Demise of Unruh Radiation

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Unruh radiation has attracted much interest, particularly because of its relationship to Hawking radiation. But its existence has been challenged by a variety of authors, including ourselves [Phys. Lett. A 158,31(1991)]. The absence of a consensus may be traced to the complex challenge of dealing with the infinite number of particles of the associated heat bath. Here we show how this problem may be obviated by using the quantum Langevin equation [Phys. Rev. A 37, 4419(1988), and J. Math. Phys. 6,504 (1965)], which enables us to present a simple argument to show that, despite the existence of an Unruh temperature, there is no Unruh radiation. The key point is that the Unruh system is an equilibrium system, in contrast to the Hawking system which is a non-equilibrium system.

I. Introductio

Following Hawking, who predicted that black holes evaporate, due to quantum particle creation, Unruh[1] used similar techniques to conclude that a system undergoing uniform acceleration with respect to a zero-temperature vacuum will thermalize at a temperature that is proportional to the acceleration, now referred to as the Unruh temperature T= $\frac{\hbar a}{2\pi ck}$ = 4x10^{-23} a where a is acceleration, and that "----a constant outward flux of particles (occurs)". Thus, a thermal effect of just 1K requires an enormous acceleration of 2.4 x 10^{22} $\frac{cm}{s^2}$ .This result, as well as the related work of Davies [2], created a lot of interest because if one chooses a to be the surface of gravity of a blackhole $\frac{GM}{r_g^2} = \frac{c^4}{4GM}$ then one obtains the Hawking temperature [3] for a radiating black hole.

However, Unruh also claimed that his system radiates. Unruh's result attracted much interest and support (see for example [4-7]) but also significant disagreement [8-11]. In [11], we re-analyzed the problem and concluded that the system does not radiate but, despite what we regarded as the exact pedagogical nature of our paper, it is clear that a number of people do not agree with our conclusions [12]. Thus, we are motivated here to present a simple argument which verifies our original conclusion.

A review of all the work on this problem leads one to the conclusion that the source of the complexity and different results stems from the treatment of the infinite number of particles in the heat bath. A similar situation arises in many problems in statistical mechanics and as it turns out, a generic solution is provided by the quantum Langevin equation [13], which can be taken as the basis of the macroscopic description of a quantum particle linearly coupled to a passive heat bath. The presence of an external force is easily incorporated into the analysis leading to an improved Abraham-Lorentz (AL) equation, from which radiation emission can be calculated. Thus, in Sect. II, we present a summary of conventional radiation theory and then, in Sect. III, we present our improved radiation theory which we then apply to the Unruh radiation problem.

II. Conventional Radiation Theory

The conventional theory [14] starts with a charged particle of mass M and charge e acted on by an external force $f(t)$ which results in the particle acquiring a velocity $\vec{v}$ and an acceleration $\dot{\vec{v}}$. Larmor calculated that the particle emits radiation at a rate

$$P_L(t) = \frac{\tau_e}{M} f^2(t), \tag{1}$$

where

$$\tau_e = \frac{2}{3}\frac{e^2}{Mc^3}. \tag{2}$$

To account for the radiation energy loss, Newton's equation of motion was modified by adding a radiative reaction force which Abraham-Lorentz (AL) calculated to give the well-known equation

$$M\dot{\vec{v}} = f(t) + M\tau\ddot{\vec{v}}. \tag{3}$$

However, the presence of the time dependence of acceleration in this equation gives rise to the well-known runaway solutions. This causality problem was resolved in a series of papers [13] which pointed out the necessity of giving structure to the electron. As a result, an exact formula was derived [15] as a replacement for the AL formula (3). This formula will be the basis for our present exposition.

III. Improved Radiation Theory

Confining our attention to one dimension, since an extension to three dimensions is trivial [13], our result for the equation of motion of a quantum particle of bare mass *m* moving in a potential *V*(*x*) and linearly couple to a passive head bath at temperature *T* is in the form of a generalized quantum Langevin equation:

$$\mathrm{m}\ddot{x}(t) + \int_{-\infty}^{t} \mathrm{d}t'\, \mu(t-t')\dot{x} + \dot{V}'(x) = F(t) + f(t)\,, \qquad (4)$$

where $F(t)$ is the operator-valued random (fluctuating) force, $f(t)$ is the external c-number force, $\mu(t)$ is the memory function and where the dot and prime denote, respectively, the derivative with respect to *t* and *x*. As we have discussed in detail, in ref. [13], the coupling to the heat bath is characterized by the fluctuation force F (t) and by the Fourier transform of the memory function, $\tilde{\mu}(\omega)$, with Im $\omega > 0$, and $\tilde{\mu}(\omega)$ is restricted to being a positive real function, i.e. it is analytic in the upper half of the plane, it has a positive real part and it obeys the reality condition $\tilde{\mu}(\omega) = \tilde{\mu}(-\omega)^{*}$.

Eq. (4) provides the foundation for a general treatment of dissipative problems in many branches of physics. For the *particular* case of the blackbody radiation heat bath, J.T. Lewis and us chose an electron form-factor [16], $\Omega^2/(\Omega^2+\omega^2)$, à la Feynman [17], with a shape convenient for calculation but arbitrary in the sense that it depends on the choice of the Ω, a large cut off frequency. This leads to the key result:

$$\mu(t) = M\Omega^2\tau_e[2\delta(t) - \Omega\exp(-\Omega t)], \qquad (5)$$

where *e* and *M* are the charge and renormalized (observed) mass of the particle, respectively. The use of this form factor is universally useful as long as one chooses Ω to be a large cutoff frequency [18].

After some further algebra [15], and in the case V=0, we obtained the result

$$M\frac{d^2x(t)}{dt^2} = f(t) + \tau_e\frac{df(t)}{dt} + G(t), \qquad (6)$$

where $G(t)$ is a function of the fluctuation force $F(t)$ which averages to zero over time and f(t) is an external force. Thus as emphasized by Jackson [Ref. 14, P. 749], this is "--- a valid equation of motion without runaway solutions or acausal behavior --- a sensible alternative to the Abraham-Lorentz equation". It is immediately clear that, for a constant external force, the second term on the right-side of (6) is zero so that no radiation occurs. This result is immediately applicable to the case of a system undergoing uniform acceleration (i.e. $\frac{df(t)}{dt}$=0) with respect to zero-temperature vacuum. It will thermalize at a finite temperature (the Unruh temperature) that is proportional to the acceleration but it does not radiate i.e. there is no Unruh radiation.

As already pointed out in our basic paper [15], a simple derivation of our main result follows from the fact that, to lowest order (in powers of τ_e) we have $M\ddot{x}(t) = f(t)$ which implies that $M\dddot{x}(t) = \dot{f}(t)$ so that equation (3) becomes

$$M\frac{d^2x(t)}{dt^2} = f(t) + \tau_e \frac{df(t)}{dt}. \quad (7).$$

The second term on the right side is clearly zero in the case of uniform acceleration.

Finally, we note that our conclusion is consistent with Feynman's, remark [17] that '-we have inherited a prejudice that an accelerating charge should radiate-the power radiated by an accelerating change [the Larmor formula] has led us astray,' and he then goes on to discuss the limited validity of the Larmor formula and the fact that '-it does not suffice to tell us' when, 'the energy is radiated'.

As a final remark, we point out that our result is consistent with general principles. First, we note that there is general agreement that the oscillator when observed at a point fixed in the accelerating system comes to equilibrium at the Unruh temperature. The question is whether the oscillator, being in an excited state, will decay emitting Unruh radiation. The answer is no because an equilibrium state is always a state of rest. Of course, there will always be microscopic fluctuations about the mean but the principle of detailed balance tells up that any emission of energy into the bath is exactly balanced by energy absorbed from the bath. An equilibrium state can change only if the external constraints, in this case the acceleration, change.

Our key conclusion is that whereas the system examined by Hawking is an non-equilibrium state (essentially caused by the gravitational field), Unruh's system (which does not contain gravity but only a constant accelerating force) is an equilibrium state.

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V. References

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