1. (20 points) For simplicity, make the (naive) assumption that all of the uranium-235 in a nuclear fission reactor undergoes its fission according to one process:

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{55}Cs + {}^{93}_{37}Rb + 2\,{}^{1}_{0}n $$

The various masses, in atomic mass units (u = 1.661 × 10^{-27} kg = 931.5 MeV) are:
n 1.00866 u, U 235.04392 u, Cs 140.91963 u, and Rb 92.92157 u

a) Calculate the energy released in a single fission event. Express your answer both in joules and in eV.

1 pts

$$\Delta m = (m_n + m_{U235}) - (2m_n + m_{Cs} + m_{Rb})$$

$$= (235.04392 - 1.00866 - 140.91963 - 92.92157)\ u$$

$$= 0.19406\ u = (.19406)(931.5\ MeV)$$

$$= \boxed{180.8\ MeV} \quad \boxed{= 1.81 \times 10^{8}\ eV}$$

$$= (180.8 \times 10^{6}\ eV)\left(1.6 \times 10^{-19}\ \frac{J}{eV}\right) = \boxed{2.89 \times 10^{-11}\ J}$$

b) Suppose that a commercial power plant completely consumed 210 kg of uranium-235 according to the above reaction. What was its total energy output, in units of kilowatt-hours (1 kW·hr = 3.6×10^6 J)?

2 pts

$$\left(\frac{210 \times 10^{3}\ g}{235\ g}\right) N_A = 5.38 \times 10^{26}\ \text{atoms fissioning}$$

$$\left(5.38 \times 10^{26}\ \text{fissions}\right)\left(2.89 \times 10^{-11}\ \frac{J}{\text{fission}}\right)\left(\frac{1\ kw\text{-}hr}{3.6 \times 10^{6}\ J}\right) =$$

$$\boxed{= 4.3 \times 10^{9}\ kw\text{-}hr}$$

c) What is the average rate, in mega-watts, at which the plant produced energy, if the fuel lasted 2 years.

6 pts

$$2\ yr \Rightarrow (2)(365.25\ d/yr)(24\ hr/d) = 1.75 \times 10^{4}\ hr$$

$$\frac{4.3 \times 10^{9}\ kw\text{-}hr}{1.75 \times 10^{4}\ hr} = (2.45 \times 10^{5}\ kw)\left(\frac{1\ MW}{1000\ kw}\right) = 245\ MW$$

→ 5 pt BONUS

2. (20 points) Ultraviolet light of wavelength 220 nm is incident on a surface coated with a thin film of metal in a photoelectric apparatus. It is observed that a potential difference of 3.25 volts will completely cutoff the photocurrent in the apparatus.

a) What is the energy, in eV, of the incident photons?

5 pts

$$E = hf = hc/\lambda = \frac{(4.14\times10^{-15}\ eV\cdot s)(3\times10^{8}\ m/s)}{220\times10^{-9}\ m}$$

$$= \underline{5.65\ eV}$$

b) What is the work function, in eV, of the metal surface?

5 pts

$$K.E._{max} = \frac{hc}{\lambda} - \phi \qquad K.E._{max} = \text{stopping potential (in eV)}$$

$$3.25\ eV = 5.65\ eV - \phi \Rightarrow \phi = \underline{2.4\ eV}$$

c) If the potential difference is set back to zero volts, what is the cutoff wavelength, that is, the longest wavelength that could still eject an electron?

5 pts E_{photon} must be at least ϕ

$$\text{so} \quad \frac{hc}{\lambda} = 2.4\ eV \Rightarrow \lambda = \frac{(4.14\times10^{-15}\ eV\cdot s)(3\times10^{8}\ m/s)}{2.4\ eV}$$

$$= \underline{518\ nm}$$

d) If the potential difference is set back to zero volts, but using 220 nm light, what is the speed of the fastest electrons emitted?

5 pts

$$K.E._{max} = 3.25\ eV = 5.2\times10^{-19}\ J = \frac{1}{2}mv^2$$

$$v = \left[\frac{(2)(5.2\times10^{-19}\ J)}{9.1\times10^{-31}\ kg}\right]^{1/2} = 1.1\times10^{6}\ m/s$$

e) What is the deBroglie wavelength of the fastest emitted electrons?

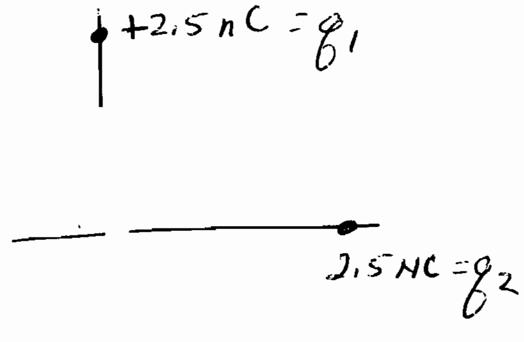
5 pts

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ J\cdot s}{(9.1\times10^{-31}\ kg)(1.1\times10^{6}\ m/s)}$$

$$= \underline{6.6\times10^{-10}}\ m \quad (6.6\ \mathring{A})$$

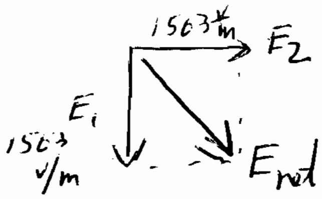
3. (10 points) In an xy coordinate plane, a positive charge of +2.5 nC (nano-coulombs) is fixed in place at coordinates (0, 12) cm. A negative charge of −2.5 nC is fixed at coordinates (12, 0) cm. The approximate magnitude of the electric field at the origin of the coordinate system is

a) 2200 V/m
b) 3100 V/m
c) 270 V/m
d) 375V/m
e) none of the above are even approximately correct



$$|E_1| = |E_2| = \left(9\times10^9 \frac{N\,m^2}{C^2}\right)\left(\frac{2.5\times10^{-9}\,C}{(.012\,m)^2}\right)$$

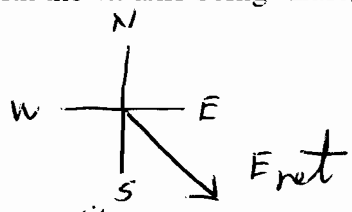
$$= 1563\ V/m$$



$$E_{net} = \sqrt{2(1563\ V/m)^2} = 2209\ V/m$$

4. (5 points) If the +y axis is taken to be the direction "North" with the +x axis being "East," the direction of the electric field found in the preceding problem is approximately

a) Northeast
b) Southwest
c) Northwest
d) Southeast
e) there is no associated direction, electric field is a scalar quantity



5. (8 points) In order for nuclear fusion to take place, a deuterium nucleus (1 proton plus 1 neutron) must become "hot enough" to overcome (or tunnel through) a coulomb force energy barrier to fuse with another deuterium nucleus. Determine the approximate height of this energy barrier (in units of electron-volts) by calculating the electric potential energy of two deuterium nuclei separated by the approximate nuclear radius of 1 fermi (10^{-15} m)

Note: By comparison, a temperature of 1 million kelvins yields an average kinetic energy of less than 100 eV

a) 2.3 MeV
b) 1.4 MeV
c) 910 KeV
d) 340 KeV
e) none of the above are even approximately correct

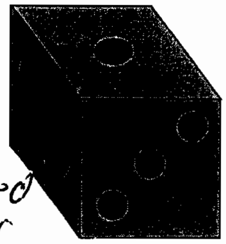
$$U = k\frac{q_1 q_2}{r} = \left(9\times10^9 \frac{N\,m^2}{C^2}\right)\left(\frac{(1.6\times10^{-19}\,C)^2}{1\times10^{-15}\,m}\right)$$

$$= 2.3\times10^{-13}\,J = 1.4\ meV$$

6. (8 points) The electric flux, in units of kN·m²/C (kilo-newton · meter squared per coulomb) through the six faces of a die (one half of a pair of dice) is given by

$$\Phi_E = (N)(-1)^N \quad kN\bullet m^2/C$$

where N represents the number of spots on a given face of the die, and $1 \leq N \leq 6$. Each edge of the cubic die measures 1.5 cm. Take inward flux as negative, and outward flux as positive. Approximately how much electric charge, in coulombs, resides inside the die?



a) zero coulombs
b) +53.1 nC
c) −1860 nC
d) +26.6 nC
e) none of the above are even approximately correct

all 6 faces form a closed Gaussian Surface

face	Φ_E, kN·m²/C
1	−1
2	+2
3	−3
4	+4
5	−5
6	+6

$\Phi_{net} = +3 \frac{kN \cdot m^2}{C}$

$\Phi_{net} = \oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$

$q = \epsilon_0 \Phi_{net}$
$= (8.854 \times 10^{-12}\ F/m)(3000\ \frac{N \cdot m^2}{C})$
$= 26.5 \times 10^{-9}\ C$

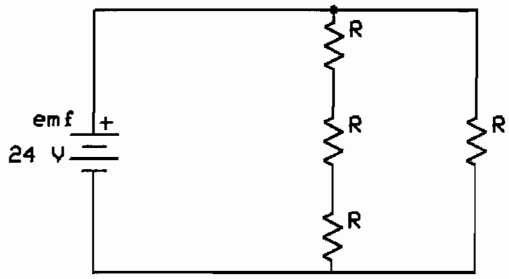
7. (5 points) A parallel plate capacitor whose plates are separated by air is charged by connecting it to a battery. The battery is then disconnected and the capacitor plates are isolated and insulated from their surroundings. Which one of the following actions would cause the magnitude of the potential difference between the plates to increase?

a) moving the plates farther apart
b) moving the plates closer together (but not touching)
c) inserting a dielectic material into the space between the plates
d) inserting a conducting material into the space between the plates (but not touching either plate)
e) none of the above, the potential difference must remain equal to the battery emf

$C = \frac{\epsilon_0 A}{d} \qquad Q = CV$

Q stays constant, C decreases,
V must increase

8. (10 points) Four resistors, each of R = 50 Ω, are connected to each other and to a battery of emf = 24 V as shown in the circuit diagram. The connecting wires have negligible resistance. The magnitude of the current through the battery is approximately



a) 360 mA
b) 480 mA
c) 160 mA
d) 520 mA
e) none of the above are even approximately correct

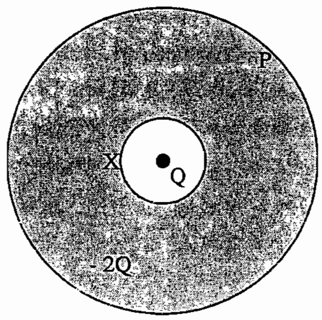
3 50Ω in series → just add them up $= 150\Omega$
50 + 50 + 50

150Ω in parallel with 50Ω

$$\frac{1}{\frac{1}{150} + \frac{1}{50}} = 37.5\,\Omega$$

$$i = \frac{24\text{ V}}{37.5\,\Omega} = .640\text{ A}$$

9. (8 points) A point charge, +Q, is located at the center of a hollowed out conducting shell of inner radius 10 cm and outer radius 50 cm. The shell itself carries a net charge of −2Q. If the electric potential is taken to be zero at an infinite distance, and the potential at point P, very near the outer surface, is measured as −300 V, then the potential at point X, very near the inner surface of the shell, is approximately



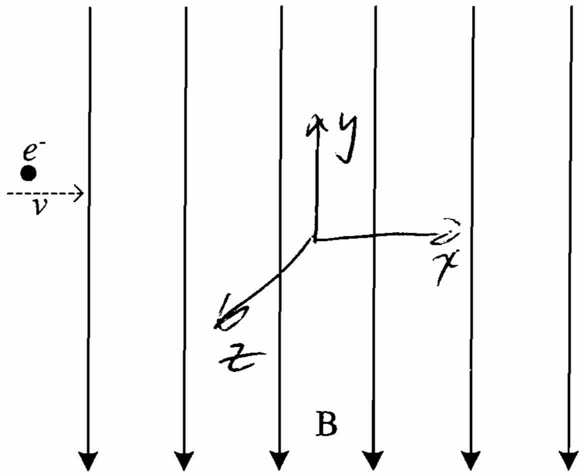
a) −300 V
b) −100 V
c) +100 V
d) +300 V
e) none of the above are even approximately correct

A conductor is an equipotential – all points in and on it are at the same potential

so $V_P = V_X = -300\text{ V}$

10. (5 points) In a region of space there is a uniform magnetic field, B, directed towards the bottom margin of the paper as shown in the figure. An electron enters the region from the left, moving towards the right margin of the paper. The electron will experience

a) a magnetic force into the paper
b) a magnetic force out of the paper
c) a magnetic force towards the top margin of the paper
d) a magnetic force towards the bottom margin of the paper
e) no magnetic force at all



$\vec{F} = q\vec{v} \times \vec{B}$

v in $+x$, B in $-y$

$\hat{i} \times (-\hat{j}) = -\hat{k}$, but q is negative

so $+\hat{k}$

11. (10 points) An elastic conducting material is stretched into a circular loop of 10.0 cm radius. It is placed with its plane perpendicular to a uniform 0.700 T magnetic field. When released, the radius of the loop starts to shrink at a rate of 100 cm/s. The emf induced in the loop at the instant immediately after release is approximately

* a) 440 mV (milli-volts)
b) 220 mV
c) 130 mV
d) zero V
e) none of the above are even approximately correct

$$\mathcal{E} = -N\frac{d\Phi_m}{dt} = -N\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

$$= -NB\frac{d}{dt}\int dA$$

$$= -NB\frac{dA}{dt} = -NB\frac{d}{dt}(\pi R^2) = -N\pi B\frac{d}{dt}R^2$$

$$= -N\pi B\left(2R\frac{dR}{dt}\right)$$

$$= -(1\text{ turn})(\pi)(.7T)(2)(.1m)\left(\frac{1m}{s}\right) = .439\text{ V}$$

For both problems 12 and 13...
Consider an interface between glass (refractive index 1.414) and air (refractive index 1.000)

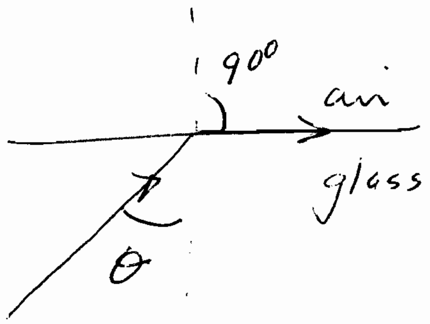
12. (5 points) In which direction should you shine a light towards the interface in order for it to experience total internal reflection (TIR)?

a) from inside the glass towards the air
b) from in the air towards the glass
c) it doesn't matter, TIR can occur in either case

Must go from high index into lower index materials

13. (8 points) At what minimum angle with respect to the normal should the light strike the interface in order that TIR takes place?

a) 30°
b) 45°
c) 60°
d) 75°
e) none of the above are even approximately correct



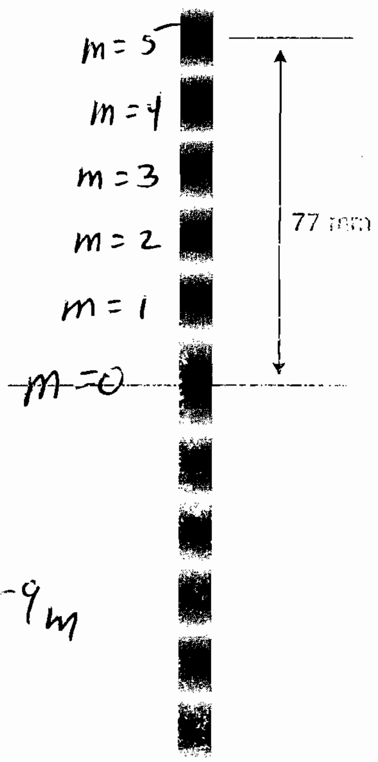
$$n_{glass} \sin\theta = n_{air} \sin 90°$$

$$\sin\theta = \frac{n_{air}}{n_{glass}} = .707$$

$$\theta = 45°$$

14. (10 points) In a double slit interference apparatus, monochromatic, coherent light from a laser is incident on two narrow parallel slits spaced 0.350 mm apart. On a screen 9.00 meters distant a pattern of light and dark fringes appears, with the fifth bright fringe, as shown in the figure, appearing at a spot 77.0 mm from the middle of the central bright fringe. The wavelength of the light is approximately

a) 368 nm
b) 499 nm
c) 599 nm
d) 748 nm
e) none of the above are even approximately correct



$$d \sin\theta = m\lambda \qquad \theta = \tan^{-1}\left(\frac{77\text{ mm}}{9000\text{ mm}}\right)$$

$$\lambda = \frac{d \sin\theta}{m}$$

$$\lambda = \frac{(.350\times10^{-3}\text{ m})\sin(4.9\times10^{-1})°}{5} = 599\times10^{-9}\text{ m}$$

Note that the picture is a ***negative image****, that is, the inked regions are the bright spots, and blank regions are dark.*

15. (10 points) An 8.0 gram sample of charcoal from an ancient fire pit has carbon-14 activity of 100 disintegrations per minute. A living tree has carbon-14 activity of 15.3 disintegrations per minute per gram, with half-life 5730 years. The age of the charcoal sample is approximately

a) ~~1850 yr~~
b) 1670 yr
c) ~~1490~~ yr
d) 1060 yr
e) none of the above are even approximately correct

$$R = R_0 e^{-\lambda t}$$

$$\frac{100 \text{ d/min}}{8 \text{ g}} = 12.5 \frac{\text{d/min}}{\text{g}} \qquad \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730 \text{ y}} = 1.21 \times 10^{-4} \text{ yr}^{-1}$$

$$12.5 \frac{\text{d/min}}{\text{g}} = 15.3 \frac{\text{d/min}}{\text{g}} e^{(-1.21\times10^{-4} \text{yr}^{-1})t}$$

$$t = 1670 \text{ yr}$$

16. (8 points) In a Compton scattering experiment, light of wavelength 3.50 pm (pico-meters) is directed at a target containing free electrons. In a few instances the light is observed to be scattered directly back towards the source, that is, at a scattering angle of 180°. The approximate wavelength of the photons observed at the 180° scattering angle is

a) 4.86 pm
b) 8.36 pm
c) 1.36 pm
d) 5.94 pm
e) none of the above are even approximately correct

$$\Delta\lambda = \frac{h}{mc}(1-\cos\phi) \qquad \phi = 180^\circ$$

$$= \frac{h}{mc}(1-\cos 180^\circ) = \frac{h}{mc}(2)$$

$$\Delta\lambda = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2)}{(9.1\times10^{-31} \text{ kg})(3\times10^{8} \text{ m/s})} = 4.9\times10^{-12} \text{ m} = 4.9 \text{ pm}$$

$$\lambda_{scattered} = \lambda_{incident} + \Delta\lambda = 3.5 \text{ pm} + 4.9 \text{ pm} = 8.4 \text{ pm}$$