Colliding black holes with perturbation theory

- Setting up the initial data and evolving: the Misner example.
- Other families of data: Brill-Lindquist, Bowen-York, Baker-Puzio-Krivan-Price.
- Results of evolutions.

We need to provide initial data to the Einstein equations. By this we mean a three dimensional metric on a three surface and its time derivative.

The initial value problem in general relativity is not free. For people unfamiliar with this, it is good to examine as an example the case of Maxwell fields. Suppose one wishes to evolve in time an electromagnetic field. As initial data one could set up the electric and magnetic fields and their time derivatives. Unfortunately not any vector field will work as initial data. In vacuum the electric field has to satisfy div E=0 at any time, in particular at the initial time. Therefore the initial data has to satisfy this (linear) equation.

In general relativity the corresponding equations are "G00" and "G0i". These equations only involve the metric and its first time derivatives and therefore constrain the initial data.

The initial value problem for general relativity is non-linear. This is physically understandable. One cannot superpose two non-trivial solutions of general relativity without taking into account "the mutual attraction". This is ubiquitous in the two black hole problem.

What one does normally do for binary situations, like two black holes that will collide? The typical attitude has been to cast the equations in such a way that the equations that govern some of the variables are linear. One then obtains a solution for those equations simply superposing known solutions for individual black holes. One then proceeds to solve the remaining equations in full non-linearity.

It is clear that this procedure may yield appropriate initial data only in certain circumstances. Since one does not have control on the non-linear equations one solves, one in the end is left with "whatever initial data the method provides"! There will never be a way out of this!

The resulting solutions usually resemble two black holes, but there usually is added spurious gravitational radiation. The amount of spurious radiation is “what we need to add in order for some of the variables of the problem to simply superpose linearly”. It is artificial.

For most solutions the amount of spurious radiation introduced decreases if the black holes are far away (the farther the black holes the more natural is to linearly superpose variables).

So in principle a strategy would be to set up initial data with the black holes far away, evolve them for a while allowing the system to “flush itself” of spurious radiation, and then one ends up with two black holes plunging into a collision under more or less realistic circumstances.

The main problem with this strategy is that it is very costly (currently it is actually impossible) to evolve binary black holes long enough to “flush out the radiation”.

In spite of these drawbacks, we will discuss here one of the most popular methods of constructing binary black hole families of initial data, the conformally flat solutions due to York, Bowen, Misner and other collaborators.