PRINT Your Name: ________________________ Instructor:___________________

Louisiana State University Physics 2102, Exam 3

April 2nd, 2009.

• Please be sure to PRINT your name and class instructor above.

• The test consists of 4 questions (multiple choice), and 4 problems (numerical).

• For the problems: Show your reasoning and your work. Note that in many of the problems, you can do parts (b) or (c) even if you get stuck on (a) or (b).

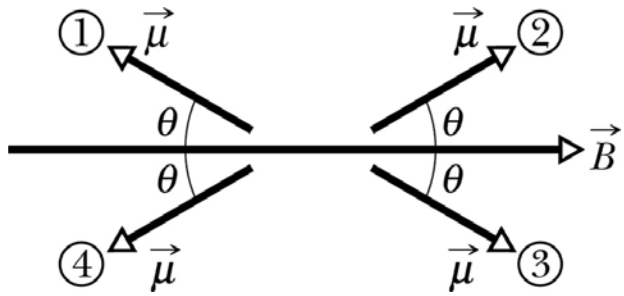
• You may use scientific or graphing calculators, but you must derive and explain your answer fully on paper so we can grade your work.

• Feel free to detach, use, and keep the formula sheet pages. No other reference material is allowed during the exam.

• **May The Force Be With You!**

Question 1 [8 points]

The figure shows four orientations, at angle θ, of a magnetic dipole moment $\vec{\mu}$ in a magnetic field $\vec{B}$.



(i) (4 pts) If the dipole moment $\vec{\mu}$ is rotated from orientation (2) to orientation (1) by an external agent, is the work done on the dipole by the agent? (circle one)

Positive Negative Zero

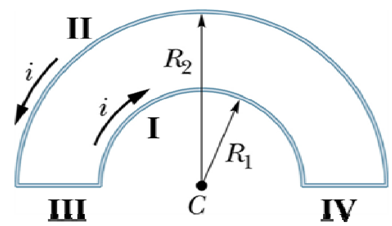
(ii) (4 pts) Rank the work done on the dipole by the agent for these three rotations, greatest first:

A: (2) → (1) B: (2) → (4) C: (2) → (3)

Problem 1 [18 points]

In the figure below, two semicircular arcs (I & II) have radii R_1 = 4.10 cm and R_2 = 6.60 cm, carry current i = 0.281 A, and share the same center of curvature C. The same current i also flows through the straight sections of wire labeled III & IV.

(a) (5 pts) What is the contribution to the magnitude of the magnetic field at point C from the two straight sections of wire III and IV? Explain your answer!



(b) (8 pts) Calculate the magnitude of the magnetic field at the point C due to all four sections of wire.

(c) (5 pts) What is the direction of the total magnetic field at the point C due to all four sections of wire? Circle one:

Out of the page (•).

Into the page ⊗.

Up towards the top of the page ↑.

Down towards the bottom of the page ↓.

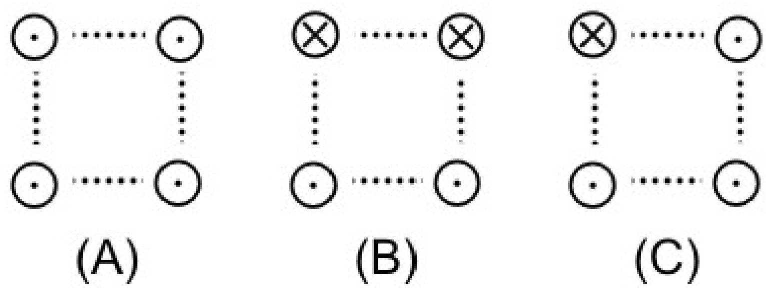
To the right of the page →.

To the left of the page ←.

The total magnetic field at C is zero and has no direction.

Question 2 [8 points]

The figure below shows four arrangements in which long parallel wires carry equal currents *i* directly into or out of the page at the corners of identical squares.



(i) (4 pts) For square B, what is the direction of the magnetic field, with respect to the page, at the center of the square? Circle one:

Out (•) *In* ⊗ *Up* ↑ *Down*↓

Left ← *Right* → *Field is Zero* $(\vec{B} = 0)$

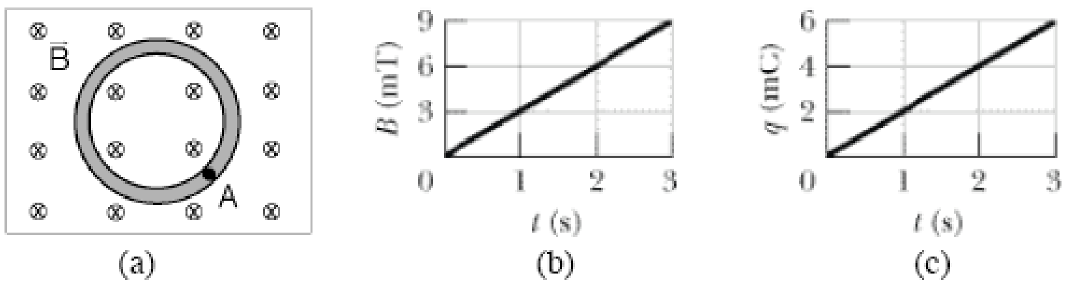
(ii) (4 pts) Rank the sections according to the magnitude of the magnitude of the magnetic field at the center of each square, greatest first. Circle one:

$B_A > B_B > B_C$ $B_C > B_B > B_A$ $B_A > B_C > B_B$

$B_B > B_C > B_A$ $B_A = B_B = B_C$

Problem 2 [14 points]

Figure (a) shows a circular conducting loop, of area 6.0×10^{-4} m^2, which is perpendicular to a uniform magnetic field. Figure (b) shows the increase of the magnitude of the magnetic field with time.



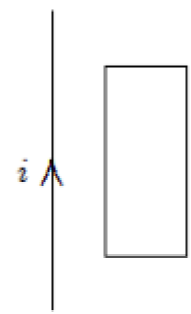
(i) (6 pts) Calculate the magnitude of the emf induced in the loop during this time:

(ii) (3 pts) What is the direction of the current induced in the loop? Answer with words or indicate on the figure.

(iii) (5 pts) Figure (c) shows the amount of charge *q* passing a point *A* on the loop, as a function of time. Calculate the resistance of the loop:

Question 3 [10 points]

A long straight wire is in the plane of a rectangular conducting loop. The straight wire carries a constant current i, as shown.



(i) (5 pts) While the wire is being moved toward the rectangle, the induced current in the rectangle is:

A. zero

B. clockwise

C. counterclockwise

D. into the page

E. out of the page

(ii) (5 pts) The wire is now held in place, and the current i is turned off. While the current is being turned off, the induced current in the rectangle is:

A. zero

B. clockwise

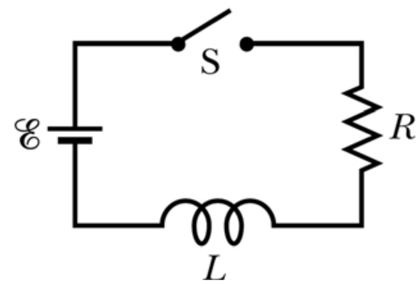
C. counterclockwise

D. into the page

E. out of the page

Problem 3 [17 points]

The figure shows an RL circuit, where R = 22 Ω, L = 1 mH, and the battery voltage $\mathcal{E}$ = 12.0 V.



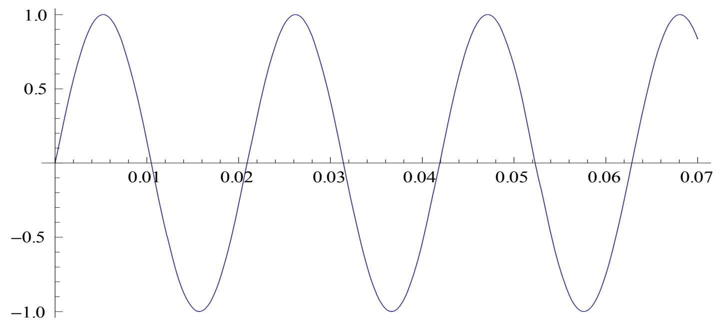
(i) (4 pts) What is the current in the circuit a long time after the switch S is closed?

(ii) (7 pts) How long does it take for the current to reach 1/2 of its maximum value?

(iii) (6 pts) At the time when current reaches 1/2 of its maximum value, what is the rate at which energy is being stored in the inductor?

Question 4 [10 points]

The figure shows an oscillating current versus time for an LC circuit where the switch is closed at time $t = 0$.



Now we double the inductance L and also double the capacitance C. Circle the correct statements below.

(ii) (4 pts) The frequency of the current oscillation is:

Doubled Remained the same Decreased by half

(ii) (2 pts) The period of the current oscillation is:

Doubled Remained the same Decreased by half

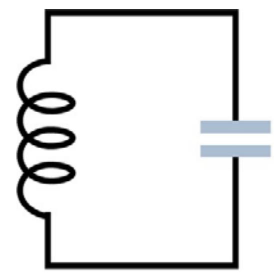
(ii) (4 pts) The amplitude i_0 of the current oscillation is:

Doubled Remained the same Decreased by half

Problem 4 [15 points]

An oscillating *LC* circuit consisting of a 1.0 μF capacitor and a 3.0 mH coil is shown in the figure. It has a maximum voltage of 3.0 V across the capacitor.

(i) (5 pts) What is the maximum charge on the capacitor?



(ii) (5 pts) If the charge on the capacitor is maximized at t = 0, calculate the first time the current in the circuit reaches its largest magnitude.

(iii) (5 pts) What is the maximum energy stored in the magnetic field of the coil?

PHYS 2102 Formula Sheet   Spring 2009.

- **Constants, definitions:**

  $\epsilon_o = 8.85 \times 10^{-12}$ C^2/Nm^2 $k = \dfrac{1}{4\pi\epsilon_o} = 8.99 \times 10^9$ Nm^2/C^2

  $\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}}$ c = 3.00 $\times 10^8$ m/s

  $e = 1.60 \times 10^{-19}$ C   1 eV = e(1V) = 1.60 $\times 10^{-19}$ J

  dipole moment: $\vec{p} = q\vec{d}$ charge densities: $\lambda = \dfrac{Q}{L}$, $\sigma = \dfrac{Q}{A}$, $\rho = \dfrac{Q}{V}$

  $m_e = 9.11 \times 10^{-31}$kg $m_p = 1.67 \times 10^{-27}$kg   gravity $g = 9.81$ m/s^2

  Area of circle: $A = \pi r^2$ Area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

- **Kinematics (constant acceleration) :**

  $v = v_0 + at$ $x - x_0 = \frac{1}{2}(v_0 + v)t$ $x - x_0 = vt - \frac{1}{2}at^2$

  $x - x_0 = v_0 t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2a(x - x_0)$

- **Coulomb's law:** $F = k\dfrac{|\, q_1 \,||\, q_2 \,|}{r^2}$

- **Force on a charge in an electric field:** $\vec{F} = q\vec{E}$

- **Electric field of a point charge:** $E = k\dfrac{|\, q \,|}{r^2}$

- **Electric field of a dipole on axis, far away from dipole:** $E = \dfrac{2kp}{z^3}$

- **Electric field of an infinite line charge:** $E = \dfrac{2k\lambda}{r}$

- **Torque on a dipole in an electric field:** $\vec{\tau} = \vec{p} \times \vec{E}$

- **Potential energy of a dipole in electric field:** $U = -\vec{p} \cdot \vec{E}$

- **Electric flux:** $\Phi = \int \vec{E} \cdot d\vec{A}$

- **Gauss' law:** $\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc}$

- **Electric field of an infinite non-conducting plane with charge density σ:** $E = \dfrac{\sigma}{2\epsilon_o}$

- **Electric field of infinite conducting plane, or close to the surface of a conductor:** $E = \dfrac{\sigma}{\epsilon_o}$

- **Electric potential, potential energy, and work:**

  $V_f - V_i = -\displaystyle\int_i^f \vec{E} \cdot d\vec{s}$,   **In a uniform field:** $\Delta V = -Ed\cos\theta$

  $\vec{E} = -\vec{\bigtriangledown} V$, $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, $E_z = -\dfrac{\partial V}{\partial z}$

  Potential of a point charge q: $V = k\dfrac{q}{r}$

  Potential of n point charges: $V = \displaystyle\sum_{i=1}^{n} V_i = k\sum_{i=1}^{n} \frac{q_i}{r_i}$

  Electric potential energy: $\Delta U = q\Delta V$ $\Delta U = -W_{field}$

  Potential energy of two point charges: $U_{12} = W_{ext} = q_2 V_1 = q_1 V_2 = k\dfrac{q_1 q_2}{r_{12}}$

- **Capacitance** definition: $q = CV$
  Capacitor with a dielectric: $C = \kappa C_{air}$
  Parallel plate: $C = \varepsilon_\circ \dfrac{A}{d}$
  Potential Energy in Cap: $U = \dfrac{q^2}{2C} = \dfrac{1}{2}qV = \dfrac{1}{2}CV^2$ Energy density of electric field: $u = \dfrac{1}{2}\kappa\varepsilon_0 E^2$
  Capacitors in parallel: $C_{eq} = \sum C_i$ Capacitors in series: $\dfrac{1}{C_{eq}} = \sum \dfrac{1}{C_i}$
- **Current:** $i = \dfrac{dq}{dt}$ **Current density:** $J = \dfrac{i}{A}$
- **Drift speed of the charge carriers:** $\vec{v}_d = \dfrac{\vec{J}}{ne}$
- **Definition of resistance:** $R = \dfrac{V}{i}$ **Definition of resistivity:** $\rho = \dfrac{E}{J}$
- **Resistance in a conducting wire:** $R = \rho\dfrac{L}{A}$
- **Power in an electrical device:** $P = iV$ **Power in a resistor:** $P = i^2 R = \dfrac{V^2}{R}$
- **Definition of** emf**:** $\mathcal{E} = \dfrac{dW}{dq}$
- **Resistors in series:** $R_{eq} = \sum R_i$ **Resistors in parallel:** $\dfrac{1}{R_{eq}} = \sum \dfrac{1}{R_i}$
- **Loop rule in DC circuits:** the sum of changes in potential across any closed loop of a circuit must be zero.
- **Junction rule in DC circuits:** the sum of currents entering any junction must be equal to the sum of currents leaving that junction.
- **Charging a capacitor in a series RC circuit:** $q(t) = C\mathcal{E}(1 - e^{-t/\tau_C})$, time constant $\tau_C = RC$
  Discharging: $q(t) = q_0 e^{-t/\tau}$
- **Magnetic Fields**
  Magnetic force on a charge q: $\vec{F} = q\vec{v} \times \vec{B}$ Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$
  Circular motion in a magnetic field: $qvB = \dfrac{mv^2}{r}$ with period: $T = \dfrac{2\pi m}{qB}$
  Magnetic force on a length of wire: $\vec{F} = i\vec{L} \times \vec{B}$
  Magnetic Dipole: $\vec{\mu} = Ni\vec{A}$
  Torque on a Magnetic Dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$
  Energy of a Magnetic Dipole: $U = -\vec{\mu} \cdot \vec{B}$

- **Generating Magnetic Fields**

  $\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{T \cdot m}}{\mathrm{A}}$

  Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^3}$

  Magnetic field of a long straight wire: $B = \frac{\mu_0}{2\pi} \frac{i}{r}$

  Magnetic field at the center of a circular arc: $B = \frac{\mu_0}{4\pi} \frac{i}{r} \phi$

  Force between parallel wires carrying currents: $F_{ab} = \frac{\mu_0 i_a i_b}{2\pi d} L$

  Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

  Magnetic field of a solenoid: $B = \mu_0 i n$

- **Induction:**

  Magnetic Flux: $\Phi = \int \vec{B} \cdot d\vec{A}$

  Faraday's law: $\mathcal{E} = -\frac{d\Phi}{dt}$ $(= -N\frac{d\Phi}{dt}$ for a coil with N turns)

  Induced Electric Field: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}$

  Motional emf: $\mathcal{E} = BLv$

  Definition of Self-Inductance: $L = \frac{N\Phi}{i}$

  Inductance of a solenoid: $L = \mu_0 n^2 A l$

  EMF (Voltage) across an inductor: $\mathcal{E}_L = -L\frac{di}{dt}$

  RL Circuit: **Rise** of current: $i = \frac{\mathcal{E}}{R}(1 - e^{-\frac{t}{\tau_L}})$ Time constant $\tau_L = L/R$

  Decay of current: $i = i_0 e^{-\frac{tR}{L}}$

  Magnetic Energy: $U_B = \frac{1}{2} L i^2$ Magnetic energy density: $u_B = \frac{B^2}{2\mu_0}$

- **LC circuits:**

  Electric energy in capacitor: $U_E = \frac{q^2}{2C} = \frac{CV^2}{2}$

  Magnetic energy in an inductor: $U_B = \frac{Li^2}{2}$

  LC circuit oscillations: $q = Q\cos(\omega t + \phi)$ $(i = \frac{dq}{dt}$ $q = CV)$ $\omega = \frac{1}{\sqrt{LC}}$ $T = \frac{2\pi}{\omega}$ $f = 1/T$

- **Transformers:**

  Transformation of voltage: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ Turns ratio: $\frac{N_p}{N_s}$ Energy conservation: $I_p V_p = I_s V_s$

  Equivalent resistance (as seen by the generator): $R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R_s$

- **Maxwell's Equations**

  $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ $\oint \vec{B} \cdot d\vec{A} = 0$ $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ $\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$

  Displacement current: $i_d = \epsilon_0 \frac{d\Phi_E}{dt}$