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BLACKBODY RADIATION: ROSETTA STONE OF HEAT BATH MODELS*

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The radiation field can be regarded as a collection of independent harmonic oscillators and, as such, constitutes a heat bath. Moreover, the known form of its interaction with charged particles provides a "rosetta stone" for deciding on and interpreting the correct interaction for the more general case of a quantum particle in an external potential and coupled to an arbitrary heat bath. In particular, combining QED with the machinery of stochastic physics, enables the usual scope of applications to be widened. We discuss blackbody radiation effects on: the equation of motion of a radiating electron (obtaining an equation of motion which is free from runaway solutions), anomalous diffusion, the spreading of a Gaussian wave packet, and decoherence effects due to zero-point oscillations. In addition, utilizing a formula we obtained for the free energy of an oscillator in a heat bath, enables us to determine all the quantum thermodynamic functions of interest (particularly in the areas of quantum information and nanophysics where small systems are involved) and from which we obtain temperature dependent Lamb shifts, quantum effects on the entropy at low temperature and implications for Nernst's law.

Keywords: Blackbody radiation; noise; runaway solutions; decoherence.

1. Introduction

The problem of a quantum particle coupled to a quantum-mechanical heat bath is fundamental to many fields of physics: statistical mechanics, condensed matter, quantum optics, atomic physics, etc. The question often arises as to the proper choice for the interaction term in the associated Hamiltonian. To be specific, consider a quantum particle with mass m in a potential $V(x)$ interacting with a heat bath consisting of an infinite number of oscillators, where m_j and ω_j refer to the mass and frequency of heat-bath oscillator j. In addition, x and p are the coordinate and momentum operators for the quantum particle and q_j and p_j are the correspond-

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ing quantities for the heat-bath oscillators. Also $f(t)$ is a c-number external force. The infinity of choices for the m_j and ω_j give this model its great generality. In the literature, one encounters a variety of choices for the interaction between the quantum particle and the heat-bath: xq_j, pq_j, xp_j, pp_j and $(ab_j^+ + a^+b_j)$, where a and b_j are the annihilation operators associated with the quantum particle and heat-bath oscillator, respectively. However, it is well-known that the blackbody radiation field (BBR) can be regarded as a collection of independent harmonic oscillators [1]. Moreover, there is a universally accepted Hamiltonian for an electron charge $-e$ interacting with the electromagnetic field in the dipole approximation which we generalized to incorporate electron structure [2, 3], by including an electron from factor (which is the Fourier transform of the electron charge distribution [4]) in the vector potential. This enabled us to obtain the exact equation of motion for the coordinate operator x in the form of a quantum Langevin equation (QLE), the solution of which was readily obtained not only for a free particle but also for an oscillator in the radiation heat bath. The solution was in terms of the generalized susceptibility $\alpha(\omega)$ for which an explicit expression was obtained. Next, using the fluctuation-dissipation therorem, the auto-correlation functions for the coordinates and the fluctuation force immediately followed.

Section 2 is devoted to fundamentals. We review our derivation of the QLE and next point out that the results for the BBR are a particular case for interaction with an arbitrary heat-bath, specifically pointing out the restrictions on the allowed forms of the interaction Hamiltonian. Section 3 discusses the solution to the problem of runaway solutions and the role played by causality considerations, which in turn we made recent use of in a discussion of the cosmological constant problem. Section 4 discusses anomalous diffusion in a BBR as well as wave packet spreading, leading in turn to an investigation of possible decoherence effects due to a BBR. Section 5 is devoted to a broad discussion of quantum thermodynamic effects, all based on our exact result for the free energy of an oscillator in an arbitrary heat bath in terms of $\alpha(\omega)$. In particular, we discuss BBR effects on atomic energy shifts, entropy and other thermodynamic functions including magnetic moments and finally, the role of quantum effects on the fundamental laws of thermodynamics.

2. Fundamentals

First, we review some basic results obtained in [2] and [3]. An electron of charge $-e$ interacting with the radiation field in the dipole approximation corresponds to the Hamiltonian [1]

$$H_{QED} = \frac{1}{2m}\left[\mathbf{p} + \frac{e}{c}\mathbf{A}\right]^2 + V(r) + \sum_{k,s}\hbar\omega_k\left(a^\dagger_{\mathbf{k},s}a_{\mathbf{k},s} + \frac{1}{2}\right), \tag{1}$$

where the vector potential, generalized to include a form factor, is given by

$$\mathbf{A} = \sum_{\mathbf{k},s}\left[\frac{2\pi\hbar c^2}{\omega_k V}\right]^{1/2} f_k \hat{e}_{\mathbf{k},s}(a_{\mathbf{k},s} + a^\dagger_{\mathbf{k},s}). \tag{2}$$

The quantity f_k is the electron form factor (Fourier transform of the electron charge distribution). Without loss of generality, we have taken the form factor as well as the polarization vector $\hat{e}_{\mathbf{k},s}$ to be real. The form factor, which is sometimes called a cutoff factor, must have the property that it is unity up to some large cutoff frequency Ω after which it falls to zero.

Introducing

$$m_k = \frac{4\pi e^2 f_k^2}{\omega_k^2 V}, \tag{3}$$

and

$$a_{\mathbf{k},s} = \frac{m_k \omega_{\mathbf{k}} q_{\mathbf{k},s} + i p_{\mathbf{k},s}}{\sqrt{2 m_k \hbar \omega_k}}, \tag{4}$$

the Hamiltonian (1) then can be written

$$\begin{aligned} H_{QED} = {} & \frac{1}{2m}\left[\mathbf{p} + \sum_{\mathbf{k},s} m_k \omega_k q_{\mathbf{k},s} \hat{e}_{\mathbf{k},s}\right]^2 + V(r) \\ & + \sum_{\mathbf{k},s}\left[\frac{1}{2m} p_{\mathbf{k},s}^2 + \frac{1}{2} m_k \omega_k^2 q_{\mathbf{k},s}^2\right]^2 . \end{aligned} \tag{5}$$

This form is referred to as the velocity-coupling model, in which the coupling is through the particle momentum interacting with the coordinates of the heat bath. Each one-dimensional component of this equation was shown [3] to be unitarily equivalent to what we referred to as the independent (IO) model, given by

$$H = \frac{p^2}{2m} + V(x) + \sum_j \left[\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 (q_j - x)^2\right]. \tag{6}$$

In fact, the IO model is a very general model of a heat bath, a particular case of H being the H_{QED}, which is our focus in this paper. We note the interaction term appears as part of a squared term as in (1), in contrast to the many linear coupling models appearing in the literature. As emphasized in [3], this is necessary to ensure that the Hamiltonian has a lower bound. Explicitly, if we require that H is a minimum with respect to q_j and x, we find that $\frac{\partial H}{\partial x} = \frac{\partial V}{\partial x}$, which does not occur in linear coupling models (where the flaw is sometimes repaired [5] by adding a "counter term" at a later stage but this only works if V is an oscillator potential). Next, starting with either (5) or (6), and using the Heisenberg equation of motion, enabled us to write down the equation of motion of an electron with charge e and bare mass m, dipole interacting with the electromagnetic field and moving in a potential $V(r)$, in the form of a generalized quantum Langevin equation [2,3] (QLE)

$$m\ddot{x}(t) + \int_{-\infty}^{t} dt' \mu(t - t')\dot{x}(t') + V'(x) = F(t) + f(t), \tag{7}$$

where m is the bare mass, $x(t)$ is the coordinate operator, $F(t)$ is the operator-valued random (fluctuating) force, $f(t)$ is the external force, $\mu(t)$ is the memory function and where the dot and prime denote, respectively, the derivative with respect to t and x. This is an <u>exact</u> result and explicit values are known for $\mu(t)$ and $F(t)$ in terms of the parameters of the heat bath (in this case the radiation field). Equation

(7) is extremely general in that the choice of the electron charge distribution has not been specified. In particular, specific choices for $\mu(t)$ lead to a variety of equations of motion. Furthermore, the corresponding classical equation has exactly the same form as (1) but now the dynamical variables are all classical quantities. It is also of interest to note that not only are the Hamiltonians given in (1) and (6) unitarily equivalent but also they are unitarily equivalent to a Hamiltonian in which the couplying is via interaction between the particle coordinate and the momenta of the heat bath [3]. Thus, all of these models are exactly solvable and lead to the same physical results. On the other hand, linear coupling models are not only defective per se (in that the associated Hamiltonians do not have a lower bound) but, despite the fact that they are obtained from the correct models by dropping terms, in general they are not solvable exactly. For example, in quantum optics, the interaction of a single-model field with a reservoir [6] is generally written in the form $(ab_j^+ + a^+b_j)$, the oft-used rotating-qave approximation (RWA), but it is not amenable to an exact solution, necessitating the use of the Weisskoff-Wigner approximation. Whereas these approximations lead to satisfactory results in many types of problems, this is not universal; for example, they lead to incorrect results, an emphasized in [7], for the power spectrum of the reservoir (obtained from auto-correlation functions for the fluctuation forces), Lamb shifts and mass renormalization. By contrast, H_{QED} contains terms ab_j and $a^+b_j^+$ terms in addition to the RWA terms but when it is written in terms of coordinate and momentum operators it displays a remarkably simple form [see, in particular the IO form given in (6) above], which leads itself to exact analysis. Finally, we stress that (6) gives H for a quantum particle in an arbitrary heat bath and in an arbitrary potential. Here, we concentrate on the particular case of an oscillator potential and a radiation heat bath, for which the corresponding values of $\mu(t)$ and $F(t)$, appearing in (7), are known.

3. Solution to the Problem of Runaway Solutions

To proceed further, it is necessary to choose a particular form factor (but, it can be shown that our results are correct to order τ_e for all reasonable choices) and we chose an electron form-factor, $\Omega^2/(\Omega^2+\omega^2)$, with a shape convenient for calculation but arbitrary in the sense that it depends on the choice of Ω, a large cut-off frequency [2, 3]. Also, as is generally the case in quantum field theory, mass renormalization is also required and we found that the renormalized mass M is given in terms of the bare mass m by the relation

$$M = m + 2e^2\Omega/3c^3 = m + \tau_e\Omega M, \tag{8}$$

where

$$\tau_e = \frac{2}{3}\,\frac{e^2}{Mc^3} = 6\times 10^{-24}s. \tag{9}$$

Based on analyticity arguments, we argued that

$$\Omega \le \tau_e^{-1}, \tag{10}$$

which from (2) implies that $m > 0$, which is a physically appealing conclusion regarding the bare mass. The choice $\Omega = \tau_e^{-1}$ corresponds to $m = 0$ and describes an electron with the smallest size consistent with causality [8]. The size is of the order of the classical electron radius [9], underlining the fact that the dipole interaction appears to be more than adequate to describe the basic physics. This choice gives

$$f_k^2 = \frac{1}{1+\omega^2\tau_e^2}, \tag{11}$$

with the result that the exact equation of motion reduces to [8]

$$M\ddot{x}(t) + V'_{eff}(x) = F_{eff}(t) + f_{eff}(t), \tag{12}$$

where now

$$f_{eff}(t) \equiv f(t) + \tau_e \dot{f}(t), \tag{13}$$

and similarly for the other "effective" quantities.

We emphasize that the simple result (12) is based on the choice $m = 0$. More generally for the whole range of allowed m and Ω values ($0 \leq m \leq M$, corresponding to $0 < \Omega \leq \tau_e^{-1}$) and for other sensible choices of electron structure, we obtain [8,10]

$$M\ddot{x}(t) = f(t) + \tau_e \dot{f}(t) + O(\tau_e^2). \tag{14}$$

Thus, for the whole range of allowed m and Ω values, (12) is an excellent approximation, being exact for the choice $m = 0(\Omega = \tau_e^{-1})$. As an aside, we mention that using the causality restriction on Ω give above in summing the zero-point energies of fluctuating quantum field in the universe (instead of the much larger value based on the Planck scale), for the estimate of the cosmological coupling constant Λ, led to a much smaller value for Λ [11].

4. Anomalous Diffusion, Wave-Packet Spreading and Decoherence

For a Brownian particle (a free particle in a heat bath), the quantites of interest in this discussion [12] are the mean-square displacement

$$s(t) = \left\langle \{x(t) - x(0)\}^2 \right\rangle, \tag{15}$$

the commutator $[x(t_1), x(t_1+t)]$ and the variance of a single wavepacket, which in general is given by

$$w^2(t) = \sigma^2 - \frac{[x(0), x(t)]^2}{4\sigma^2} + s(t), \tag{16}$$

where σ^2 is the initial variance. These quantities are evaluated by use of the quantum Langevin equation (7), leading to the results [12]

$$s(t) = \frac{2\hbar}{\pi}\int_0^\infty d\omega \mathrm{Im}\{\alpha(\omega + i0^+)\} \coth\frac{\hbar\omega}{2kT}(1-\cos\omega t), \tag{17}$$

and

$$[x(t_1), x(t_1+t)] = \frac{2i\hbar}{\pi}\int_0^\infty d\omega \mathrm{Im}\{\alpha(\omega + i0^+)\}\sin\omega t, \tag{18}$$

where the response function (generalized susceptibility) for a free particle has the general form

$$\alpha(z) = \frac{1}{-mz^2 - iz\tilde{\mu}(z)}, \tag{19}$$

with

$$\tilde{\mu}(z) = \int_0^\infty dt \mu(t) e^{izt}, \qquad \text{Im}\{z\} > 0, \tag{20}$$

where $\tilde{\mu}(z)$ is the Fourier transform of the memory function $\mu(t)$ appearing in the quantum Langevin equation. These expressions are valid for arbitrary temperature and arbitrary dissipation.

In the case of a nonrelativistic electron coupled to the radiation field, the Fourier transform of the memory function in QED can be written in the form: [2, 3]

$$\tilde{\mu}(z) = \frac{2e^2}{3c^3} \frac{z\Omega^2}{z + i\Omega}, \tag{21}$$

where Ω is the large cut-off frequency, given in (8). With this (18) becomes [13]

$$\alpha(z) = -\frac{1}{mz^2} \quad \frac{z + i\Omega}{z + i\Omega'}. \tag{22}$$

where

$$\Omega' \equiv \frac{M - m}{m\tau_e} = \frac{M}{m}\ \Omega, \tag{23}$$

from which we see that

$$\infty \geq \Omega' \geq \Omega \geq 0. \tag{24}$$

Thus, we obtain [13]

$$s(t) = \frac{2\hbar\tau_e}{\pi M} V(\Omega' t), \tag{25}$$

where

$$V(z) = \int_0^\infty dy\ \frac{z^2(1 - \cos y)}{y(y^2 + z^2)}, \tag{26}$$

and, in addition

$$[x(0), x(t)] = \frac{i\hbar}{M}\{t + \tau_e(1 - e^{-\Omega' t})\}. \tag{27}$$

Using these results in (16), taking the temperature $T = 0$ and also taking $t >> \tau_e$, leads to

$$\begin{aligned} w^2(t) &= \sigma^2 + \frac{\hbar^2 t^2}{4M^2\sigma^2} + s(t) \\ &= \sigma^2 + \frac{\hbar^2 t^2}{4M^2\sigma^2} + \frac{2\hbar\tau_e}{\pi M}\{\log \Omega' t + \gamma_E\}. \end{aligned} \tag{28}$$

Some comments are now in order. First, we note that $s(t)$, for long times, given by the third term on the right side of (27), depends logarithmically on time, in contrast to normal (Einstein diffusion) which corresponds to a linear time dependence for long times and vanishes for $T = 0$. In the present case, we have obtained a finite result at $T = 0$, which is a quantum phenomena (noting that $s(t) \sim \hbar$). Secondly, we note from (22) that Ω' depends on m, the bare mass. This is a rather striking conclusion since all the well-known QED phenomena do not involve m whereas here it is manifest in diffusion and wave packet spreading. Thirdly, applying these results to a calculation of the decoherence of a free electron moving in one dimension that is placed in an initial superposition state ("Schrödinger cat" state) corresponding to a pair of Gaussian wave packets, we find [13] that, for $m = 0$ we obtain almost infinite spreading of the initial wave packets (which is clearly ruled out by experiments) whereas for $m \neq 0$ the electron essentially behaves as a free electron subject only to the "uncertainty principle" spreading.

5. Quantum Thermodynamic Effects

There has been a lot of recent interest in this general area, including four international conferences. Here, I will confine myself to a brief review of our activities in this area. The foundation for our work goes back to our publication [2] where we considered the system of an oscillator coupled to a heat bath in thermal equilibrium at temperature T, which has a well-defined free energy. The free energy ascribed to the oscillator, $F(T)$, is given by the free energy of the system minus the free energy of the heat bath in the absence of the oscillator, leading to the result [2]

$$F(T) = \frac{1}{\pi}\int_0^{\infty} d\omega f(\omega, T)\mathrm{Im}\left\{\frac{d\ \log\alpha(\omega + i0^+)}{d\omega}\right\}, \tag{29}$$

where $f(\omega, T)$ is the free energy of a single oscillator of frequency ω, given by

$$f(\omega, T) = kT\log[1 - \exp(-\hbar\omega/kT)]. \tag{30}$$

Here the zero-point contribution ($\hbar\omega/2$) has been omitted. Next, we argued that, for atoms coupled to a heat bath in thermal equilibrium at temperature T, the quantity actually measured by experiment is the free energy (as distinct from the energy). Then, using (29) in the high temperature limit, we obtained a result for the shift in free energy, in agreement with experiment [14]. Later, we used (29) to examine various claims in the literature that quantum effects lead to violations of the second and third laws of thermodynamics. In the latter case, we showed, by using F to calculate the entropy, that Nernst's Third Law is valid even when quantum effects are taken into account [15, 16]. In the former case, we examined the case of an atom in contact with a zero-temperature heat bath. The atom is necessarily in an excited state and there was speculation that this could be used to extract work form the bath. However, we pointed out that it takes work to couple the atom to the bath and that this work must exceed that obtained from the atom [17]. This led us to the conclusion that there is no violation of the Second Law due to quantum effects.

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