the theory in a completely unphysical domain?

• Don't you get too many observables?

• GR contains situations where there is chaos (Bianchi IX), how could you claim to have observables in such situations?

• What do you mean by approximate quantum states?

• Isn't dealing with a quantum field theory harder than a perturbed quantum mechanical system?

To help in clarifying some of these points, we considered some simplified examples:

- Bianchi models.

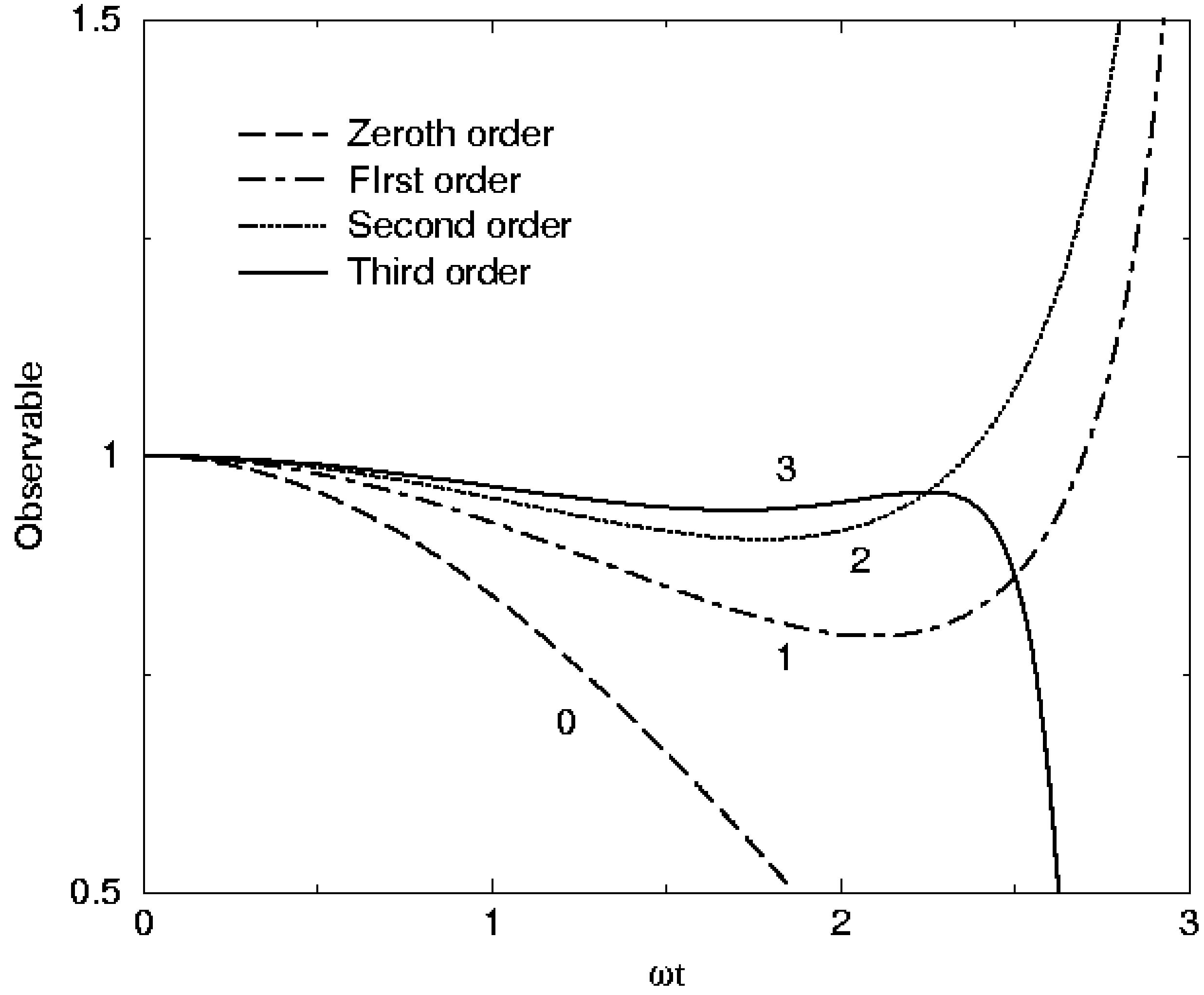
- The heavy symmetric top.

- Pathological coupled harmonic oscillators (Hajicek)

R. Gambini, JP gr-qc/0008031,0008032.

Due to lack of time, I cannot discuss all of these in detail, but let me highlight some aspects of what we learnt.

If one now evaluates the observables on the exact solution, something remarkable occurs: since the solution depends on Lambda, it turns out that the Lambda dependence of the power series actually **drops out**. One has $O=O^0+O^1+O^2+\ldots$ without any information on Lambda!



Rough sketch of how to find states in full 4D quantum GR:

$$\left(< \psi^{(0)}| + \Lambda^{-1} < \psi^{(1)}|\right) \left(\hat{V}(M) + \Lambda^{-1}\hat{H}(M)\right) = \left(< \psi^{(0)}| + \Lambda^{-1} < \psi^{(1)}|\right) \left(\epsilon^{(0)} + \Lambda^{-1}\epsilon^{(1)}\right)$$

At zeroth order: $< \psi^{(0)}|V(M) = \epsilon^{(0)}(M) < \psi^{(0)}|$

V(M) is very closely related to the volume operator. We can then draw on the machinery of Ashtekar, Lewandowski, Thiemann to compute its eigenstates and eigenvalues. The zeroth order state (schematically) is therefore given by,

$$< \psi^{(0)}_{\epsilon^{(0)}(M)}| = \sum_s C(s) < s_{V(M)}|$$

Where the $<s_{V(M)}|$ are spin networks with the same "smeared volume" V(M).

As we did in the case of the oscillators, we now consider the first order portion of the equation and project it on $<s_{V(M)}|$

$$< \psi^{(1)}|\hat{V}(M)|s_{V(M)} > + < \psi^{(0)}_{\epsilon^{(0)}(M)}|\hat{H}(M)|s_{V(M)} >= \epsilon^{(1)}(M) < \psi^{(0)}_{\epsilon^{(0)}(M)}|s_{V(M)} > + \epsilon^{(0)}(M) < \psi^{(1)}|s_{V(M)} >$$

First and last term cancel, therefore one is left with,

$$< \psi^{(0)}_{\epsilon^{(0)}(M)}|\hat{H}(M)|s_{V(M)} >= \epsilon^{(1)}(M) < \psi^{(0)}_{\epsilon^{(0)}(M)}|s_{V(M)} >$$

And one can use this to compute the first order correction to the energy, given a concrete proposal for H(M). From here one could compute the value of the cosmological constant that makes the constraint eigenvalue $\epsilon^{(0)} + \Lambda^{-1}\epsilon^{(1)} = 0$

To obtain the first order correction to the state, we need to project the first order equation on the space of states orthogonal to the zeroth order ones. Naively, we will call these states $|s_{V'(M)}>$, but this shoulnd only be viewed as a "toy calculation". We then get,

$$< \psi^{(1)}|\hat{V}(M)|s_{V'(M)} > + \Lambda^{-1} < \psi^{(0)}_{\epsilon^{(0)}(M)}|\hat{H}(M)|s_{V'(M)} >=$$

$$\epsilon^{(0)}(M) < \psi^{(0)}_{\epsilon^{(0)}(M)}|s_{V'(M)} > + \Lambda^{-1}\epsilon^{(0)}(M) < \psi^{(1)}|s_{V'(M)} >,$$

And since $,\hat{V}(M)|s_{V'(M)} >= \epsilon'^{(0)}(M)s_{V'(M)} >$

$$< \psi^{(1)}|s_{V'(M)} >= \frac{<\psi^{(0)}_{\epsilon^{(0)}(M)}|\hat{H}(M)|s_{V'(M)}>}{\epsilon'^{(0)}(M)-\epsilon^{(0)}(M)}.$$

So, very roughly, if one has a Hamiltonian like Thiemann's, which adds a link at intersections, one sees that the first order corrected state will have an additional link. Higher order states will involve repeated actions of the Hamiltonian, so there will be further extra links at intersections. The repeated action of the Hamiltonian is reminiscent of the "group averaging" method of solution.

So, at this level of roughness, we seem to reproduce the quantum states found by Thiemann.

It should be noted that if one looks at trivalent intersections only the method collapses to requesting that the states be solutions of the Lambda=0 constraint, so one recovers the states found by Thiemann.

We see that a more detailed set of results will require careful evaluation of the operators on at least four-valent states.

Final summary:

- The early rough findings motivated a lot of interest in this approach.
- The subsequent activity has put the field on a very strong mathematical footing.
- We are currently in position to start addressing in a rigorous way physical issues.
- Many current avenues of research: Hamiltonian, approximation methods, semiclassical states, path integrals.