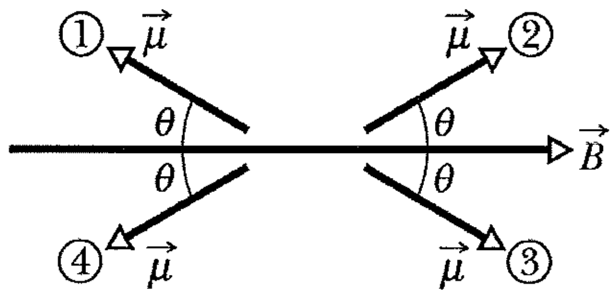
Question 1 [8 points]

The figure shows four orientations, at angle θ, of a magnetic dipole moment $\vec{\mu}$ in a magnetic field $\vec{B}$.



(i) (4 pts) If the dipole moment $\vec{\mu}$ is rotated from orientation (2) to orientation (1) by an external agent, is the work done on the dipole by the agent? (circle one)

Positive Negative Zero

(ii) (4 pts) Rank the work done on the dipole by the agent for these three rotations, greatest first:

A: (2) → (1) B: (2) → (4) C: (2) → (3)

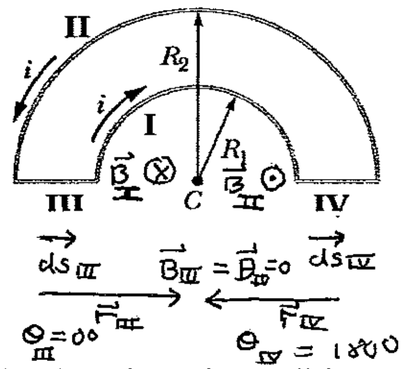
A = B > C

$$\frac{\mu_0 i}{4}\left[\frac{R_2 + R_1}{R_1 R_2}\right] = 3.49\times10^{-6}\ \mathrm{T} \qquad B_I = \frac{\mu_0 i}{4R_1} = 2.15\times10^{-6}\ \mathrm{T}$$

$$B_{II} = \frac{\mu_0 i}{4R_2} = 1.34\times10^{-6}\ \mathrm{T}$$

Problem 1 [18 points]

In the figure below, two semicircular arcs (I & II) have radii R_1 = 4.10 cm and R_2 = 6.60 cm, carry current i = 0.281 A, and share the same center of curvature C. The same current i also flows through the straight sections of wire labeled III & IV.



(a) (5 pts) What is the contribution to the magnitude of the magnetic field at point C from the two straight sections of wire III and IV? Explain your answer!

$\boxed{\text{ZERO FROM BOTH!}}$ 3

FROM BIOT-SAVART

$$|d\vec{B}_{III}| = \frac{\mu_0 |i| ds_{III}\, r_{III}}{4\pi\, r_{III}^2} \overbrace{\sin 0^0}^{0} \qquad 2$$

$$|d\vec{B}_{IV}| = \frac{\mu_0}{4\pi}|i|\frac{ds_{IV}\, r_{IV}}{r_{IV}^2} \overbrace{\sin 180^0}^{0}$$

(b) (8 pts) Calculate the magnitude of the magnetic field at the point C due to all four sections of wire.

$B_{III} = B_{IV} = 0$ 2 FROM (a)

$\vec{B}_I$ INTO PAGE ⊗ $\qquad \vec{B}_{II}$ OUT ⊙ RIGHT HAND RULE

SO $\vec{B}_{II}$ and $\vec{B}_I$ cancel! (somewhat) $\boxed{|B_I| > |B_{II}| \text{ SINCE } R_1 < R_2}$

2

$$|\vec{B}_{TOT}| = |\vec{B}_I| - |\vec{B}_{II}| + |\vec{B}_{III}| + |\vec{B}_{IV}|$$

$$= \frac{\mu_0 i \pi}{4\pi R_1} - \frac{\mu_0 i \pi}{4\pi R_2} + 0 + 0 = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad 2$$

$$= \frac{\mu_0 i}{4}\left[\frac{R_2 - R_1}{R_1 R_2}\right] = 0.25 \left| \frac{4\pi\times10^{-7}\ \mathrm{T\,m}}{\mathrm{A}} \right| 0.281\ \mathrm{A} \left| \frac{2.50\times10^{-2}\ \mathrm{m}}{2.71\times10^{-3}\ \mathrm{m^2}} \right.$$

$$= \boxed{8.14\times10^{-7}\ \mathrm{T}} \quad 2$$

(c) (5 pts) What is the direction of the total magnetic field at the point C due to all four sections of wire? Circle one:

~~Out of the page (•).~~ 2 −3 $\qquad$ Into the page ⊗. +5 5

~~Up towards the top of the page ↑.~~ −5 0 $\qquad$ Down towards the bottom of the page ↓. −5 0

To the right of the page →. −5 0 $\qquad$ To the left of the page ←. −5 0

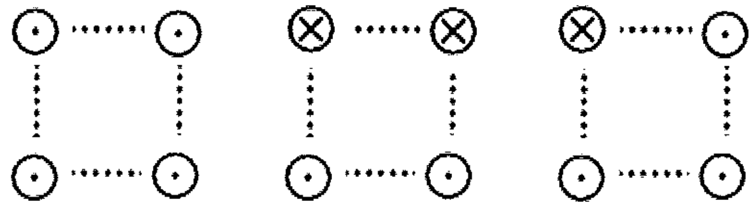
The total magnetic field at C is zero and has no direction.

SINCE $|B_I| > |B_{II}|$ and $\vec{B}_{II} = ⊙$ / $\vec{B}_I = ⊗$

TOTAL IS ⊗ SINCE $\vec{B}_I$ IS STRONGER

Question 2 [8 points]

The figure below shows four arrangements in which long parallel wires carry equal currents *i* directly into or out of the page at the corners of identical squares.



(A) (B) (C)

(i) (4 pts) For square B, what is the direction of the magnetic field, with respect to the page, at the center of the square? Circle one:

Out (•) *In* ⊗ *Up* ↑ *Down* ↓

Left ← *Right* → *Field is Zero* ($\vec{B} = 0$)

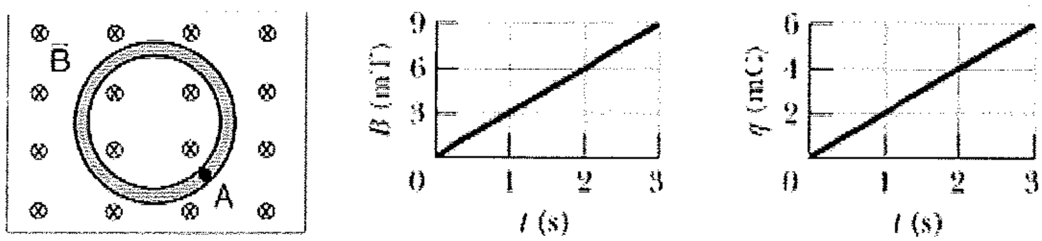
(ii) (4 pts) Rank the sections according to the magnitude of the magnitude of the magnetic field at the center of each square, greatest first. Circle one:

$B_A > B_B > B_C$ $B_C > B_B > B_A$ $B_A > B_C > B_B$

$B_B > B_C > B_A$ $B_A = B_B = B_C$

Problem 2 [14 points]

Figure (a) shows a circular conducting loop, of area 6.0×10^{-4} m^2, which is perpendicular to a uniform magnetic field. Figure (b) shows the increase of the magnitude of the magnetic field with time.



(a) (b) (c)

(i) (6 pts) Calculate the magnitude of the emf induced in the loop during this time:

$$A = 6.0 \times 10^{-4} (m^2) \qquad \frac{dB}{dt} = 3 \left(\frac{mT}{s}\right)$$

$$\frac{d\Phi_B}{dt} = A \cdot \frac{dB}{dt} = 1.8 \times 10^{-6} \left(\frac{Tm^2}{s} = V\right)$$

$$\therefore \text{ mag. of emf} = \left| -\frac{d\Phi_B}{dt} \right| = 1.8 \times 10^{-6} (V)$$

(ii) (3 pts) What is the direction of the current induced in the loop? Answer with words or indicate on the figure.

counter clockwise.

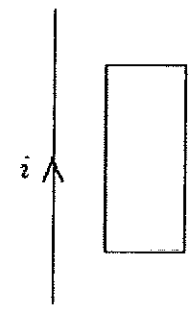
(iii) (5 pts) Figure (c) shows the amount of charge q passing a point A on the loop, as a function of time. Calculate the resistance of the loop:

$$i = \frac{\Delta q}{\Delta t} = 2 \left(\frac{mC}{s}\right) = 2 \times 10^{-3} (A)$$

$$R = \frac{|emf|}{i} = \frac{1.8 \times 10^{-6} (V)}{2 \times 10^{-3} (A)} = 9.0 \times 10^{-4} (\Omega).$$

Question 3 [10 points]

A long straight wire is in the plane of a rectangular conducting loop. The straight wire carries a constant current *i*, as shown.



(i) (5 pts) While the wire is being moved toward the rectangle, the induced current in the rectangle is:

A. zero

B. clockwise

C. counterclockwise

D. into the page

E. out of the page

(ii) (5 pts) The wire is now held in place, and the current *i* is turned off. While the current is being turned off, the induced current in the rectangle is:

A. zero

B. clockwise

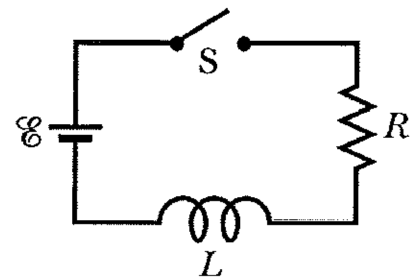
C. counterclockwise

D. into the page

E. out of the page

Problem 3 [17 points]

The figure shows an RL circuit, where R = 22 Ω, L = 1 mH, and the battery voltage $\mathcal{E}$ = 12.0 V.



(i) (4 pts) What is the current in the circuit a long time after the switch S is closed?

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

$$i(t \to \infty) = \frac{\mathcal{E}}{R} = \frac{12.0\,V}{22\,\Omega} = 5.5 \times 10^{-1}\ (A)$$

(ii) (7 pts) How long does it take for the current to reach 1/2 of its maximum value?

$$i(t = t_{1/2}) = \frac{\mathcal{E}}{R}\left(1 - e^{-t_{1/2}/\tau}\right) = \frac{\mathcal{E}}{R}\cdot\frac{1}{2}.$$

$$e^{-t_{1/2}/\tau} = \frac{1}{2}.$$

$$-t_{1/2}/\tau = -\ln 2$$

$$\therefore\ t_{1/2} = \tau \ln 2 = \left(\frac{L}{R}\right)\ln 2 = \frac{1\times10^{-3}H}{22\,\Omega}\cdot \ln 2$$

$$= 3.15 \times 10^{-5}\ (sec).$$

(iii) (6 pts) At the time when current reaches 1/2 of its maximum value, what is the rate at which energy is being stored in the inductor?

$$u_B = \frac{1}{2}Li^2$$

$$\frac{du_B}{dt} = \frac{di}{dt}\cdot Li$$

$$\left.\frac{du_B}{dt}\right|_{t=t_{1/2}} = \left.\frac{di}{dt}\right|_{t=t_{1/2}}\cdot L \cdot i(t_{1/2}) \quad ; \quad i(t_{1/2}) = \frac{1}{2}\frac{\mathcal{E}}{R}.$$

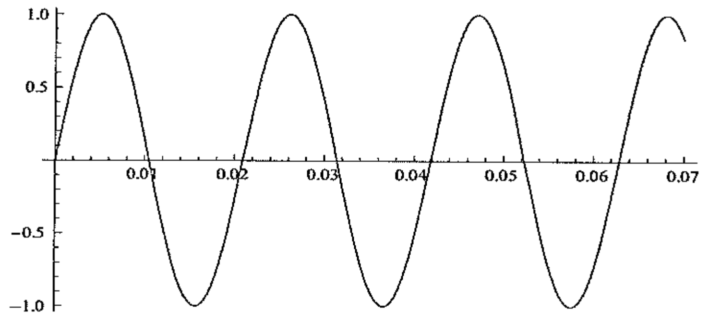
$$\left.\frac{di}{dt}\right|_{t=t_{1/2}} = \frac{\mathcal{E}}{R}\cdot\left(\frac{1}{\tau}\right)e^{-t_{1/2}/\tau} = \frac{\mathcal{E}}{R}\cdot\frac{R}{L}\cdot\frac{1}{2} = \frac{1}{2}\frac{\mathcal{E}}{L}$$

$$\therefore \left.\frac{du_B}{dt}\right|_{t=t_{1/2}} = \frac{1}{2}\cdot\frac{\mathcal{E}}{L}\cdot L\cdot\frac{1}{2}\frac{\mathcal{E}}{R} = \frac{1}{4}\frac{\mathcal{E}^2}{R} = \frac{1}{4}\frac{(12.0\,V)^2}{22\,\Omega}$$

$$= 1.64\ (W)$$

Question 4 [10 points]

The figure shows an oscillating current versus time for an LC circuit where the switch is closed at time $t = 0$.



Now we double the inductance L and also double the capacitance C. Circle the correct statements below.

(ii) (4 pts) The frequency of the current oscillation is:

Doubled   Remained the same   Decreased by half

(ii) (2 pts) The period of the current oscillation is:

Doubled   Remained the same   Decreased by half

Both correct.

(ii) (4 pts) The amplitude i_0 of the current oscillation is:

Doubled   Remained the same   Decreased by half

Charging with the same battery

$q_0' = C'V$

$= 2CV$

$= 2q_0$

$i_0' = \omega' q_0' = \frac{1}{2}\omega\, 2q_0$

$= i_0$

assuming q_0 is the same.

$i_0' = \omega' q_0$

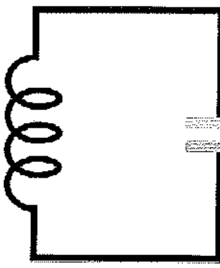
$= \frac{1}{2}\omega q_0$

$= \frac{1}{2} i_0$

Problem 4 [15 points]

An oscillating LC circuit consisting of a 1.0 μF capacitor and a 3.0 mH coil is shown in the figure. It has a maximum voltage of 3.0 V across the capacitor.

(i) (5 pts) What is the maximum charge on the capacitor?



$$q_0 = CV_0 = (1.0 \times 10^{-6} F)(3.0\,V)$$

$$= 3.0 \times 10^{-6} (C)$$

(ii) (5 pts) If the charge on the capacitor is maximized at t = 0, calculate the first time the current in the circuit reaches its largest magnitude.

$$q = q_0 \cos(\omega t + \phi)$$

at $t=0$ $\quad q = q_0 \quad \therefore \phi = 0$

Then $i = \frac{dq}{dt} = -\omega q_0 \sin \omega t$

For i to be its largest magnitude for the first time,

$$t = \frac{\pi}{2}\frac{1}{\omega}$$

$$\therefore \; t = \frac{\pi}{2} \cdot \sqrt{LC} = \frac{\pi}{2}\sqrt{(3.0 \times 10^{-3} H)(1.0 \times 10^{-6} F)} = 8.6 \times 10^{-5} (sec).$$

(iii) (5 pts) What is the maximum energy stored in the magnetic field of the coil?

$$U_{B,max} = U_{E,max}$$

$$= \frac{1}{2}\frac{q_0^2}{C}$$

$$= \frac{1}{2}\frac{(3.0 \times 10^{-6} C)^2}{1.0 \times 10^{-6} F}$$

$$= 4.5 \times 10^{-6} (J).$$