PRINT Your Name: SOLUTION Instructor:________________

Louisiana State University Physics 2102, Exam 2,

March 5th, 2009.

- Please be sure to PRINT your name and class instructor above.

- The test consists of 4 questions (multiple choice), and 4 problems (numerical).

- For the problems: Show your reasoning and your work. Note that in many of the problems, you can do parts (b) or (c) even if you get stuck on (a) or (b).

- You may use scientific or graphing calculators, but you must derive and explain your answer fully on paper so we can grade your work.

- Feel free to detach, use, and keep the formula sheet pages. No other reference material is allowed during the exam.

- **May The Force Be With You!**

Question 1 [8 points]

A parallel plate capacitor has a dielectric material of dielectric constant $\kappa > 1$ between the plates. The capacitor is charged by first connecting it to a 12 V battery and then disconnecting it from the battery. After it is disconnected, the dielectric is pulled out. Compared to before the dielectric is removed, what happens after it is removed? (Circle the correct answer for each part.)

✓

(i) The electric potential across the capacitor:

decreases   stays the same   [✓ increases] V INC

Q

(ii) The charge on the capacitor:

decreases   [✓ stays the same]   increases Q CONST

C

(iii) The capacitance:

[✓ decreases]   stays the same   increases C DEC

U

(iv) The potential energy stored in the capacitor:

decreases   stays the same   [✓ increases] U INC

DISCONNECTED $\Rightarrow$ Q = CONST

FORMULA SHEET : $C' = KC = K\varepsilon_0 A/d$

$K > 1$ SO REMOVING DIELECTRIC C DECREASES

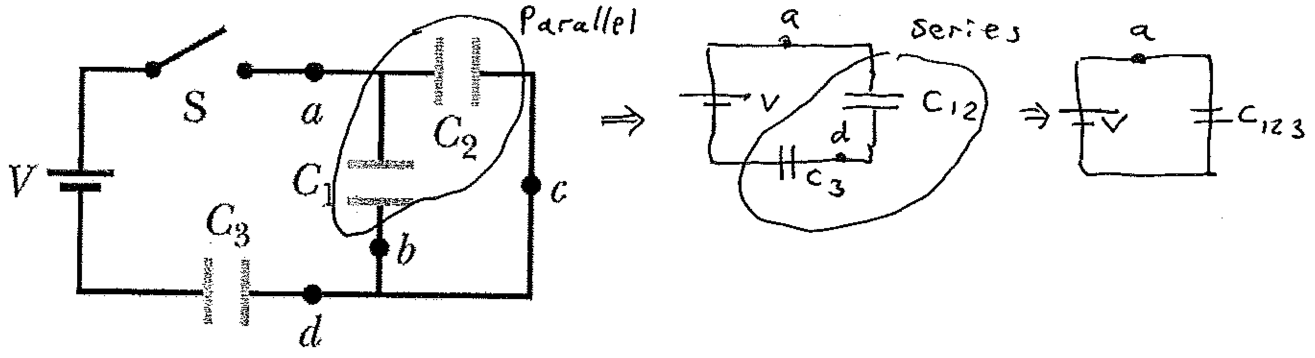
$C_{AFTER} < C_{BEFORE}$

$U_{AFTER} = \frac{Q^2}{2C_A} > \frac{Q^2}{2C_B} = U_{BEFOR} \therefore U$ INC.

$V_{AFTER} = \frac{Q}{C_{AFTER}} > \frac{Q}{C_B} = V_B \therefore V$ INC

Problem 1 [18 points]

Consider the circuit in the figure below. The capacitors have capacitance values $C_1 = 0.75$ pF, $C_2 = 0.50$ pF, and $C_3 = 1.00$ pF. The battery voltage is 12 V.



(a) (7 pts) What is the equivalent capacitance of this circuit?

$$C_{12}^{PAR} = C_1 + C_2 = 0.75\,pF + 0.50\,pF = 1.25\,pF$$

$$C_{123}^{SER} = \frac{C_{12}\ C_3}{C_{12}+C_3} = \frac{(1.25\,pF)(1.00\,pF)}{(1.25\,pF)+(1.00\,pF)} = \boxed{\begin{array}{l} 0.55\,pF \\ = C_{eq} \end{array}}$$

ANS!

(b) (6 pts) Long after the switch is closed, how many electrons have traveled through point *a*?

$$Q_{123} = VC_{eq}^{123} = (0.55\times10^{-12}F)(12V)$$

$$= 6.66\times10^{-12}\,C$$

$$\#\ \text{electrons} = \frac{Q}{e} = \frac{6.66\times10^{-12}\,C}{1.60\times10^{-19}\,C} = \boxed{4.17\times10^{7}\ \text{electrons}}$$

↑ charge per electron

ANS!

(c) (5 pts) After the capacitors are charged, what is the voltage drop across capacitor C_1?

series: $Q_{123} = Q_{12} = Q_3$ CHARGE SAME

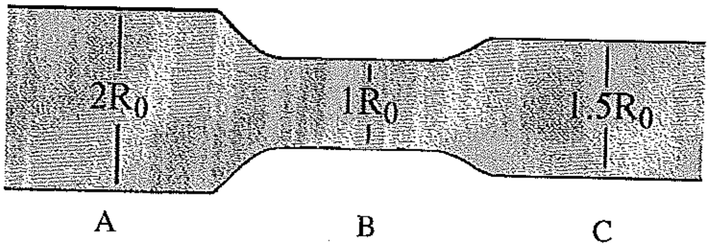
$$\Rightarrow\ V_{12} = Q_{12}/C_{12} = 6.66\times10^{-12}\,C\,/\,1.25\times10^{-12}\,F$$

$$= 5.33V$$

PARALLEL: $V_1 = V_2 = V_{12} = \boxed{5.33V}$ ANS!

Question 2 [9 points]

The figure shows a current-carrying wire consisting of three sections, which have different radii, as marked on the figure:



(a) (3 pts) Rank the sections according to the magnitude of the current, greatest first (circle one):

$i_A > i_B > i_C$ $\quad$ $i_C > i_B > i_A$ $\quad$ $i_B > i_C > i_A$

$i_A > i_C > i_B$ $\quad$ [✓ all tie] $\quad$ CURRENT IS CONSERVED

(b) (3 pts) Rank the sections according to the magnitude of the current density, greatest first (circle one):

$J_A > J_B > J_C$ $\quad$ $J_C > J_B > J_A$ $\quad$ [✓ $J_B > J_C > J_A$]

$J_A > J_C > J_B$ $\quad$ all tie

CURRENT DENSITY $= J = i/A$
IS LARGEST IN NARROWEST PIPE

(c) (3 pts) Rank the sections according to the magnitude of the electric field, greatest first (circle one):

$E_A > E_B > E_C$ $\quad$ $E_C > E_B > E_A$ $\quad$ [✓ $E_B > E_C > E_A$]

$E_A > E_C > E_B$ $\quad$ all tie

$E = \rho J$

But ρ const

Hence E tracks J

Problem 2 [18 points]

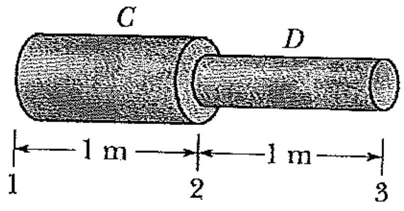
ρ_c ρ_D A_c

Wire *C* and wire *D* have lengths $L_C = L_D = 1.0$ m. Wire *C* is made of copper and has a resistivity of 1.69×10^{-8} Ω•m and a cross-sectional area of 4.50×10^{-6} m². Wire *D* is made of aluminum and has a resistivity of 2.75×10^{-8} Ω•m and a cross-sectional area of 1.33×10^{-6} m². The wires are joined as shown in the figure, and a current of 2.0 A is sent down the two wires.

i

A_D

WIRES ARE IN SERIES



$$R_{eq} = R_C + R_D$$

$$= \left[\rho_c \frac{L_c}{A_c} + \rho_D \frac{L_D}{A_D}\right]$$

✓

(a) (6 pts) Calculate the electric potential difference between points 1 and 3:

$$\Rightarrow R_{eq} = \left[(1.64\times10^{-8}\,\Omega\text{m})\frac{(1.0\text{m})}{(4.5\times10^{-6}\text{m}^2)} + (2.75\times10^{-8}\,\Omega\cdot\text{m})\frac{(1.0\text{m})}{(1.33\times10^{-6}\text{m}^2)}\right]$$

$$= 0.245\,\Omega$$

$$V = i R_{eq} = (2.0\text{A})(0.245\,\Omega) = \boxed{0.49\text{ V}}$$ ANS!

$$\boxed{P = iV = i^2R = \frac{V^2}{R}}$$

C

i const

(b) (6 pts) Calculate the rate at which energy is dissipated between points 1 and 2:

$$P_c = i^2 R_c = i^2 \left[\rho_c \frac{L_c}{A_c}\right]$$

ANS!

$$= \frac{(2\text{A})^2(1.64\times10^{-8}\,\Omega\cdot\text{m})(1.0\text{m})}{4.5\times10^{-6}\text{m}^2} = \boxed{0.15\text{W}}$$

Watts

n_c

(c) (6 pts) Calculate the drift speed of the electrons in wire *C*, given that copper has a charge carrier density of 8.49×10^{28} m^{-3}:

$$v_D = \frac{J_c}{n_c e} \qquad J_c = i/A_c = (2.0\text{A})/(4.50\times10^{-6}\text{m}^2)$$

$$= 4.44\times10^{5}\text{ A/m}^2$$

ANS!

$$v_D = \frac{J_c}{n_c e} = \frac{(4.44\times10^5\text{ A/m}^2)}{(8.49\times10^{28}/\text{m}^3)(1.60\times10^{-19}\text{C})} = \boxed{3.27\times10^{-5}\text{ m/s}}$$

Question 3 [8 points]

In the figure below to the left, the switch is closed to point A at time $t = 0$. After closing the switch, there is a current i through resistor R. In the figure below (to the right) it shows this current i, as a function of time t, for four sets of values of resistance R and capacitance C.

CHARGING $\underline{RC}$

Set (1): $R = 2.5\ \Omega$ and $C = 3.0\ \mu F$

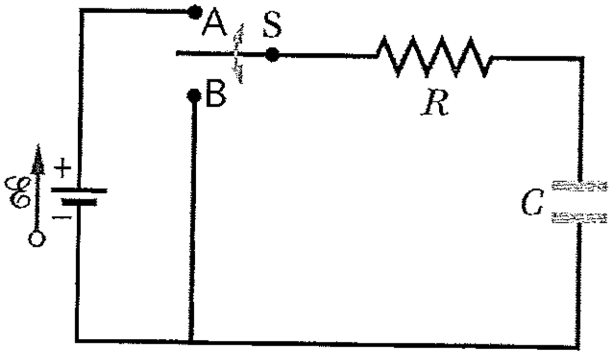
~~Set (2): $R = 5.0\ \Omega$ and $C = 3.0\ \mu F$~~

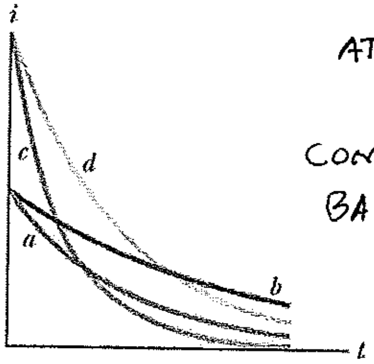
~~Set (3): $R = 2.5\ \Omega$ and $C = 6.0\ \mu F$~~

Set (4): $R = 5.0\ \Omega$ and $C = 6.0\ \mu F$

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau})$$

$$\frac{dq}{dt} = i = \frac{C\mathcal{E}e^{-t/\tau}}{\tau} = \frac{\cancel{C}\mathcal{E}e^{-t/\tau}}{R\cancel{C}} = \boxed{\frac{\mathcal{E}}{R}}e^{-t/\tau}$$





AT $t = 0$ $i = \mathcal{E}/R$

CONNECTED TO BATT $\Rightarrow$ $\mathcal{E} = V$

$= \underline{\text{CONST}}$

(a) (4 pts)

In set (1) above, the R and C values go with which curve (circle one):

$\mathcal{E} = V = \text{const}$

$i(0) = \mathcal{E}/R_a$

$R_a = 2.5\Omega > 5.0\Omega$

$R \Rightarrow$ SMALL

$\underline{i(0) \Rightarrow \text{LARGE}}$

SHORT TIME & LARGE $i(0)$

(i) curve (a) (ii) curve (b)

[✓ (iii) curve (c)] (iv) curve (d)

$\tau = RC = 7.5\mu s \Rightarrow$ $\underline{\text{SHORT TIME}}$

(b) (4 pts)

In set (4) above, the R and C values go with which curve (circle one):

(i) curve (a) [✓ (ii) curve (b)]

(iii) curve (c) (iv) curve (d)

$RC = \tau = 30\mu s$

$\underline{\text{LONG TIME}}$

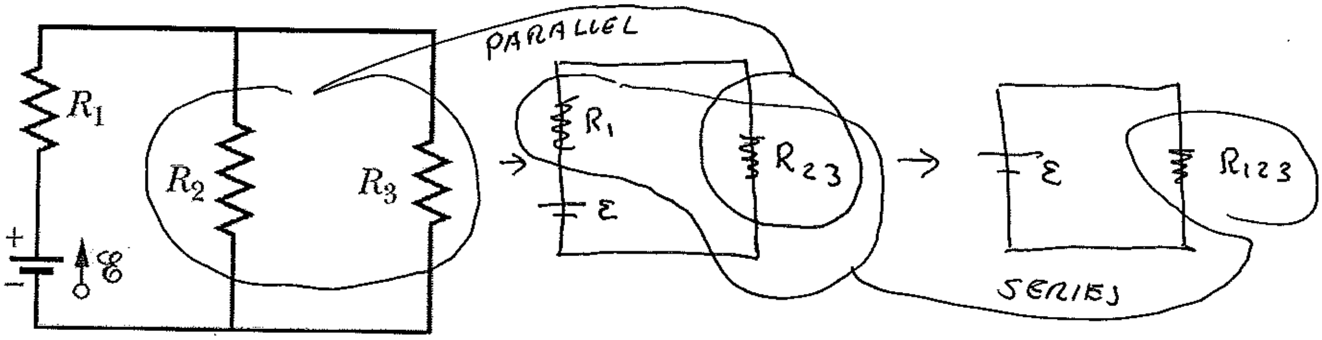
LONG TIME & SMALL $i(0)$

$\mathcal{E}/R_b < \mathcal{E}/R_a$

$$i(0)_B = \frac{\mathcal{E}}{5.0\Omega} < \frac{\mathcal{E}}{2.5\Omega} = i(0)_A$$

Problem 3 [18 points]

In the figure below, the resistors have resistance values of $R_1 = 1.2\ \Omega$, $R_2 = 2.5\ \Omega$, $R_3 = 10.0\ \Omega$, and the ideal battery has an EMF of $\mathcal{E}$=3.0 mV.



(a) (5 pts) Find the equivalent resistance for R_2 and R_3. PARALLEL

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \Rightarrow R_{23} = \frac{R_2 R_3}{R_2 + R_3}$$

$$= \frac{(2.5\Omega)(10.0\Omega)}{2.5\Omega + 10.0\Omega} = \boxed{2.0\Omega = R_{23}}$$ ANS!

(b) (6 pts) What is the current through the battery? $V_{123} = \mathcal{E} = i R_{123}$

$$R^{\text{SERIES}}_{123} = R^{EQ}_{123} = R_1 + R_{23} = 1.2\Omega + 2.0\Omega = 3.2\Omega$$

ANS!

$$i = \frac{V_{123}}{R_{123}} = \frac{\mathcal{E}}{R_{123}} = \frac{3.0\times10^{-3}V}{3.20\Omega} = \boxed{9.38\times10^{-4}A}$$

(c) (7 pts) What is the power being dissipated in the resistor $\underline{R_2}$?

$$P_2 = i_2 V_2 = i_2^2 R_2 = \boxed{V_2^2/R_2}$$

SERIES: $i_1 = i_{23} = i_{123} = V_{123}/R_{123} = \mathcal{E}/R_{123}$

$= 9.38\times10^{-4}A \Rightarrow V_{23} = i_{23}R_{23} = (9.38\times10^{-4}A) \times (2.0\Omega) = 1.88\times10^{-3}V$

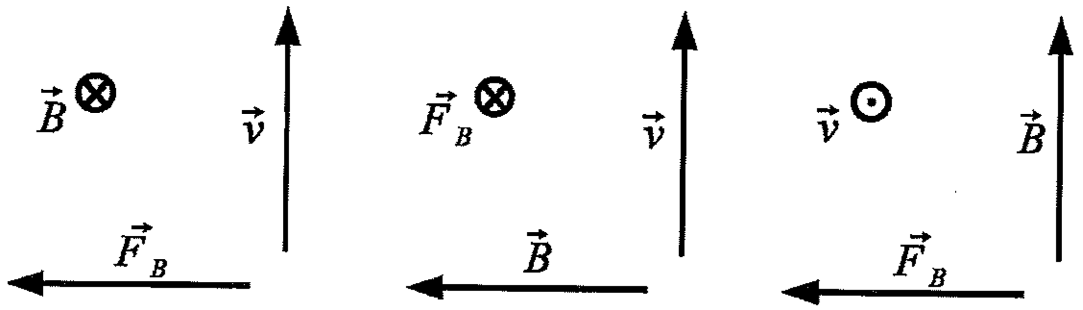
Parallell: $\underline{V_2} = V_3 = V_{23} = 1.88\times10^{-3}V$

$$\Rightarrow P_2 = V_2^2/R_2 = (1.88\times10^{-3}V)^2/(2.5\Omega) = \boxed{1.41\times10^{-6}\text{ WATTS}}$$

ANS!

Question 4 [8 points]

In the figure you see three situations in which a *negatively* charged particle moves at velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ and experiences a force $\vec{F}_B$.



(a) (b) (c)

Which of the depicted situations are correct? Circle one:

(i) (a)

[✓ (ii) (b)]

(iii) (c)

(iv) (a) and (b)

(v) (b) and (c)

(vi) (a) and (c)

(vii) (a) and (b) and (c)

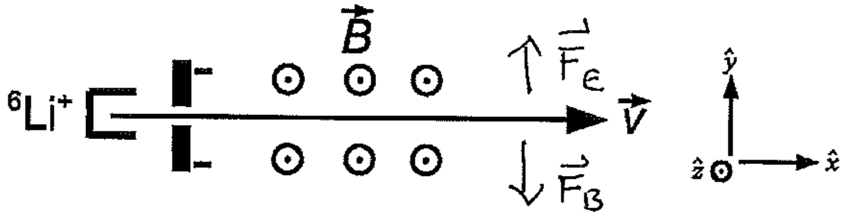
ONLY B CONSISTENT WITH RIGHT HAND RULE & NEG CHARGE!

ⓐ and ⓒ CONSISTENT W/ POSITIVE CHARGE

$$(1e)\times(1\,\text{Volt}) = 1\,\underline{\text{eV}}$$

Problem 4 [13 points]

An ion source S is producing positively charged ^{6}Li ions, which have a charge $+e$ and a mass of 9.99×10^{-27} kg, as shown in the figure.



(a) (3 pts) The ions are accelerated by a potential difference of 10 kV. What is their final speed?

$$KE = (1e)\times(10\,\text{kV}) = 10\,\text{k}\cancel{\text{eV}} \;\Big|\; \frac{1.60\times10^{-19}\,\text{J}}{\cancel{\text{eV}}}$$

$$= 10\times10^{3}\times 1.60\times10^{-19}\,\text{J}$$

$$= 1.60\times10^{-15}\,\text{J} = K$$

CONVERT eV TO JOULES!

$$\tfrac{1}{2}mv^2 = KE = K \Rightarrow v = \sqrt{\frac{2K}{m}}$$

$$= \sqrt{\frac{2(1.60\times10^{-15}\,\text{J})}{(9.99\times10^{-27}\,\text{kg})}} = \boxed{5.66\times10^{5}\,\text{m/s}}$$ ANS!

(b) (5 pts) The ^{6}Li ions enter horizontally a region in which there is a uniform magnetic field with magnitude $B = 1.2$ T, pointing out of the page as shown. What is the direction and magnitude of the force $\vec{F}_B$ on the ions due to the magnetic field? (Draw the direction on the figure or use the coordinate system to specify it.)

RHR FOR Q POSITIVE ⇒ $\vec{F}_B$ IS DOWN $= \boxed{-\hat{\jmath}}$ ANS!

$$F_B = qvB = (1.60\times10^{-19}\,\text{C})(5.66\times10^{5}\,\text{m/s})(1.2\,\text{T}) = \boxed{1.09\times10^{-13}\,\text{N}}$$ ANS!

(c) (5 pts) What is the direction and magnitude of an electric field necessary to create a force, which balances $\vec{F}_B$ such that the ^{6}Li ions pass un-deflected through region with the two fields? (Draw the direction on the figure or use the coordinate system to specify it.)

$\vec{F}_E = q\vec{E}$ is parallel to $\vec{E}$ and must point up so $\vec{E}$ points up $\boxed{+\hat{\jmath}}$ ANS!

$$|\vec{F}_E| = |\vec{F}_B| \Rightarrow \cancel{q}E = \cancel{q}vB \Rightarrow E = vB$$ ANS!

$$= (5.66\times10^{5}\,\text{m/s})(1.2\,\text{T}) = \boxed{6.79\times10^{5}\,\text{V/m}}$$