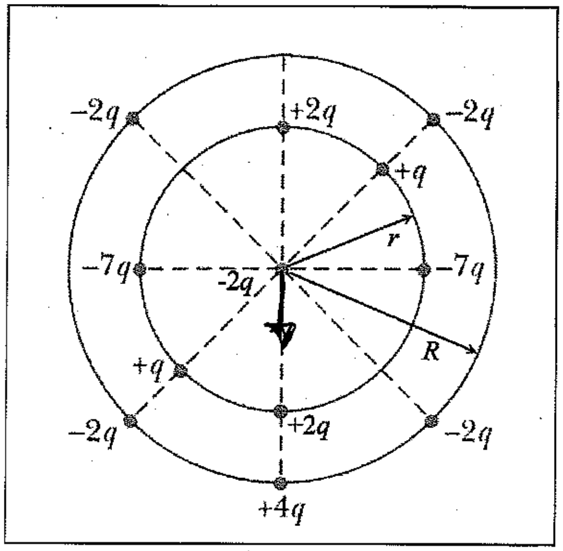
Question 1 [8 points]

The figure shows a central particle of charge –2q, surrounded by two circular rings of charged particles. The radii of the rings are r and R, as shown.



(i) (4 pts) What is the magnitude of the net electrostatic force on the central particle due to the other particles? Circle the right answer.

(a) $4kq/R^2$

(b) $4kq^2/R^2$

(c) $8kq/R^2$

(d) $8kq^2/R^2$

(e) zero

Attractive force. All charges cancel except +4q at the bottom.

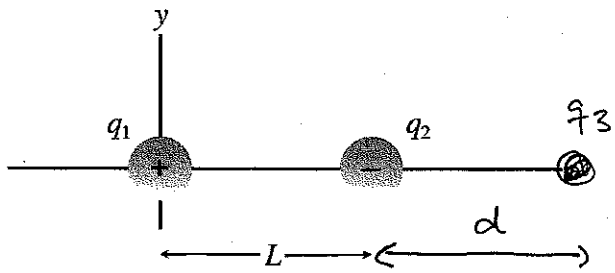
$$F = k\frac{q_1 q_2}{R^2} = k\frac{(2q)(4q)}{R^2}$$

(ii) (4 pts) What is the direction of the net electrostatic force on the central particle due to the other particles? (circle the right answer):

(a) rightward
(b) upward
(c) leftward
(d) downward
(e) force is zero, no direction

Problem 1 [17 points]

The figure shows two point charges $q_1 = +4Q$ and $q_2 = -Q$ which are held at separation L on an x-axis.



(i) (4 pts) Where on the x-axis can a charge $q_3 = -Q$ be placed so that it is in equilibrium? (Circle the right answer)

(a)To the left of q_1

(b) Between q_1 and q_2

(c) To the right of q_2

(ii) (3 pts) Briefly explain your answer in (i)

- Must be further away from bigger charge ($q_1 = +4Q$)
- Must be outside charges because q_1 and q_2 have opposite signs

(iii) (10 pts) Give an expression for the x-coordinate of this point, in terms of L:

$$F_{31} = F_{32} \quad (=) \quad \cancel{k}\frac{\cancel{q_3}q_1}{(L+d)^2} = \cancel{k}\frac{\cancel{q_3}q_2}{d^2}$$

$$(=) \quad q_2(L+d)^2 = q_1 d^2 \qquad |q_1| = 4Q, \quad |q_2| = Q$$

$$(=) \quad \cancel{Q}(L^2 + 2Ld + d^2) = 4\cancel{Q}d^2$$

$$(=) \quad 3d^2 - 2Ld - L^2 = 0$$

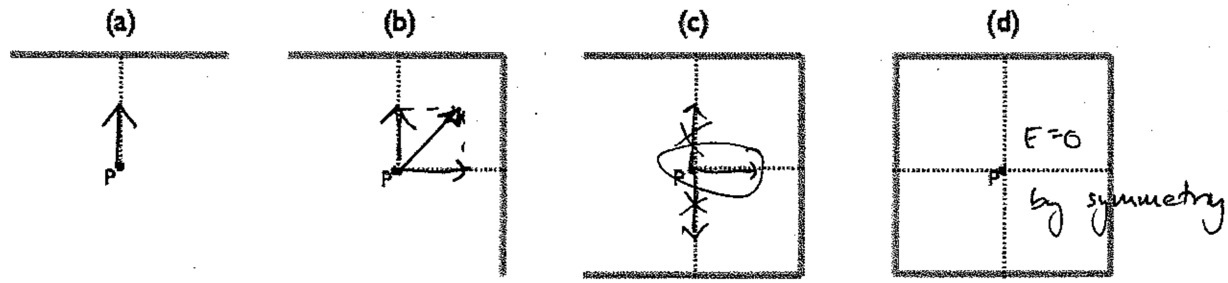
$$(=) \quad d = \frac{+2L \pm \sqrt{4L^2 - 4\cdot 3(-L^2)}}{6} = \frac{L}{3} \pm \frac{1}{6}\sqrt{16 L^2}$$

$$= \frac{L}{3} \pm \frac{2L}{3} = \begin{cases} L \\ \cancel{-L/3} \end{cases} \quad d \text{ is positive by definition.}$$

So x-coordinate $\boxed{x = L + d = 2L}$

Question 2 [8 points]

Figure (a) shows a plastic rod which has charge $-Q$ uniformly spread along its length. In Figures (b), (c), and (d) more plastic rods are added to form sides of a square, each rod has the same negative charge $-Q$. The point P is at the center of the square.



(i) (3 pts) Indicate the direction of the electric field at point P, for figure (c) (circle the right answer:

$\rightarrow$ $\qquad$ $\nearrow$ $\qquad$ $\uparrow$ $\qquad$ $\leftarrow$ $\qquad$ $\downarrow$

(ii) (5 pts) Rank the scenarios according to the magnitude of the electric field at point P (circle the right answer):

$E_a > E_b > E_c > E_d$ $\qquad$ $E_b = E_c > E_a > E_d$ $\qquad$ All tie

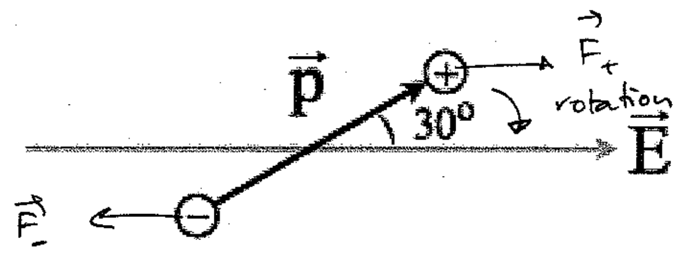
$E_b > E_a = E_c > E_d$ $\qquad$ $E_d > E_c > E_b > E_a$

$E_d = 0$, $E_a = E_c$ field from one side

E_b larger because fields add up

Problem 2 [17 points]

An electric dipole, with equal and opposite charges of magnitude 5.1 μC that are separated by 450 nm, is located in a uniform electric field of magnitude 10 N/C. The dipole is initially held at 30° with respect to the electric field, as indicated in the figure:



(a) (4 pts) Calculate the magnitude of the electric dipole moment:

$$|\vec{p}| = qd = 5.1 \cdot 10^{-6}\,C \cdot 450 \cdot 10^{-9}\,m = \underline{\underline{2.3 \cdot 10^{-12}\ Cm}}$$

(b) (3 pts) If the dipole were not held in place, in which direction would it rotate? Answer with words or indicate on the figure.

Clockwise

(c) (10 pts) Instead, the dipole is rotated from an angle of 30° to an angle of 115°. Calculate the change in the potential energy of the dipole.

U increases, since we are rotating dipole against where it wants to go.

$$\Delta U = U_f - U_i$$

$$= -pE\cos 115° - (-pE\cos 30°)$$

$$= pE(-\cos 115° + \cos 30°)$$

$$= 2.3 \cdot 10^{-12}\,Cm \cdot 10\,\frac{N}{C} \cdot (-\cos 115° + \cos 30°)$$

$$= \underline{\underline{2.96 \cdot 10^{-11}\,J}}$$

Question 3 [10 points]

An isolated conductor of arbitrary shape has a net charge of +10 μC. Inside the conductor is a cavity within which is a point charge q = +3.0 μC.

(i) (5 pts) What is the charge on the wall of the cavity? Circle the right answer.

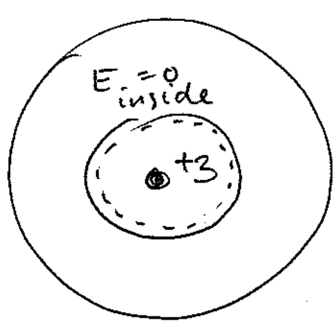
+3.0 μC   –3.0 μC   +10 μC

–10 μC   +13 μC

(ii) (5 pts) What is the charge on the outer surface of the conductor? Circle the right answer:

+3.0 μC   –3.0 μC   +10 μC

+7 μC   +13 μC



(i) Use Gauss' law, with surface just inside conducting material

$q_{encl} = \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = 0$

since $E = 0$ inside metal.

So $q_{encl} = q_{point} + q_{inner} = 0$

$\Rightarrow q_{inner} = -q_{point} = \underline{\underline{-3\ \mu C}}$

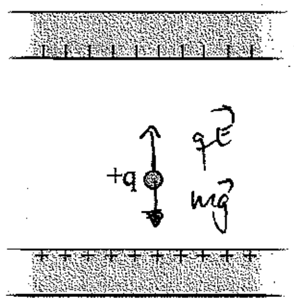
(ii) By charge conservation

$q_{TOT,\ SHELL} = 10\ \mu C = q_{inner} + q_{outer}$

$\Rightarrow q_{outer} = 10\ \mu C - q_{inner} = 10\ \mu C - (-3\ \mu C) = \boxed{+13\ \mu C}$

Problem 3 [17 points]

The figure shows (sections of) two very large conducting plates which have equal and opposite charge densities. A neon nucleus of charge $q = +1.6 \times 10^{-18}$ C and mass $m = 3.4 \times 10^{-26}$ kg is at rest in between the two plates, because the electrostatic force exactly balances the gravitational force. The electric field between the plates is uniform and is not altered by the presence of the neon nucleus.



(a) (5 pts) Calculate the magnitude of the electric field between the plates:

$$mg = |q||E| \Leftrightarrow E = \frac{mg}{q}$$

$$= \frac{3.4 \cdot 10^{-26}\ \text{kg} \cdot 9.8\ \text{m/s}^2}{1.6 \cdot 10^{-18}\ \text{C}} = \underline{\underline{2.1 \cdot 10^{-7}}}\ \frac{\text{N}}{\text{C}}$$

(b) (7 pts) Calculate the surface charge density on each of the plates:

E-field from conductor $E = \frac{\sigma}{\varepsilon_0}$, where σ is the charge on <u>each</u> plate. (We do <u>not</u> add E from + and − plates separately – we have already taken that into account because the other plate pulls all charge to the inside)

$$\sigma = \varepsilon_0 E$$

$$= 8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2} \cdot 2.1 \cdot 10^{-7} \frac{\text{N}}{\text{C}}$$

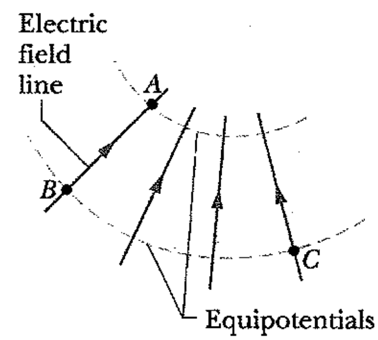
$$= \underline{\underline{1.9 \cdot 10^{-18}}}\ \text{C/m}^2$$

(c) (5 pts) What is the magnitude of the electric field below the bottom plate:

$\boxed{E = 0}$ outside plates because E-field cannot penetrate conducting material, and there is no charge on the outside of plates.

Question 4 [10 points]

When an electron moves from A to B along the electric field line in the figure below, the electric field does $+3.20 \times 10^{-19}$ J of work on it.



(i) (4pts.) What is the electric potential difference $V_B - V_A$?

a) 0 V
b) 2 V
c) –2 V
d) None of the above are even approximately correct

V decreases in direction of $\vec{E}$

$$\Delta V = \frac{\Delta U}{q} = \frac{-W_{field}}{q} = \frac{-3.2\cdot 10^{-19}\ \text{J}}{-1.6\cdot 10^{-19}\ \text{C}} = +2\,\text{V}$$

(ii) (3pts.)What is the electric potential difference $V_B - V_C$?

a) 0 V
~~b) 2 V~~
c) –2 V
d) None of the above are even approximately correct

B, C on same equipotential surface

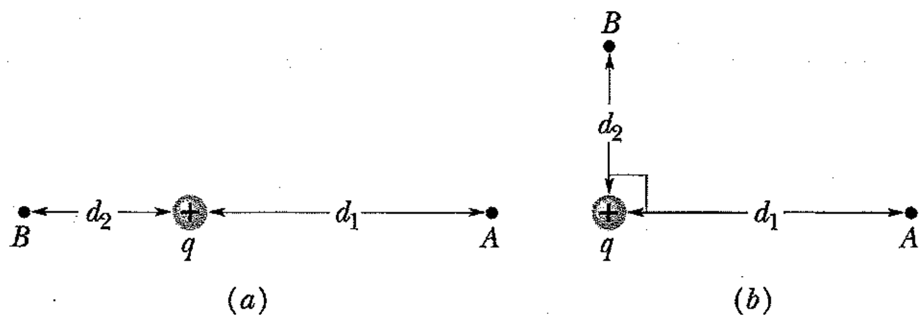
(iii) (3pts.) What is the electric potential difference $V_A - V_C$?

a) 0 V
b) 2 V
c) –2 V
d) None of the above are even approximately correct

$$V_A - V_C = V_A - V_B = -(V_B - V_A)$$

Problem 4 [13 points]

Consider a point charge $q = 3.0$ nC, point A at distance $d_1 = 5.0$ cm from q, and point B at distance $d_2 = 2.5$ cm.



(i) (4 pts.) Calculate the electric potential at point A, relative to $V = 0$ at infinity:

$$V_A = k\frac{q}{d_1} = 8.99 \cdot 10^9 \frac{Nm^2}{C^2} \cdot \frac{3.0 \cdot 10^{-9}\,C}{5.0 \cdot 10^{-2}\,m} = \underline{\underline{539\ V}}$$

(ii) (5 pts) If A and B are opposite to each other as shown in figure (a), what is the electric potential difference $V_A - V_B$?

$$V_A - V_B = V_A - k\frac{q}{d_2}$$

$$= 539\ V - 8.99 \cdot 10^9 \frac{Nm^2}{C^2} \cdot \frac{3.0 \cdot 10^{-9}\,C}{2.5 \cdot 10^{-2}\,m}$$

$$= \cancel{-839.41} \quad \underline{\underline{-539\ V}}$$

(iii) (4 pts.) What is the electric potential difference $V_A - V_B$, if A and B are located as in figure (b)?

Same - distances are still the same, geometry does not matter.