Student Name: Answer Key **Student No:**____________________

Louisiana State University Physics 2102, Exam 3,
6:00PM Thursday April 15, 2010.

Please, circle your section:

1 & 6 (Giammanco)

2 (Vekhter)

3 (Rupnik)

4 (Dowling)

5 (Rupnik)

• Please be sure to write your name and student number and circle your instructor above.

• Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct.

• On problems that are not multiple choice, be sure to show all of your work since *no credit will be given for an answer without explanation or work.* These problems will be graded in full, and you are expected to show all relevant steps that lead to your answer. Note that you can often do parts (b) or (c) even if you get stuck on part (a).

• You may use scientific or graphing calculators but you must derive and explain your answer fully on paper so we can grade your work.

• Please be sure that *all* numerical quantities include appropriate units. Points will be deducted if the units are absent. In addition to magnitude, vector quantities must contain sufficient information to tell us which way the vector points.

• Feel free to detach, use, and keep the formula sheet pages. No other reference material is allowed during the exam.

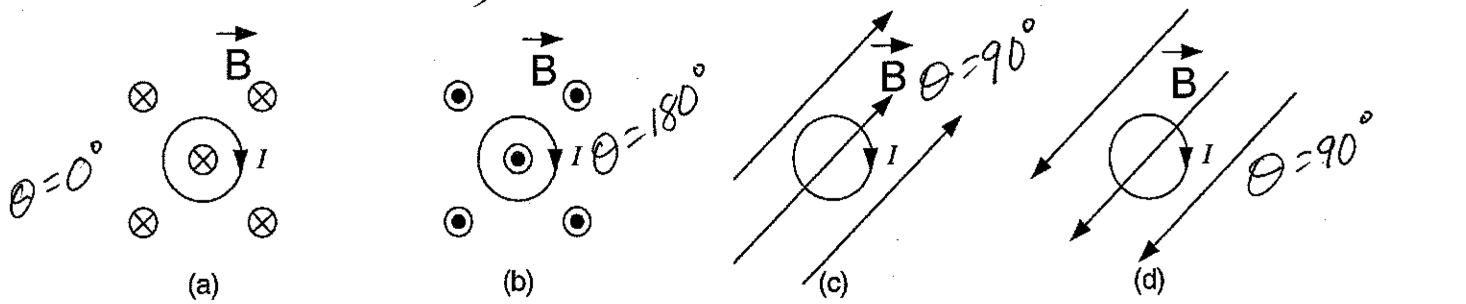
• The only electronic devices to be used during the exam are standard or graphing calculators. All cell phones should be turned off and put away. Cell phones are not to be used as calculators. Any cell phone heard or seen during the exam will be confiscated.

• **May the Force be With You!**

Question 1 [8 points] Taken from a practice exam

The figure below depicts four identical wire loops lying in the plane of this paper, each carrying identical clockwise currents, I, as shown. Each loop is located in a region of uniform external magnetic field, $\boldsymbol{B}$, in the directions indicated. The magnitude of $\boldsymbol{B}$ is the same in each case, only the directions differ.

Since I is CW, take the normal to the loop as <u>into</u> <u>page</u>



θ is angle between $\vec{B}$ and normal to loop's plane.

i) Which loop(s) experience the greatest magnitude of torque due to the external magnetic field, B?

3 pts

(a) (b) (c) (d)

$\vec{\tau} = \vec{\mu} \times \vec{B} = [\mu B \sin\theta]\hat{u}$ so max torque when $\theta = 90° \Rightarrow$ (c) and (d)

ii) Which loop(s) experience zero torque due to the external magnetic field, B?

3 pts

(a) (b) (c) (d)

when $\theta = 0°$ $\vec{\mu} \times \vec{B} = 0 \Rightarrow$ (a) and (b)

2 pts

iii) Which loop(s) have the least potential energy due to their orientation in the external magnetic field, B?

Note that a negative potential energy would be considered to be smaller than either a positive energy or zero energy.

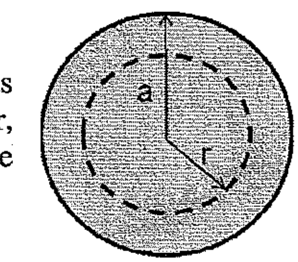
stable equilibrium condition

(a) (b) (c) (d)

<u>minimum</u> U when $\vec{\mu}$ is parallel to $\vec{B} \Rightarrow$ (a)

max U when $\vec{\mu}$ is anti-parallel to B

Problem 1 [17 points] adapted from HW7, #12



The figure shows a cross section across a long solid cylindrical conductor of radius a. The current density in the conductor is directed parallel to the axis of the cylinder, and is dependent upon the radial distance, r, from the center according to the relation

$J = \beta r$ where J is in amperes per square meter, and β is in amperes per cubic meter, and $0 \le r \le a$.

5 pts

i) Obtain an expression, in terms of β and r, for the current flowing through a cylindrical portion of the conductor whose radius, r, is smaller than a.

$$\int \vec{J} \cdot d\vec{A} = I \qquad \underbrace{dA = 2\pi r\,dr \qquad J = \beta r}_{\vec{J} \text{ is parallel to } d\vec{A}}$$

$$\int_0^r (\beta r)(2\pi r\,dr) = 2\pi\beta \int_0^r r^2\,dr \qquad I, \text{ for } 0 \le r \le a, = \frac{2\pi\beta r^3}{3}$$

5 pts

ii) Use Ampere's Law and your expression for the current from part (a) to obtain an expression for the magnitude of the magnetic field, B, at that distance, r, from the center.

Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$, use I from above

$$\oint \vec{B} \cdot d\vec{s} = B(\cancel{2\pi r}) = \mu_0 \left(\frac{\cancel{2\pi} \beta r^{\cancel{3}\,2}}{3}\right) \Rightarrow B = \frac{\mu_0 \beta r^2}{3} = \left(\frac{\mu_0 \beta}{3}\right) r^2$$

B increases as r^2 for $0 \le r \le a$

5 pts

iii) Use Ampere's Law to obtain an expression, now in terms of β, a, and r, for the magnitude of the magnetic field at distances, r, from the center that are larger than a.

for $r > a$, $I = 2\pi\beta \int_0^a r^2\,dr = \dfrac{2\pi\beta a^3}{3}$

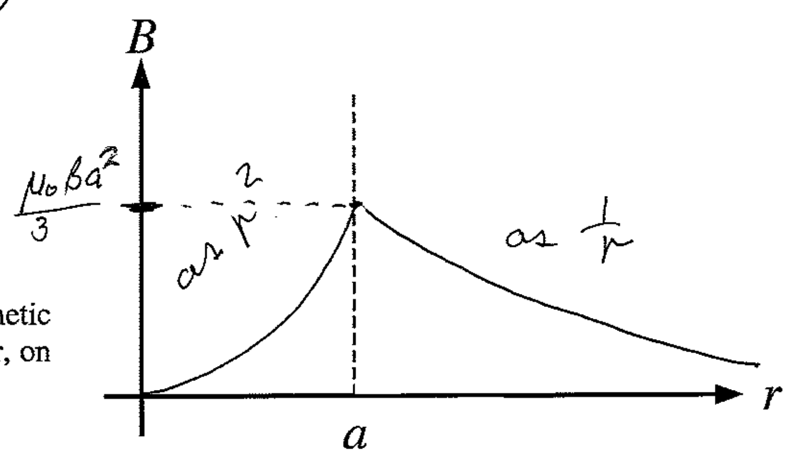
$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 \left(\frac{2\pi\beta a^3}{3}\right)$$

$$B = \frac{\mu_0 \beta a^3}{3r} = \left(\frac{\mu_0 \beta a^3}{3}\right)\frac{1}{r}$$

B diminishes as $\frac{1}{r}$ for $r > a$

2 pts

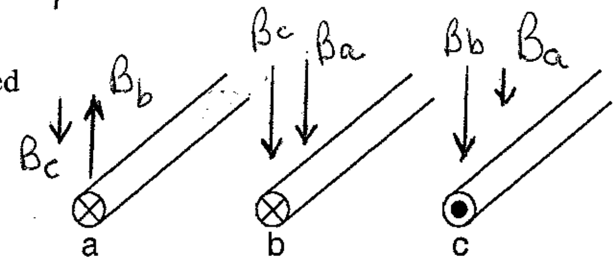
iv) Make a sketch of the magnitude of the magnetic field, as a function of radial distance from the center, on the axes drawn at right.



Field Directions; RHR

Question 2 [8 points] from Checkpoint 1, Ch29

The figure here shows three long, straight, parallel, equally spaced wires running perpendicular to this page, with identical currents either into or out of the paper as indicated by the dots or crosses.



4 pts

i) Rank the wires according to the magnitude of the force on each due to the currents in the other two wires. Rank fror greatest force to least force.

use: $\vec{f} = I\vec{l} \times \vec{B}$

a = b > c

b > a = c

note: the directions and relative magnitudes of the fields as drawn above

b > c > a

c > b > a

(b) experiences two "strong" forces to the right due to two "strong" fields

(c) experiences two forces to the left, but one is a "weak" force

(a) experiences a "weak" and a "strong" force, but in opposite directions

ii) Which wire(s) experience a net force directed towards the right?

4 pts

only a

(a) net field is ↑

only b

(b) and (c) net field is ↓

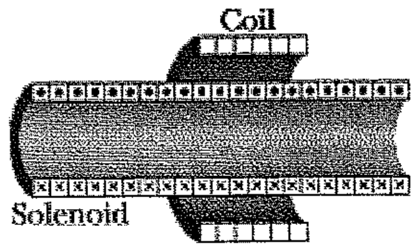
b & c

a & c

use $\vec{f} = I\vec{l} \times \vec{B}$ and RHR

(a) and (c) experiences rightward force

Problem 2 [17 points] from HW 8 #3



In the figure at right a 20 turn coil of radius 4.0 cm and resistance 10 Ω is coaxial with a solenoid with 50 turns/cm and radius 2.0 cm. The current in the solenoid increases linearly from 0 A to 50 A in a time interval Δt of 15 ms.

i) What is the magnitude of the magnetic field in the interior of the solenoid at the end of the 15 ms interval?

5 pts

$$B_{sol} = \mu_0 n i = (\mu_0)\left(50\ t/cm\right)\left(\frac{100\ cm}{1\ m}\right)(50\ A)$$

$$= \underline{\underline{0.31\ T}}$$

ii) How much magnetic flux (magnitude) does the current in the <u>solenoid</u> produce within the interior of the <u>coil</u>?

5 pts

$$\phi = \int \vec{B} \cdot d\vec{A} \qquad B \text{ is uniform over } A$$

$$= B \int dA = B(\pi r^2) = (.31T)(\pi)(.02m)^2$$

$$= \underline{\underline{3.9 \times 10^{-4}\ T \cdot m^2}}$$

iii) How much current (magnitude) is induced in the coil during the 15 ms interval?

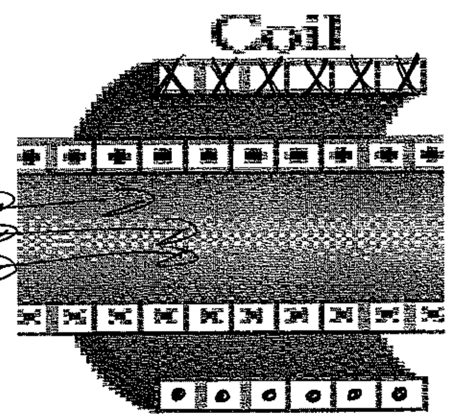
5 pts

$$|\mathcal{E}| = N\left|\frac{d\phi}{dt}\right| = N\frac{\Delta\phi}{\Delta t} = \left(\frac{3.9 \times 10^{-4} T - 0\ T}{.015\ s}\right)(20\ turns)$$

$$= .53\ V$$

$$I = \mathcal{E}/R = \frac{.53\ V}{10\ \Omega} = \underline{\underline{.053\ A}}$$

iv) Annotate the figure at right by adding "x" or "·" as appropriate to indicate the direction of the current which is induced in the windings of the coil. Be sure to show direction both on top and bottom. Explain the reason for your choice.



B_{sol}

2 pts

since B_{sol} is increasing and to the right, current in coil must produce flux to left (to oppose changing ϕ)

Question 3 [8 points]

An ideal inductor consisting of a solenoid coil of length ℓ, cross section A, and turns per unit length n, has been drawing a steady constant current, I, for a long time.

If we then doubled the number of turns per unit length in the solenoid but kept the current, I, cross sectional area, A, and length, ℓ, the same, then waited again for a long time, what has happened to

(i) the magnetic energy stored by the inductor

4 pts

a) it remains the same

b) it is twice as great as before

c) it is half as great as before

d) it is four times as great as before

e) it is one-fourth as great as before

$L_{sol} = \mu_0 n^2 A \ell$

$U_L = \frac{1}{2} L I^2$

if n is doubled,
L becomes 4X as much

so U_L is 4X larger

4 pts

(ii) the voltage drop across the inductor

a) it remains the same

b) it is twice as great as before

c) it is half as great as before

d) it is four times as great as before

e) it is one-fourth as great as before

for inductors carrying steady (constant with time) current

$V_L = L \cancel{\frac{di}{dt}}^{\,0}$

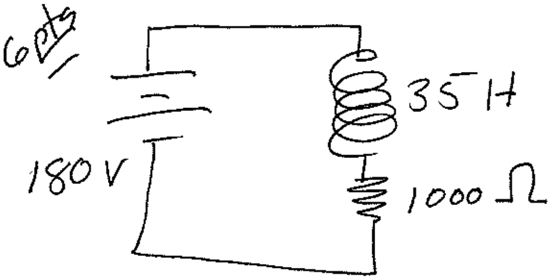
so zero in both steady current cases

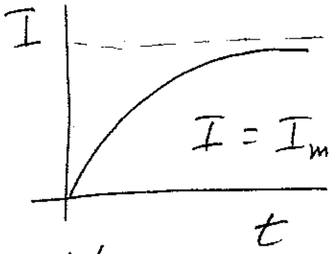
Problem 3 [17 points] from HW 9 #7

A coil with an inductance of 35 H and a resistance of 1000 Ω is suddenly connected to an ideal battery of 180 volts. At the moment exactly 35 ms after the connection was made...

i) At what rate is energy being delivered by the battery?

6 pts





$$I_{MAX} = \frac{180\,V}{1000\,\Omega} = .18\,A$$

$$I = I_{MAX}\left(1 - e^{-t/\tau}\right)$$

$$\tau = L/R = .035\,s$$

$$I\,(t = 35\,ms) = (.18\,A)\left[1 - e^{-t/.035s}\right] \quad \text{where } t = .035\,s$$

$$I = (.18A)\left[1 - e^{-1}\right] = 0.11\,A \qquad P_{BATT} = VI = \underline{19.8\,W}$$

ii) At what rate is energy being converted into heat by the resistor?

6 pts

$$P_R = I^2 R = (.11\,A)^2 (1000\,\Omega) = \underline{12.1\,W}$$

5 pts

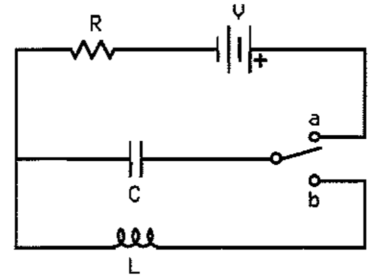
iii) How much total energy has the battery delivered during those first 35 ms?

$$U = \int_0^{t_1} P_{BATT}\,dt = \int_0^{t_1} (V_{BATT})\, I(t)\,dt = \int_0^{.035\,s} (180V)\left[.18A\left(1 - e^{-t/.035s}\right)\right]dt$$

$$= (3.24\ V\cdot A) \int_0^{.035\,s} \left(1 - e^{-t/.035s}\right)dt = .42\ J$$

Question 4 [8 points] adapted from HW 10. #3

In the figure at right, the switch has been in position **a** for a very long time, and is thrown to position **b** at time $t = 0$ s.



i) Just the tiniest moment after the switch is moved to position b, the current throu~~gh the cap~~acitor will be

4 pts

a) zero
b) V/R
c) V/(RC)
d) V/(LC)
e) 2V/L

with switch at "b" L and C have the same current

The capacitor had been charged to V while the switch was at "a"
when the switch moves to "b" the capacitor tries to start discharging through L, but L will not allow an instantaneous change in current
L's current <u>was</u> 0 before the switch so will remain 0 "right after"

ii) After the switch is moved to position b the current in the circuit will oscillate at some frequency, f_0. That frequency, f_0, will be determined by the values of

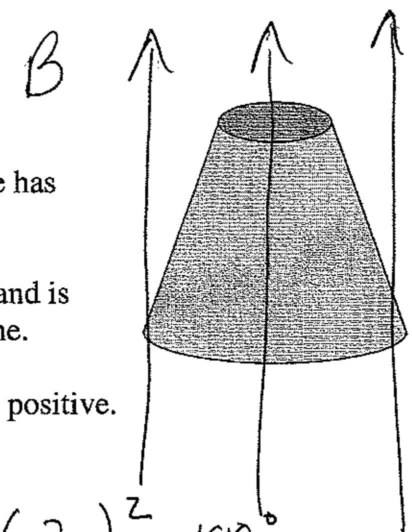
4 pts

a) V and R only
b) R and C only
~~c) R and L only~~
d) L and C only
~~e) R, L, and V only~~
f) R, L, C, and V

V and R are out of the circuit now, so only L and C determine the oscillation frequency

Adapted from HW 10 # 10 and 11

Problem 4 [17 points] take directions of surface vector, $d\vec{A}$, as outward



A Gaussian surface is in the shape of a truncated cone. The bottom of the cone has radius 20 cm and the top has radius 10 cm. The height of the cone is 30 cm.

A magnetic field of 250 mT is constant with time, uniform throughout space, and is directed upwards, passing perpendicular to the top and bottom faces of the cone.

To specify flux directions, assume flux pointing out of the Gaussian surface is positive.

i) What magnetic flux passes through the bottom of the cone?

4pts $\Phi_{Bott} = \int_{BOTT} \vec{B}\cdot d\vec{A} = B A_{BOTT} \cos 180^\circ = (.25T)(\pi)(.2m)^2 \cos 180^\circ$

$= -3.14 \times 10^{-2}\ T\cdot m^2$ inward direction

so $-3.14 \times 10^{-2}\ T\cdot m^2$

ii) What magnetic flux passes through the top of the cone?

4pts $\Phi_{TOP} = \int_{TOP} \vec{B}\cdot d\vec{A} = B A_{TOP} = (.25T)(\pi)(.1m)^2 \cos 0^\circ$

$= +7.85 \times 10^{-3}\ T\cdot m^2$ "+" means outward

iii) What magnetic flux passes through the sloping sides of the cone?

6pts since $\oint \vec{B}\cdot d\vec{A}$, Gauss's Law for B $\quad \Phi_{TOP} + \Phi_{BOTT} + \Phi_{SIDES} = 0$

$\Phi_{SIDES} = -\Phi_{TOP} - \Phi_{BOTT} = -7.85 \times 10^{-3}\ T\cdot m^2 - (-3.14 \times 10^{-2}\ T\cdot m^2)$

$= +2.35 \times 10^{-2}\ T\cdot m^2$ (outward, "+")

iv) Would your answers for (i) through (iii) above change is the cone were made twice as tall? Explain.

3pts NO — Φ_{TOP} & Φ_{BOTT} would not change

so Φ_{sides} remains the same

since $\Phi_{TOP} + \Phi_{BOTT} + \Phi_{SIDES}$ still $= 0$