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Free Electron Motion in an Electromagnetic Field at Zero Temperature and the Dependence on Its Rest Mass

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Abstract—Efforts to achieve quantum computation, teleportation, and communication depend on minimizing decoherence, which is the destruction of a quantum interference pattern. Here, we examine effects arising from the universal zero-point (temperature $T = 0$) oscillations of the electromagnetic field on a free electron in a Schrödinger cat superposition state. A unique conclusion is that the spreading of an electron wavepacket and the rate of decay of decoherence depend on the bare mass m of the electron. However, only for $m = 0$ does decoherence occur and the fact that it occurs almost instantly is ruled out by electron interference experiments. For $m \neq 0$, the electron essentially behaves as a free particle.

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A quantum particle in a superposition (Schrödinger cat) state is subject to decoherence in the presence of an environment and the decay rate, τ_d say, of decoherence depends on whether the particle and the environment are not coupled initially [1, 2] or entangled for all time [3]. Efforts to tackle the $T = 0$ case are, thus, doomed to failure if one employs methods such as master equations [2] which assume initial decoupling of the particle and environment. The reason is that the zero-point oscillations have been entangled with all charged particles for all time, a characteristic feature of Dirac's groundbreaking paper [4] on the quantization of the electromagnetic field. Thus, we use what we believe is the simplest method introduced so far [3] to analyze entanglement for all time. In [3], we presented general results for all T and arbitrary environments, but confined the application of same to the case of high T and the oft-used ohmic environment. A further application considered the case of $T = 0$, but again for an Ohmic environment [5] in which case τ_d depended on a dissipative parameter characterizing the nature of the *specific* reservoir being studied. Here, we again consider $T = 0$, but choose the *universal* blackbody radiation heat bath as the source of decoherence.

As in our previous discussions [3], we consider decoherence in terms of the simple problem of a free particle moving in one dimension that is placed in an initial superposition state ("Schrödinger cat" state) corresponding to a pair of Gaussian wavepackets, each with variance σ^2 and separated by a distance $d \gg \sigma$. We consider the particle to be coupled to an arbitrary reservoir such that in the distant past the complete system is in thermal equilibrium at temperature T. For such a state, the probability distribution at time t can be shown to be of the form [3]

$$P(x, t) = \frac{1}{2(1+e^{-d^2/8\sigma^2})}\left\{P_0\left(x-\frac{d}{2}, t\right) + P_0\left(x+\frac{d}{2}, t\right) + 2e^{-d^2/8\omega^2(t)}a(t)P_0(x, t)\cos\frac{[x(0), x(t)]xd}{4i\sigma^2 w^2(t)}\right\} \equiv P_1 + P_2 + 2P_I\cos\theta(t), \tag{1}$$

where P_0 is the probability distribution for a single wavepacket, given by

$$P_0(x, t) = \frac{1}{\sqrt{2\pi w^2(t)}}\exp\left\{-\frac{x^2}{2w^2(t)}\right\}. \tag{2}$$

Here and in (1), $w^2(t)$ is the variance of a single wavepacket, which in general is given by

$$w^2(t) = \sigma^2 - \frac{[x(0), x(t)]^2}{4\sigma^2} + s(t), \tag{3}$$

where σ^2 is the initial variance, $[x(0), x(t)]$ is the commutator, and

$$s(t) = \langle\{x(t) - x(0)\}^2\rangle \tag{4}$$

is the mean square displacement. The temperature dependence enters only in $s(t)$. Also, the attention factor $a(t)$, which can be defined as the ratio of the factor multiplying the cosine in the interference term to twice the geometric mean of the first two terms [3], is given by the following exact general formula [3]

$$a(t) = \exp\left\{-\frac{s(t)d^2}{8\sigma^2 w^2(t)}\right\}. \quad (5)$$

We note that our definition of $a(t)$ corresponds to what Mandel and Wolf call "a true measure of the 'sharpness' of the interference effects" [6]. However, there are other possible measures, such as the concept of "visibility" V, originally introduced by Michelson [7], and used by Zernike [8] and many others. In fact, V is proportional to $a(t)$ and these quantities are actually equal when $P_1 = P_2$ [9], as in our case.

The quantities appearing in (3) and (4) are evaluated by use of the quantum Langevin equation [10], leading to the results [3]

$$s(t) = \frac{2\hbar}{\pi}\int_0^\infty d\omega \operatorname{Im}\{\alpha(\omega + i0^+)\} \times \coth\frac{\hbar\omega}{2kT}(1-\cos\omega t) \quad (6)$$

and

$$[x(t_1), x(t_1+t)] = \frac{2i\hbar}{\pi}\int_0^\infty d\omega \operatorname{Im}\{\alpha(\omega + i0^+)\}\sin\omega t, \quad (7)$$

where the response function for a free particle has the general form

$$\alpha(z) = \frac{1}{-mz^2 - iz\tilde{\mu}(z)}, \quad (8)$$

with

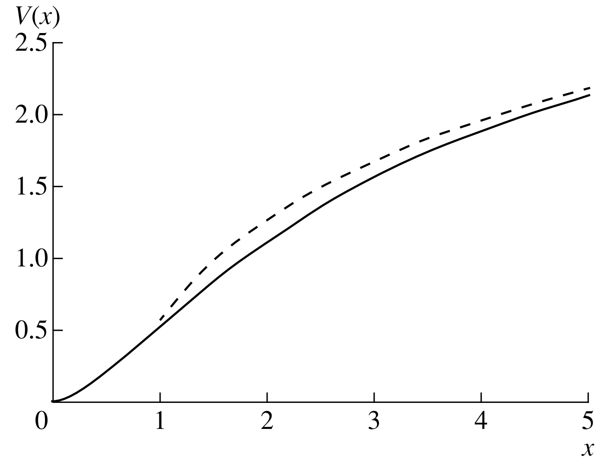
$$\tilde{\mu}(z) = \int_0^\infty dt \mu(t) e^{izt}, \quad \operatorname{Im}\{z\} > 0, \quad (9)$$

where $\tilde{\mu}(z)$ is the Fourier transform of the memory function $\mu(t)$ appearing in the quantum Langevin equation. These expressions are valid for arbitrary temperature and arbitrary dissipation.

In the case of a nonrelativistic electron coupled to the radiation field, the Fourier transform of the memory function in QED can be written in the form [10, 11]

$$\tilde{\mu}(z) = \frac{2e^2}{3c^3}\frac{z\Omega^2}{z+i\Omega}, \quad (10)$$

where Ω is a large cutoff frequency related to the electron form factor. With this, the form (8) for the generalized susceptibility for a free particle becomes [12]



$V(x)$, as defined in Eq. (20), is represented by the solid line. The dashed curve represents the asymptotic value, as given by Eqs. (19) and (22).

$$\alpha(z) = \frac{z+i\Omega}{-mz^3 - iM\Omega z^2}, \quad (11)$$

where m is the bare mass and

$$M = m + \frac{2e^2\Omega}{3c^3} \quad (12)$$

is the renormalized (physical) mass. Thus,

$$\Omega = \frac{M-m}{M\tau_e}, \quad 0 \le m \le M, \quad (13)$$

where

$$\tau_e = \frac{2e^2}{3Mc^3} \cong 6.25 \times 10^{-24}\ \text{s}. \quad (14)$$

From (13), we note that m can be viewed as a measure of the strength of coupling, with $0 \le m \le M$. The limit $m = M$ corresponds to no interaction (i.e., free particle), and the limit $m = 0$ corresponds to maximal coupling where the cutoff has its largest value consistent with causality ($\Omega = \tau_e^{-1}$).

It is also convenient to introduce the parameter

$$\Omega' \equiv \frac{M-m}{m\tau_e} = \frac{M}{m}\Omega, \quad (15)$$

from which we see that

$$\infty \ge \Omega' \ge \Omega \ge 0. \quad (16)$$

Also, it follows that (11) can be written in the form

$$\alpha(z) = -\frac{1}{mz^2}\frac{z+i\Omega}{z+i\Omega'}. \quad (17)$$

Hence, we obtain [12]

$$\mathrm{Im}\{\alpha(\omega+i0^+)\} = -\frac{\pi}{M}\delta'(\omega)+\frac{\tau_e}{M\omega}\frac{\Omega'^2}{\Omega'^2+\omega^2}, \quad (18)$$

where δ' is the derivative of the delta function. Thus, substituting (18) in (6) and taking $T = 0$ leads to the result

$$s(t) = \frac{2\hbar\tau_e}{\pi M}V(\Omega' t), \quad (19)$$

where

$$V(z) = \int_0^\infty dy \frac{z^2(1-\cos y)}{y(y^2+z^2)}. \quad (20)$$

A plot of $V(x)$ is given in Fig. 1. The dashed curve is the asymptotic form [see (22) below] and we note that it is a good representation for values of x of order unity. We next consider the commutator, given by (7), which we note is temperature independent. Hence, using (18) in (7), we obtain [12]

$$[x(0), x(t)] = \frac{i\hbar}{M}\{t+\tau_e(1-e^{-\Omega' t})\}. \quad (21)$$

These results all depend crucially on the bare mass m. Thus, we now turn to some speculations as to the magnitude of (m/M) and related implications. From (15), we see that large m corresponds to small Ω'. However, the extreme case $m = M$ is of little interest since it corresponds to no interaction. On the other hand, if we take $m = (1 - \lambda)M$, where $\lambda \ll 1$, then $\Omega' > \lambda\tau_e^{-1} = (1.6 \times 10^{23}\lambda)\ s^{-1}$. With respect to how large m is compared to M, as Feynman [13], among others, has noted, it "cannot be determined theoretically." However, taking into account retardation and relativistic effects, most estimates conclude that $\lambda \approx \alpha$ [14], where α is the fine-structure constant. Hence, except for very short times, we can take $\Omega' t \gg 1$. This leads to the results [12]

$$s(t) = \frac{2\hbar\tau_e}{\pi M}\{\log\Omega' t+\gamma_E\} \quad (22)$$

and

$$[x(0), x(t)] = \frac{i\hbar}{M}(t+\tau_e). \quad (23)$$

Next, using (22) and (23) in (3), for $t \gg \tau_e$, gives

$$\begin{aligned}\omega^2(t) &= \sigma^2+\frac{\hbar^2t^2}{4M^2\sigma^2}+s(t)\\ &= \sigma^2+\frac{\hbar^2t^2}{4M^2\sigma^2}+\frac{2\hbar\tau_e}{\pi M}[\log\Omega' t+\gamma_E].\end{aligned} \quad (24)$$

Since Ω' depends on m, it is clear that $s(t)$ and $\omega^2(t)$ also depends on the bare mass m. This is quite unique since all the well-known QED phenomena do not involve m, whereas here it is manifest in wavepacket spreading and decoherence.

Noting that $s(t)$ and hence $\omega^2(t)$ increases logarithmically with Ω', we see that the choice $m = 0$ ($\Omega' = \infty$) leads to an almost instant infinite spreading of the initial wavepackets, and consequently

$$a(t) = \exp\left\{-\frac{d^2}{8\sigma^2}\right\}, \quad (25)$$

which is independent of t. (In fact, it is worth remarking that (25) is actually a model-independent result whenever the mean square displacement $s(t)$ dominates in expression (3) for the variance.) This behavior is clearly ruled out by electron interference experiments [15]. On the other hand, for $m \neq 0$, it is clear that the second term in (24), the "uncertainty principle spreading" term [12], dominates and the electron essentially behaves as a free particle for which $a(t) = 1$.

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