Student Name: SOLUTION **Student No:** 11 MAR 10

Louisiana State University Physics 2102, Exam 2,
6:00PM Thursday March 11, 2010.

Please, circle your section:

1 & 6 (Giammanco)

2 (Vekhter)

3 (Rupnik)

4 (Dowling)

5 (Rupnik)

• Please be sure to write your name and student number and circle your instructor above.

• Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct.

• On problems that are not multiple choice, be sure to show all of your work since *no credit will be given for an answer without explanation or work.* These problems will be graded in full, and you are expected to show all relevant steps that lead to your answer. Note that you can often do parts (b) or (c) even if you get stuck on part (a).

• You may use scientific or graphing calculators but you must derive and explain your answer fully on paper so we can grade your work.

• Please be sure that *all* numerical quantities include appropriate units. Points will be deducted if the units are absent. In addition to magnitude, vector quantities must contain sufficient information to tell us which way the vector points.

• Feel free to detach, use, and keep the formula sheet pages. No other reference material is allowed during the exam.

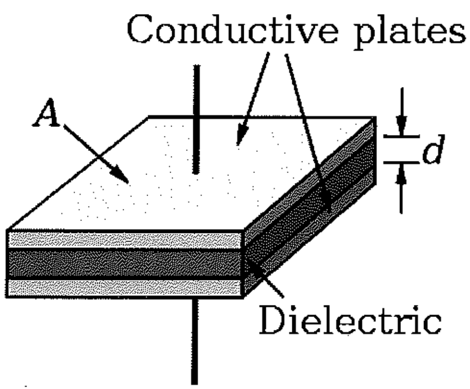
• The only electronic devices to be used during the exam are standard or graphing calculators. All cell phones should be turned off and put away. Cell phones are not to be used as calculators. Any cell phone heard or seen during the exam will be confiscated.

• **May the Force be With You!**

Question 1 [8 points] The figure shows a square parallel plate capacitor of area A, plate separation d, and dielectric of constant κ. The capacitor is connected to a battery of voltage V (battery not shown) and is fully charged to a charge Q.

V = CONSTANT

$Q \neq$ CONSTANT



(i) While remaining connected to the battery, the dielectric of constant κ is replaced with a new dielectric of constant 2κ, with the distance d remaining the same. What happens to the charge Q? Check the right answer.

(a) ☐ The charge Q remains the same Q.

(b) ☐ The charge Q is halved to $Q/2$.

(c) ☑ The charge Q doubles to $2Q$.

(d) ☐ The charge Q goes to zero $Q = 0$.

$C = K C_{AIR} \longrightarrow 2K C_{AIR} = 2C$

$V =$ CONSTANT (CONNECTED TO BATTERY)

$Q \neq$ CONSTANT

$Q = VC \longrightarrow V\underline{2}C = 2Q$

UNITS $[V] = \left[\frac{kQ}{r}\right] = \left[\frac{N\cdot m^2}{C^2}\Big|\frac{C}{m}\right] = \left[\frac{N\cdot m}{C}\right]$

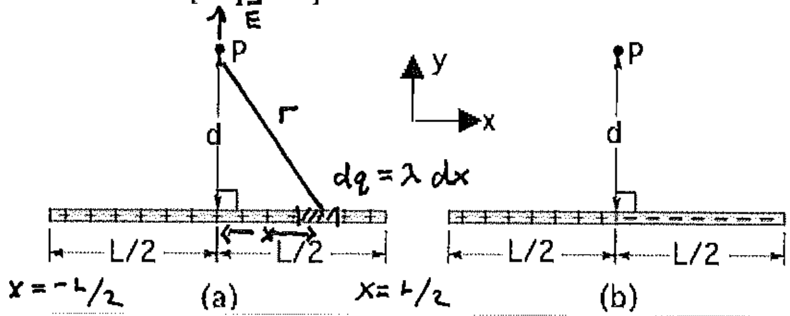
Problem 1 [17 points]

$dq = \lambda\, dx$

$dV = \frac{k\,dq}{r}$

$r = \sqrt{x^2+d^2}$

PYTHAGORAS



$dV = \frac{k\lambda\, dx}{\sqrt{x^2+d^2}}$

0.07 m

(i) (5 pts) Fig. (a) shows a non-conducting rod of length L = 7.00 cm and uniform linear charge density λ = +*3.24* pC/m. Take $V = 0$ at infinity. What is V at point P at distance d = 8.00 cm along the rod's perpendicular bisector? Hint: $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$.

$\lambda = +3.24\times10^{-12}$ C/m $d = 0.08$ m

$$V = \int dV = k\lambda \int_{-L/2}^{L/2} \frac{dx}{\sqrt{x^2+d^2}} = k\lambda \ln\left[x+\sqrt{x^2+a^2}\right]\Big|_{-L/2}^{L/2}$$

$$V = \left(8.99\times10^9 \frac{N\,m^2}{C^2}\right)\left(3.24\times10^{-12}\frac{C}{m}\right)\left[\ln\left(\tfrac{.07}{2}+\sqrt{(\tfrac{.07}{2})^2+(.08)^2}\right) - \ln\left(-\tfrac{.07}{2}+\sqrt{(\tfrac{.07}{2})^2+(.08)^2}\right)\right]$$

$= 2.47\times10^{-2}$ Nm/C $= \boxed{0.025\text{ V}}$ ANS! $[V] = \left[\frac{N\cdot m}{C}\right]$ ✓ UNIT CHECK

(ii) (4 pts) Write down a formula for the *approximate* potential V at a distance $D \gg L$ along the rod's perpendicular bisector in Fig. (a). Use this formula to compute V on the bisector for D = 10 m.   FOR $D \gg L$ ROD ACTS LIKE POINT CHARGE OF $Q = \lambda L$

$$V_{POINT} = \frac{kQ}{R} = \boxed{\frac{k\lambda L}{D}} = \frac{8.99\times10^9\,N\cdot m^2}{C^2}\Big|\frac{3.24\times10^{-12}\,C}{m}\Big|\frac{0.07\,m}{10\,m} = \boxed{2.04\times10^{-4}\text{ V}}$$

ANS!   ANS!

(iii) (3 pts) What is the *direction* of the *electric field* **E** at point *P* at distance d = 8.00 cm along the rod's perpendicular bisector in Fig. (a)? Check one:

- [ ] +x direction
- [ ] –x direction
- [x] +y direction
- [ ] –y direction
- [ ] No Direction: $\boldsymbol{E} = 0$.

BY SYMMETRY X-COMPONENTS CANCEL & FIELD POINTS UP AWAY FROM ⊕ CHARGE.

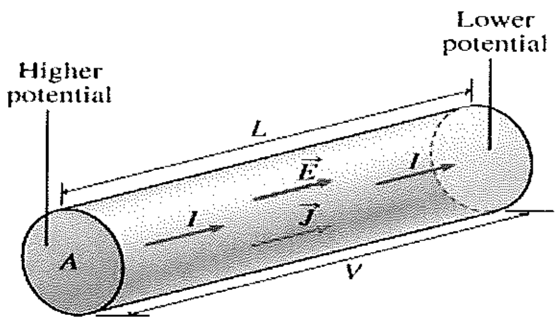
(iv) (5 pts) Fig. (b) shows an identical rod, *except that one half is now negatively charged.* Both halves have a linear charge density of *magnitude* $|\lambda|$ = 4.04 pC/m. With $V = 0$ at infinity, what is V at P? *You must explain your reasoning to get full credit!*

EACH $dV = \frac{k|\lambda|\,dx}{r}$ ON LEFT CANCELS EACH

$-dV = -\frac{k|\lambda|\,dx}{r}$ ON RIGHT $\boxed{\text{BY SYMMETRY!}}$ ANS!

$\boxed{V = 0\text{ V}}$ ANS!

Question 2 [8 points] The figure shows a current-carrying cylindrical rod resistor of constant area A, length L, resistance R, made from a material of resistivity ρ, carrying a constant current I, and connected to an ideal battery (no internal resistance) of voltage V (battery not shown).



$V = \text{CONSTANT}$

(i) (4 pts) While remaining connected to the battery, the rod is replaced with an identically shaped rod but now with a material of resistivity 2ρ. What happens to the current I? Check one.

(a) ☐ The current I remains the same I.

(b) ☑ The current I is halved to $I/2$.

(c) ☐ The current I is doubled to $2I$.

(d) ☐ The current I goes to zero $I = 0$.

$R = \rho L/A \rightarrow 2\rho L/A = 2R$

$V = \text{CONST.}$

$I \neq \text{CONST.}$

ANS!

$I = \frac{V}{R} \rightarrow \frac{V}{2R} = \boxed{\frac{I}{2}}$

(ii) (4 pts) Now the original rod of resistivity ρ *is cut in half so its length is L/2* and then reattached to the same battery. What happens to the potential difference V *across the new resistor?* Check one.

(a) ☑ The voltage V remains the same.

(b) ☐ The voltage V is halved to $V/2$.

(c) ☐ The voltage V doubles to $2V$.

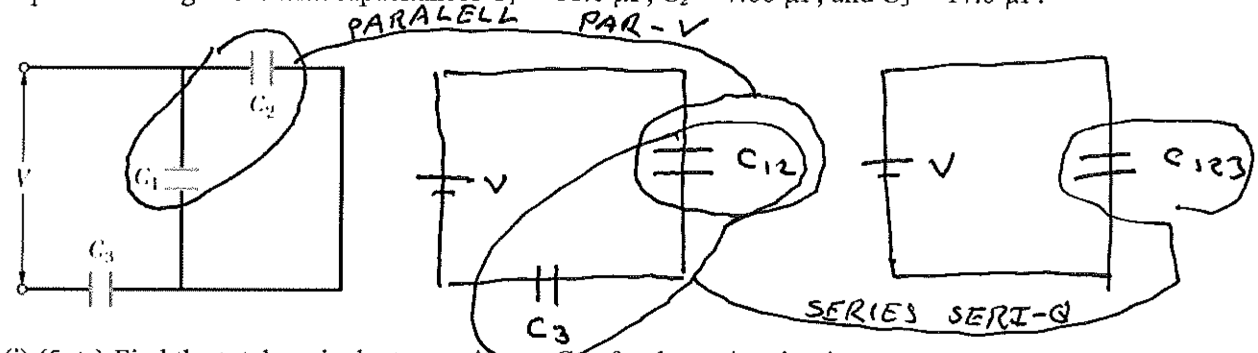
(d) ☐ The voltage V goes to zero $V=0$.

CONNECTED TO BATTERY:

V IS CONSTANT!

$V_R = V_{BATT} = \text{CONSTANT}$

Problem 2 [17 points] In the figure below, a potential difference $V = 200$ V is applied across a capacitor arrangement with capacitances $C_1 = 11.0$ μF, $C_2 = 7.00$ μF, and $C_3 = 17.0$ μF.



(i) (5pts) Find the total equivalent capacitance C_{123} for the entire circuit.

$$C_{12}^{PAR} = C_1 + C_2 = 18.0\,\mu F$$ IN PARALLEL $\underline{V}$ SAME : $\underline{\text{PAR-V}}$

$$\frac{1}{C_{123}^{SER}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{18.0\,\mu F} + \frac{1}{17.0\,\mu F}$$ IN SERIES $\underline{Q}$ SAME : SERI$\underline{Q}$

$$\Rightarrow \boxed{C_{123}^{EQ} = 8.74\,\mu F} \text{ ANS.}$$

(ii) (4pts) Find the charge on capacitor 3.

SERI-Q Q'S SAME $\Rightarrow$ $Q_{123} = Q_{12} = Q_3$ $V_{123} = V$

$$Q_{123} = V C_{123} = [200\,V][8.74\,\mu F] = 1748\,\mu C$$

$$Q_3 = Q_{123} = \boxed{1.748\,mC} \text{ ANS!} = \boxed{1.75 \times 10^{-3}\,C}$$

(iii) (4pts) Find the potential difference across capacitor 2. PARALLEL $\Rightarrow$ V SAME: PAR-$\underline{V}$

PAR-V $\boxed{V_2 = V_1 = V_{12}}$ BUT $Q_{12} = C_{12} V_{12}$

$$\Rightarrow V_{12} = \frac{Q_{12}}{C_{12}} = \frac{1748\,\mu C}{18.0\,\mu F} = \boxed{97.1\,V = V_2} \text{ ANS!}$$

U_1

(iv) (4pts) Find the stored energy for capacitor 1.

$$V_1 = V_2 = V_{12} = 97.1\,V$$

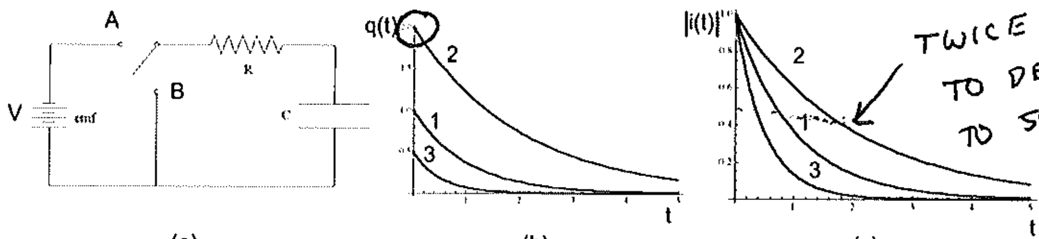
$$\boxed{U_1 = \tfrac{1}{2} C_1 V_1^2} = \tfrac{1}{2}(11.0\,\mu F)(97.1\,V)^2$$

$$= 51870\,\mu J$$

$$= \boxed{51.9\,mJ} \text{ ANS!}$$

$$= 0.052\,J$$

Question 3 [8 points] A RC circuit in Fig. (a) with a switch is shown below with an EMF of $V = 1.0$ V, $R = 1.0\ \Omega$, and $C = 1.0$ F. For time a long time the switch is at point A fully charging the capacitor. At $t = 0$ the switch is thrown to B causing the capacitor to discharge through the resistor. In Fig. (b) the curve 1 shows the charge $q(t)$ on the capacitor and in Fig. (c) the curve 1 shows the magnitude of the current $|i(t)|$ through the resistor as a function of time.



TWICE AS LONG TO DECAY TO 50% $|i(0)|$

(a) (b) (c)

(i) (4 pts) We repeat this experiment with a new capacitor with double the capacitance: $C = 2.0$ F. (We keep $V = 1.0$ V and $R = 1.0\ \Omega$.) What is the new charge curve $q(t)$ on the capacitor as a function of time in Fig. (b)? Check one answer.

$q_0 = q(0) = VC$

(a) ☐ The function $q(t)$ remains the same curve 1.

$C \to 2C$

(b) ☑ The function $q(t)$ is now curve 2.

$q_0 \to V\underline{2C} = 2q_0$

(c) ☐ The function $q(t)$ is now curve 3.

Y-INTERCEPT DOUBLES

(d) ☐ None of the curves $q(t)$ in Fig. (b) are correct.

(ii) (4 pts) With the same new capacitor with double the capacitance: $C = 2.0$ F, keeping $V = 1.0$ V and $R = 1.0\ \Omega$, what is the new current curve $|i(t)|$ through the resistor as a function of time in Fig. (c)? Check one answer.

(a) ☐ The function $|i(t)|$ remains the same curve 1.

$\tau = RC$

$C \to 2C$

(b) ☑ The function $|i(t)|$ is now curve 2.

$\tau \to R2C = 2\tau$

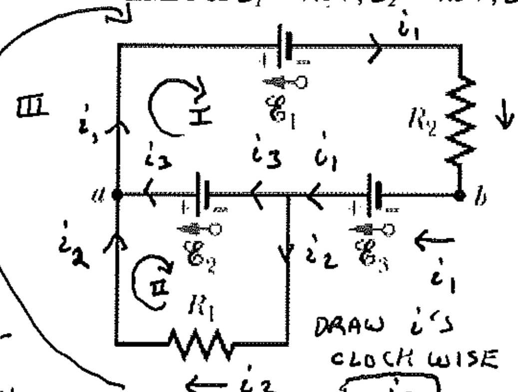
(c) ☐ The function $|i(t)|$ is now curve 3.

CURRENT TAKES TWICE AS LONG TO DECAY TO 50% $|i(0)|$.

(d) ☐ None of the curves $|i(t)|$ in Fig. (c) are correct.

TOO MANY BATTERIES? USE LOOP & JUNCTION RULES!

Problem 3 [17 points] In the figure below, $R_1 = 150\Omega$, $R_2 = 85\Omega$, and the ideal batteries have EMFs of $\mathcal{E}_1 = 7.0\text{V}$, $\mathcal{E}_2 = 4.5\text{V}$, $\mathcal{E}_3 = 4.0\text{V}$.



WALK CLOCKWISE EACH LOOP

$\boxed{0}$ $i_1 = i_2 + i_3$ JUNCTION RULE

LOOP RULES: START (a) END (a)

(I) $-\mathcal{E}_1 - i_1 R_2 + \mathcal{E}_3 + \mathcal{E}_2 = 0$

(II) $-\mathcal{E}_2 - i_2 R_1 = 0$

(III) $-\mathcal{E}_1 - i_1 R_2 + \mathcal{E}_3 - i_2 R_1 = 0$

(a) (6 pts) Find the $\boxed{\text{current}}$ (i_2) in resistor 1. USE (II) THAT HAS ONLY i_2

$$\text{(II)} \Rightarrow i_2 R_1 = -\mathcal{E}_2 \Rightarrow \boxed{i_2 = -\mathcal{E}_2 / R_1} = -\frac{4.5\text{V}}{150\Omega}$$

$$\Rightarrow \boxed{i_2 = -0.03\text{A}} \text{ ANS!}$$

$\ominus$ SIGN MEANS i_2 IS REALLY COUNTERCLOCKWISE

(b) (6 pts) Find the $\boxed{\text{current}}$ (i_1) in resistor 2. USE (I) ONLY HAS i_1

$$\text{(I)} \Rightarrow \mathcal{E}_3 + \mathcal{E}_2 - \mathcal{E}_1 = i_1 R_2 \Rightarrow \boxed{i_1 = \frac{\mathcal{E}_3 + \mathcal{E}_2 - \mathcal{E}_1}{R_2}}$$

$$\Rightarrow i_1 = \frac{(4.0 + 4.5 - 7.0)\text{V}}{85\Omega} = \frac{1.5\text{V}}{85\Omega}$$

$$\Rightarrow \boxed{i_1 = +0.0177\text{A}} \text{ ANS!}$$

$\oplus$ MEANS i_1 IS REALLY CLOCKWISE

(c) (5 pts) Find the total power (P_{TOT}) dissipated by the entire circuit.

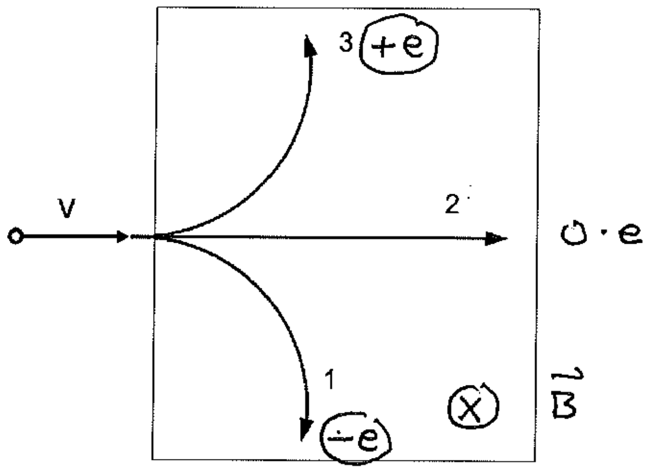
ONLY R_1 & R_2 CAN DISSIPATE (HEAT) POWER

CONSERVATION OF ENERGY $\Rightarrow$

$$P_{TOT} = P_1 + P_2$$
$$= i_2^2 R_1 + i_1^2 R_2$$
$$= (-0.03\text{A})^2 (150\Omega) + (0.0177\text{A})^2 (85\Omega)$$
$$= \boxed{0.162\text{W}} \text{ ANS!}$$

NOTE: POWER $i_1 \mathcal{E}_1$ IS NOT DISSIPATED BUT STORED IN BATTERY.

Question 4 [8 points] Three particles of equal mass and equal velocity are launched into a box containing a uniform and homogeneous magnetic field as shown below.



BY SYMMETRY

③ = +e IF

① = −e

RIGHT HAND RULE!

(i) (4 pts) The particle that takes curve 1 is negatively charged with $q = -e$. What is the direction of the magnetic field in the box? Check one.

APPLY R.H.R.
TO ③ = ± e
TO SIMPLIFY

(a) ☐ Up towards the top of the page $(\uparrow)$.

(b) ☐ Down towards the bottom of the page $(\downarrow)$.

(c) ☑ Into the page $(\otimes)$.

(d) ☐ Out of the page $(\odot)$.

(ii) (4 pts) What is the charge of the particle that takes curve 2? Check one.

NO DEFLECTION!

$F = qvB = 0$

$v \neq 0$

$B \neq 0$

$q = 0$!

(a) ☐ $q = -e$

(b) ☐ $q = +e$

(c) ☑ $q = 0$

(d) ☐ None of the above.

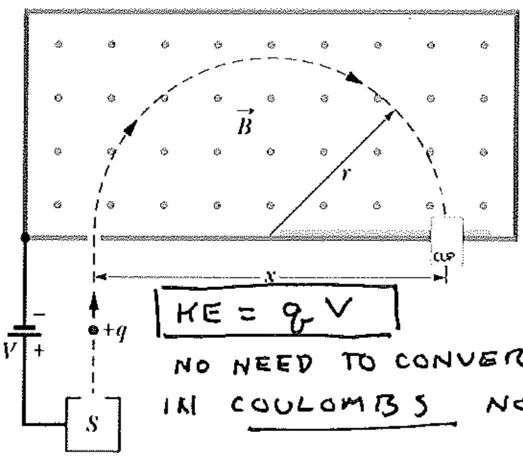
IF ② HAS SAME MASS AS ① & ③ IT WOULD DEFLECT FOR ANY $|q| \neq 0$. $\Rightarrow q = 0$

v = velocity $\neq V$ = Volts

UNITS: $F = vqB \Rightarrow [B] = \left[\frac{N}{} \middle| \frac{s}{m} \middle| \frac{}{C}\right] = \left[\frac{kg \cdot m}{s^2} \middle| \frac{s}{m} \middle| \frac{}{C}\right] = \left[\frac{kg}{s \cdot C}\right] = [T]$

Problem 4 [17 points] A certain commercial mass spectrometer is used to separate fissionable uranium-235 ions of mass 3.92×10^{-25} kg and charge 3.20×10^{-19} C from non-fissionable uranium-238. The uranium-235 ions are accelerated through a potential difference of 120 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 2.00 m. After traveling through 180° and passing through a slit of width 3.00 mm and height 5.00 cm, they are collected in a cup.

m, q, V, r



$F = ma = \frac{mv^2}{r}$

$F = qvB$

$\Rightarrow qvB = \frac{mv^2}{r}$

$\Rightarrow \boxed{B = \frac{mv}{qr}}$ FIND v!

$\boxed{KE = qV}$

NO NEED TO CONVERT eV to J SINCE q IS GIVEN IN COULOMBS NOT e's!

ALSO: $B = \frac{mv}{qr} = \frac{m}{qr}\sqrt{\frac{2qV}{m}} = \boxed{\sqrt{\frac{2mV}{qr^2}}}$

(i) (10 pts.) What is the magnitude of the (perpendicular) magnetic field in the separator?

$KE = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2KE}{m}}$

KINETIC ERG = $KE = qV = (3.20 \times 10^{-19} C)(120 \times 10^3 V) = \boxed{3.84 \times 10^{-14} J}$

$v = \sqrt{\frac{2(3.84 \times 10^{-14} J)}{3.92 \times 10^{-25} kg}} = \boxed{4.43 \times 10^5 m/s}$

$B = \frac{mv}{qr} = \frac{3.92 \times 10^{-25} kg}{3.20 \times 10^{-19} C} \left| \frac{4.43 \times 10^5 \cancel{m}}{s} \right| \frac{}{2.0 \cancel{m}} = \boxed{0.27T}$ ANS!

(ii) (7 pts) If the machine is used to separate out 110 mg of material per hour calculate the current of the desired ions in the machine.

$M = 110 \times 10^{-6}$ kg $T = 3600$ s

$I = Q/T$

Q = TOTAL CHARGE = (CHARGE / PARTICLE) x (# PARTICLES)

q

PARTICLES = (TOTAL MASS) / (MASS / PARTICLE)

$= M/m$

$I = \frac{Q}{T} = \frac{q \cdot \#}{} \Big| \frac{}{T}$ $\boxed{= 0.025A}$ ANS

$= \frac{q}{} \left| \frac{M}{m} \right| \frac{}{T} = \frac{3.20 \times 10^{-19} C}{} \left| \frac{110 \times 10^{-6} kg}{3.92 \times 10^{-25} kg} \right| \frac{}{3600 s}$

Formula Sheet for LSU Physics 2102, Exam 2, Spring '10

- **Constants, definitions:**

  $\epsilon_o = 8.85 \times 10^{-12}\,\mathrm{C^2/Nm^2}$ $\quad k = \dfrac{1}{4\pi\epsilon_o} = 8.99 \times 10^9\,\mathrm{Nm^2/C^2}$ $\quad \mu_o = 4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}$

  $c = 3.00 \times 10^8\,\mathrm{m/s}$ $\quad e = 1.60 \times 10^{-19}\,\mathrm{C}$ $\quad 1\ \mathrm{eV} = \mathrm{e(1V)} = 1.60{\times}10^{-19}\,\mathrm{J}$

  dipole moment: $\vec{p} = q\vec{d}$ $\quad$ charge densities: $\lambda = \dfrac{Q}{L}$, $\sigma = \dfrac{Q}{A}$, $\rho = \dfrac{Q}{V}$

  $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$ $\quad m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$ $\quad g = 9.8\,\mathrm{m/s^2}$

  Area of a circle: $A = \pi r^2$ $\quad$ Area of a sphere: $A = 4\pi r^2$ $\quad$ Volume of a sphere: $V = \frac{4}{3}\pi r^3$

- **Kinematics (constant acceleration) :**

  $v = v_o + at$ $\quad x - x_o = \frac{1}{2}(v_o + v)t$ $\quad x - x_o = vt - \frac{1}{2}at^2$

  $x - x_o = v_o t + \frac{1}{2}at^2$ $\quad v^2 = v_o^2 + 2a(x - x_o)$

- **Coulomb's law:** $F = k\dfrac{|q_1||q_2|}{r^2}$
- **Force on a charge in an electric field:** $\vec{F} = q\vec{E}$
- **Electric field of a point charge:** $E = k\dfrac{|q|}{r^2}$
- **Electric field of a dipole on axis, far away from dipole:** $E = \dfrac{2kp}{z^3}$
- **Electric field of an infinite line charge:** $E = \dfrac{2k\lambda}{r}$
- **Torque on a dipole in an electric field:** $\vec{\tau} = \vec{p} \times \vec{E}$
- **Potential energy of a dipole in electric field:** $U = -\vec{p} \cdot \vec{E}$
- **Electric flux:** $\Phi = \int \vec{E} \cdot d\vec{A}$
- **Gauss' law:** $\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc}$
- **Electric field of an infinite non-condncting plane with a charge density σ:** $E = \dfrac{\sigma}{2\epsilon_o}$
- **Electric field of infinite conducting plane or close to the snrface of a conductor:** $E = \dfrac{\sigma}{\epsilon_o}$
- **Electric potential, potential energy, and work:**

  $V_f - V_i = -\displaystyle\int_i^f \vec{E} \cdot d\vec{s}$ $\quad$ In a uniform field: $\Delta V = -Ed\cos\theta$

  $\vec{E} = -\vec{\nabla} V$, $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, $E_z = -\dfrac{\partial V}{\partial z}$

  Potential of a point charge q: $\quad V = k\dfrac{q}{r}$

  Potential of n point charges: $\quad V = \displaystyle\sum_{i=1}^{n} V_i = k\sum_{i=1}^{n} \frac{q_i}{r_i}$

  Electric potential energy: $\boxed{\Delta U = q\Delta V}$ $\Delta U = -W_{\text{field}}$

  Potential energy of two point charges: $U_{12} = W_{\text{ext}} = q_2 V_1 = q_1 V_2 = k\dfrac{q_1 q_2}{r_{12}}$

- **Capacitance definition:** $q = CV$

  Capacitor with a dielectric: $C = \kappa C_{air}$

  Parallel plate: $C = \varepsilon_o \dfrac{A}{d}$

  Potential Energy in Cap: $U = \dfrac{q^2}{2C} = \dfrac{1}{2}qV = \dfrac{1}{2}CV^2$ $\quad$ Energy density of electric field: $u = \dfrac{1}{2}\kappa\varepsilon_o|\vec{E}|^2$

  Capacitors in parallel: $C_{eq} = \sum C_i$ $\quad$ Capacitors in series: $\dfrac{1}{C_{eq}} = \sum \dfrac{1}{C_i}$

- **Current:** $i = \frac{dq}{dt}$ **Current density:** $J = \frac{i}{A}$
- **Drift speed of the charge carriers:** $\vec{v}_d = \frac{\vec{J}}{ne}$
- **Definition of resistance:** $R = \frac{V}{i}$ **Definition of resistivity:** $\rho == \frac{|\vec{E}|}{|\vec{J}|}$
- **Resistance in a conducting wire:** $R = \rho\frac{L}{A}$ **Temperature dependence:** $\rho - \rho_o = \rho_o\alpha(T - T_o)$
- **Power in an electrical device:** $P = iV$ **Power in a resistor:** $P = i^2R = \frac{V^2}{R}$
- **Definition of** *emf*: $\mathcal{E} = \frac{dW}{dq}$
- **Resistors in series:** $R_{eq} = \sum R_i$ **Resistors in parallel:** $\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$
- **Loop rule in DC circuits: the sum of changes in potential across any closed loop of a circuit must be zero.**
- **Junction rule in DC circuits: the sum of currents entering any junction must be equal to the sum of currents leaving that junction.**
- **Charging a capacitor in a series RC circuit:** $q(t) = C\mathcal{E}(1-e^{-\frac{t}{\tau_C}})$, time constant $\tau_C = RC$ **Discharging:** $q(t) = q_o e^{-\frac{t}{\tau_C}}$
- **Magnetic Fields**
  bf Magnetic force on a charge q: $\vec{F} = q\vec{v} \times \vec{B}$ **Lorentz force:** $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$
  Circnlar motion in a magnetic field: $qv_\perp B = \frac{mv_\perp^2}{r}$ with period: $T = \frac{2\pi m}{qB}$
  Magnetic force on a length of wire: $\vec{F} = i\vec{L} \times \vec{B}$
  Magnetic Dipole: $\vec{\mu} = Ni\vec{A}$
  Torque on a Magnetic Dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$
  Energy of a Magnetic Dipole: $U = -\vec{\mu} \cdot \vec{B}$