Exam 1
Physics 2102 Spring 2010

February 11, 2010

Name: Solution ID #

Please, circle your section: 1 & 6 (Giammanco)
2 (Vekhter)
3 (Rupnik)
4 (Dowling)
5 (Rupnik)

Answer all questions (7).

Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct.

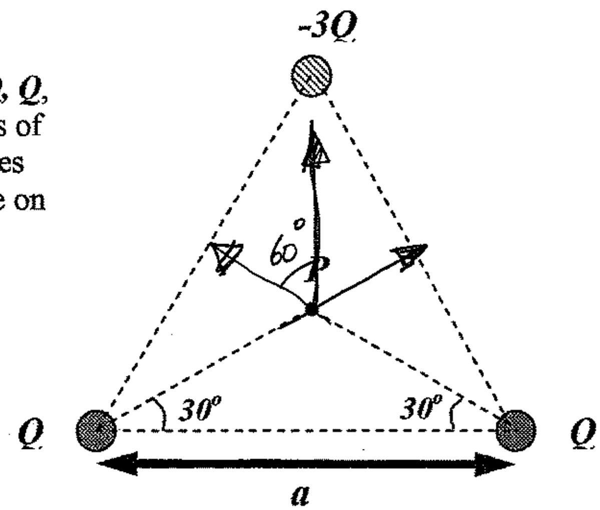
On problems that are not multiple choice, be sure to show all of your work since *no credit will be given for an answer without explanation or work.* These problems will be graded in full, and you are expected to show all relevant steps that lead to your answer.

Please be sure that *all* numerical quantities include appropriate units. Points will be deducted if the units are absent.

The only electronic devices to be used during the exam are standard or graphing calculators. All cell phones should be turned off and put away. Cell phones are not to be used as calculators.

1. (20 points)

Three charges, with magnitudes ***Q***, ***Q***, and ***-3Q***, are located at the vertices of an equilateral triangle with the sides of length ***a***, as shown in the Figure on the left.



a) (3 points) Sketch the electric field due to each of the three charges at point ***P*** at the center of the triangle.

b) (8 points) Determine the magnitude of the net electric field at point ***P***. The distance from P to each of the vertices is $a/\sqrt{3}$.

Horizontal components cancel.
Vertical component

$$E = k\frac{3Q}{(a/\sqrt{3})^2} + 2k\frac{Q}{(a/\sqrt{3})^2}\cos 60° =$$

$$= k\frac{9Q}{a^2} + k\frac{3Q}{a^2} = k\frac{12Q}{a^2}$$

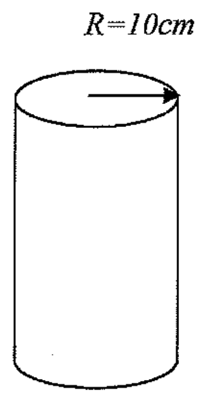
c) (3 points) What is the direction of the net electric field at point P?

(i) towards the top margin of the page;
ii) towards the bottom margin of the page;
iii) towards the right margin of the page;
iv) towards the left margin of the page ;
v) towards the right +Q charge;
vi) towards the left +Q charge;
vii) away from the right +Q charge;
viii) away from the left +Q charge.

d) (6 points) Compute the electrostatic potential at point P.

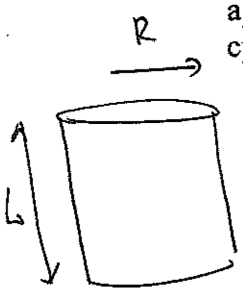
$$V = -k\frac{3Q}{a/\sqrt{3}} + 2k\frac{Q}{a/\sqrt{3}} = -k\frac{\sqrt{3}Q}{a}$$

2. (20 points) HW03 #7



A long, non-conducting, solid cylinder of radius $R=10\ cm$ has a non-uniform volume charge density $\rho = ar^2$, a function of the radial distance r from the cylinder axis. The constant $a=10\mu C/cm^5$

a) What is the linear charge density (charge per unit length) along the length of the cylinder?

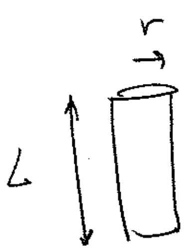


$$Q = \int \rho\, dV = \int \rho(r)\, r\, dr\, d\varphi\, dz =$$

$$= \int_0^{2\pi} d\varphi \int_0^{L} dz \int_0^{R} r\, dr\, ar^2 = 2\pi L a R^4/4$$

$$\lambda = \frac{Q}{L} = \frac{\pi}{2} a R^4 = 1.57 \cdot 10^{-5} \cdot 10^4 = 0.157\ C/cm$$

b) What is the magnitude of the electric field at a radial distance of 5 cm from the axis of the cylinder?



$$\Phi = \oint \vec{E} \cdot d\vec{A} = E \cdot 2\pi r L = Q_{encl}/\epsilon_0$$

$$Q_{encl} = 2\pi L a r^4/4, \quad \text{so} \quad E = \frac{ar^3}{4\epsilon_0}$$

$$r = 5\ cm \qquad a = 10\mu C/cm^5 = 10^{-5} C / 10^{-10} m^5 = 10^5\ C/m^5$$

$$E = \frac{10^5 \cdot 125 \cdot 10^{-6}}{4 \cdot 8.85 \cdot 10^{-12}} = 3.53 \cdot 10^{11}\ N/C$$

Units

$$\frac{[C]}{[m]^5} \cdot [m]^3 \frac{N \cdot [m]^2}{C^2} = \left[\frac{N}{C}\right]$$

(cont.on next page)

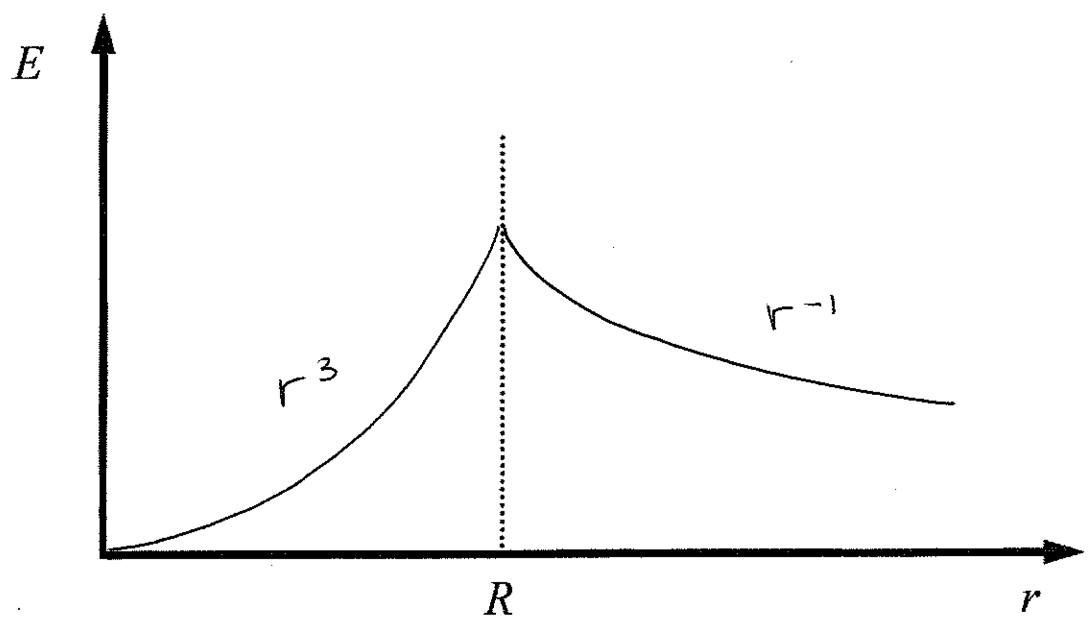
c) What is the magnitude of the electric field at a radial distance of 15 cm from the axis of the cylinder?

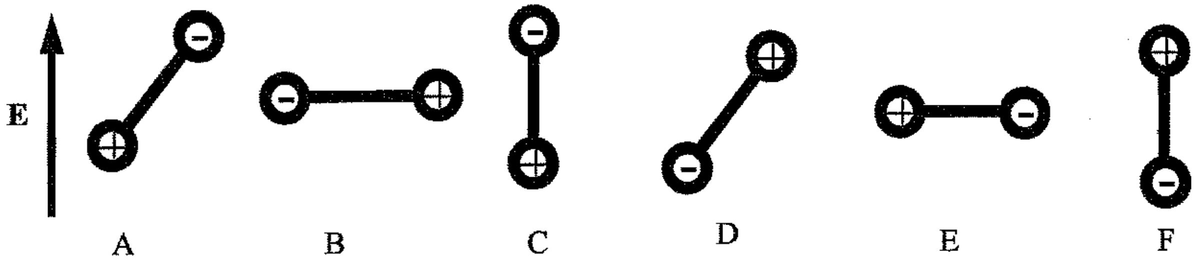
$$E \cdot 2\pi r L = \lambda L/\epsilon_0 \qquad \lambda = \frac{\pi}{2} a R^4$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2 \cdot 9 \cdot 10^9 \cdot \frac{0.157}{10^{-2}} \frac{1}{15 \cdot 10^{-2}} =$$

$$= 1.88 \cdot 10^{12} \ N/C$$

d) Sketch the dependence of the magnitude of the electric field on the radial distance *r*. Indicate the power law for the field as a function of *r* in each region of space.





3. (15 points) A dipole is placed in a *uniform electric field* directed towards the top margin of the page as shown above.

a) For which of the situations A through F is the magnitude of the net force on the dipole *maximal*?

i) B and E;
ii) C and F;
iii) A and D;
iv) C;
v) B;
vi) the same for all situations;

Uniform field => forces on +q and -q canceled $\vec{F} = 0$ always

HW02 #4

b) For which of the situations A through F is the magnitude of the torque on the dipole *maximal*?

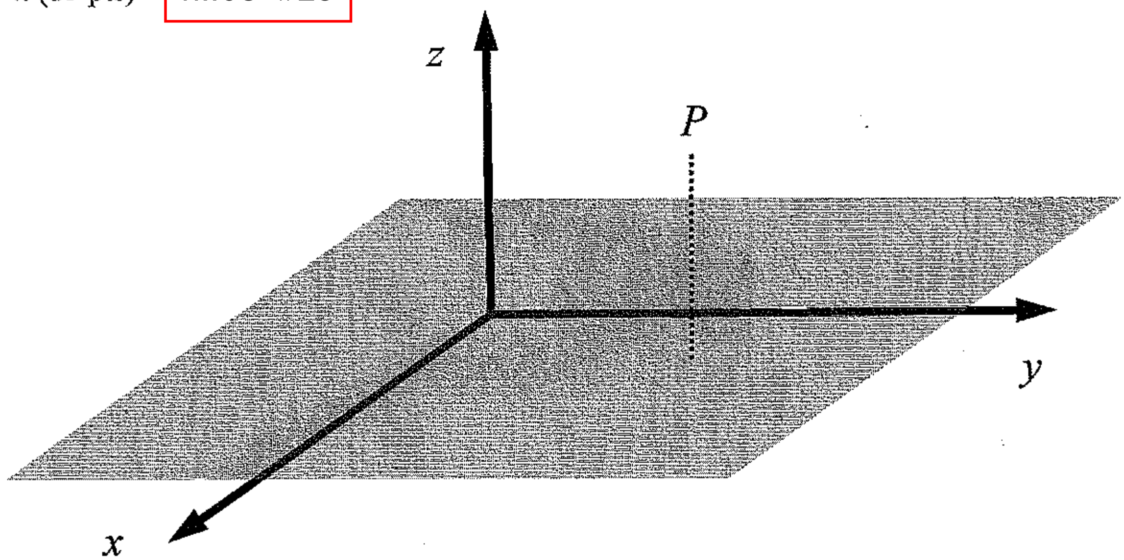
i) B and E
ii) C and F;
iii) A and D;
iv) C;
v) B;
vi) the same for all situations;

torque ≠ force ≠ potential energy

c) For which of the situations A through F is the potential energy of the dipole *maximal*?

i) E ;
ii) F;
iii) A;
iv) C;
v) B;
vi) the same for all situations;

4. (15 pts) HW03 #15



An infinite non-conducting thin sheet with uniform surface charge density of 25 nC/cm^2 covers the entire *xy* plane.

i) What is the flux through a spherical Gaussian surface centered on the origin and having a radius of 10 cm?

Charge inside the surface $\sigma \cdot \pi r^2 = 25 \cdot 10^{-9} \cdot 3.14 \cdot 10^2$

$= 7.85 \cdot 10^{-6}$ C

$$\Phi = \frac{q_{encl}}{\epsilon_0} = \frac{7.85 \cdot 10^{-6}}{8.85 \cdot 10^{-12}} = 8.9 \cdot 10^5 \ \frac{Nm^2}{C}$$

ii) What is the electric field in unit vector notation at the point *P* with the coordinates *x=2 cm, y=10 cm, z=10 cm*?

$$E = \frac{\sigma}{2\epsilon_0} = \frac{25 \cdot 10^{-9}}{10^{-4} \cdot 2 \cdot 8.85 \cdot 10^{-12}} = 1.41 \cdot 10^7 \ N/C$$

$$\vec{E} = E\hat{k}$$

iii) Suppose, instead of the sheet, the lower half-space ($z<0$) is filled with a metal, and that metal has the same charge density of 25 nC/cm^2 distributed over the x-y surface? What would the electric field be in unit vector notations just above the x-y surface of such a metal? Provide a brief explanation of your answer.

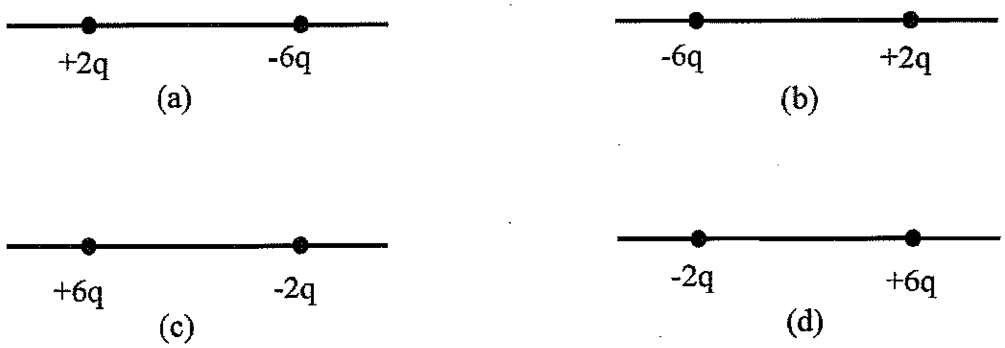
$$E = \frac{\sigma}{\epsilon_0} = 2.8 \cdot 10^7 \ N/C$$

For the surface of the metal, field is twice that of the charged sheet since the field inside the metal $\vec{E} = 0$.

$$\vec{E} = \vec{E}\,\hat{k}$$

HW01 #18

5. (8 points) The figure below shows four situations in which charged particles are fixed in place on an axis.



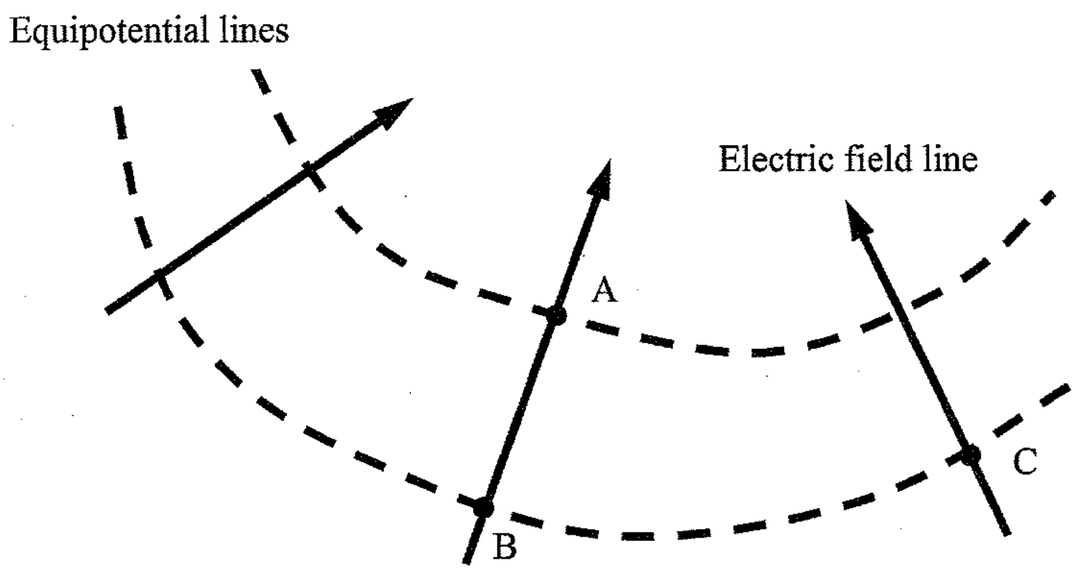
In which situations is there a point to the right of the particles where an electron will be in equilibrium? (Select all that apply)

(a)
(b)
(c)
(d)

Equilibrium : $\vec{E} = 0$: need to be closer to the charge of smaller magnitude.

HW03 #18

6. (9pts) When an electron moves from A to B along an electric field line in the figure below, the electric field does $4.80\text{x}10^{-19}$ J of work on it.



What are the electric potential differences:

(i) $V_B - V_A = \frac{4.8 \cdot 10^{-19}}{1.6 \cdot 10^{-19}} = 3\,V$ B @ higher potential

(ii) $V_C - V_A =$ $3\,V$ since $V_C = V_B$

(iii) $V_C - V_B =$ 0 on the same equipotential lines

7. (13 pts)

HW01 #11

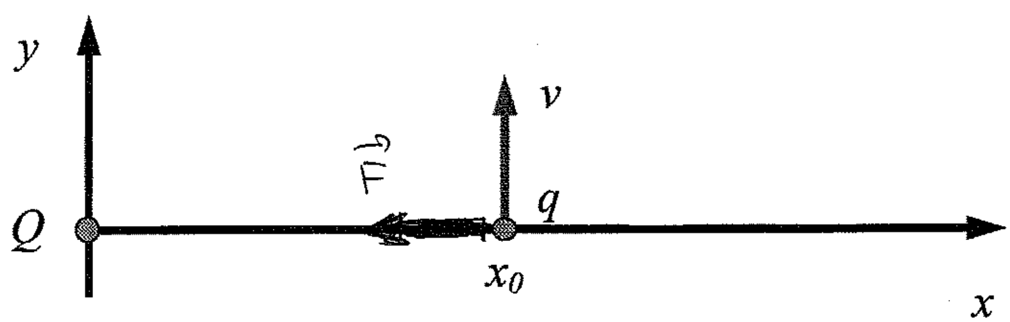
a) (6 points) A current of 0.250 A through your chest can send your heart into fibrillation. If that current persists for 2.50 min, how many conduction electrons pass through your chest?

$$i = \frac{dq}{dt} \qquad q = it = 2.5 \cdot 10^{-1} A \cdot 150 \text{ sec} = 37.5 C$$

$$N = \frac{q}{e} = \frac{37.5 C}{1.6 \cdot 10^{-19} C} = 2.3 \cdot 10^{20} \text{ electrons}$$

HW01 #14

b) (7 points) A charged particle of unknown charge Q is fixed at the origin of an xy coordinate system. At $t = 0$ a particle of mass m and a *positive* charge q is located on the x axis at position $x = x_0$, moving with a speed v in the positive y direction. Find an expression for Q *(including sign)*, in terms of m, q, v and x_0 for which the moving particle executes circular motion. (Neglect the gravitational force on the particle.)



$$F = m\frac{v^2}{x_0} = |Q| q \cdot k \frac{1}{x_0^2}$$

$$|Q| = \frac{mv^2 x_0}{kq}$$

Force needs to be attractive

$$Q = -\frac{mv^2 x_0}{kq}$$

PHYS-2102 Spring of 2010, Formula Sheet for theFirst Examination

$\varepsilon_0 = 8.854\times10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}\ or\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}$ $\quad c = 3.00\times10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}$ $\quad G = 6.67\times10^{-11}\ \frac{\mathrm{N\cdot m}^2}{\mathrm{kg}^2}$ $\quad g = 9.80\frac{\mathrm{m}}{\mathrm{s}^2}$

$\mu_0 = 1.26\times10^{-6}\ \frac{\mathrm{H}}{\mathrm{m}}$ $\quad e = 1.6\times10^{-19}\ \mathrm{C}$ $\quad \frac{1}{4\pi\varepsilon_0} = 8.99\times10^{9}\ \frac{\mathrm{N\cdot m}^2}{C^2}$ $\quad N_A = 6.02\times10^{23}$ particles

$m_e = 9.109\times10^{-31}\ \mathrm{kg}$ $\quad m_p = 1.673\times10^{-27}\ \mathrm{kg}$ $\quad m_n = 1.675\times10^{-27}\ \mathrm{kg}$ $\quad 1u = 1.66\times10^{-27}\ \mathrm{kg} = 931.5\ \mathrm{MeV}$

$h = 6.63\times10^{-34}\ \mathrm{J\cdot s} = 4.14\times10^{-15}\ \mathrm{eV\cdot s}$

$A_{\text{sphere}} = 4\pi R^2$ $\quad V_{\text{sphere}} = \frac{4}{3}\pi R^3$ $\quad A_{\text{circle}} = \pi R^2$ $\quad C_{\text{circle}} = 2\pi R$ $\quad \vec{v} = \vec{v}_0 + \vec{a}t$ $\quad \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$$F_{\mathrm{E}} = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{|q_1||q_2|}{r^2}\right) \qquad \vec{E} = \frac{\vec{F}}{q} \qquad d\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{dq}{r^2}\right)\hat{r} \qquad \vec{\tau} = \vec{p}\times\vec{E} \qquad U = -\vec{p}\bullet\vec{E}$$

$$\varepsilon_0\oint \vec{E}\bullet d\vec{A} = \varepsilon_0\Phi_{\mathrm{E}} = q_{\text{enclosed}} \qquad V = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{r}\right) \qquad V = \frac{U}{q} \qquad \Delta U = -W_{\text{field}} = W_{\text{applied}}$$

$$\Delta V = -\int \vec{E}\bullet d\vec{s} \qquad V = \left(\frac{1}{4\pi\varepsilon_0}\right)\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad dV = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{dq}{r}\right) \qquad \vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{\mathrm{i}} - \frac{\partial V}{\partial y}\hat{\mathrm{j}} - \frac{\partial V}{\partial z}\hat{\mathrm{k}}$$