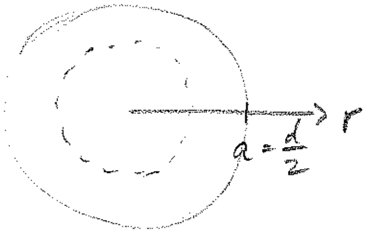
Problem 1 [20 points]

A long cylindrical wire of diameter 1.0 cm carries a total current of 75 A uniformly distributed over its cross section.

(a) [5 pts] Calculate the current density in the wire:

$$J = \frac{i}{A} = \frac{75\ A}{\pi\left(\frac{1.0\cdot 10^{-2}m}{2}\right)^2} = 9.55\cdot 10^5\ \frac{A}{m^2}$$

(b) [8 pts] At what distance from the axis of the wire, but still **inside the wire**, does the magnetic field have a magnitude of 1.0 mT? (Hint: think Ampere's Law)



$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{encl}$$

Use circle w. radius r for path.

$|B|$ const., always same dir. as $d\vec{s}$:

$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = B\cdot 2\pi r$$

$$\mu_0 i_{encl} = \mu_0 \int_{encl.} J\,dA = \mu_0 J\cdot A_{encl} = \mu_0 J \pi r^2$$

$$B\cdot 2\cancel{\pi}\cancel{r} = \mu_0 J \cancel{\pi} r^2$$

$$\Rightarrow r = \frac{2B}{\mu_0 J} = \frac{2\cdot 1.0\cdot 10^{-3}\ T}{4\pi\cdot 10^{-7}\ \frac{Tm}{A}\cdot 9.55\cdot 10^5\ \frac{A}{m^2}} = \underline{\underline{1.7\ mm}} \quad (< a = 5mm)$$

(c) [7 pts] At what distance from the axis of the wire, but now **outside the wire**, does the field once again have a magnitude of 1.0 mT? (Hint: can be done independently of part (b)).

Outside $\quad B(r) = \frac{\mu_0 i}{2\pi r}$

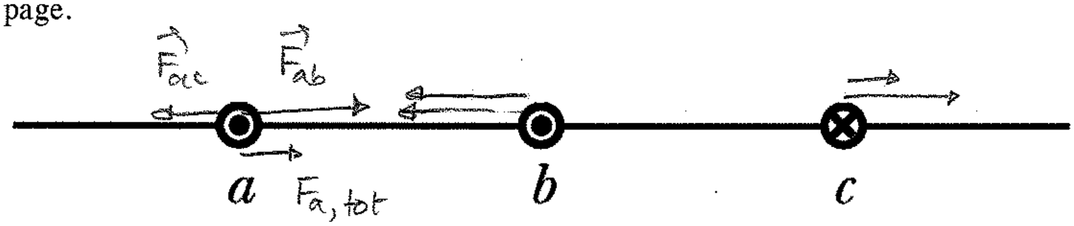
$$\Rightarrow r = \frac{\mu_0 i}{2\pi B}$$

$$= \frac{4\pi\cdot 10^{-7}\ \frac{Tm}{A}\cdot 75\ A}{2\pi\cdot 1.0\cdot 10^{-3}\ T}$$

$$= \underline{\underline{1.5\ cm}} \quad (> a = 0.5\ cm)$$

Question 1 [10 points]

The figure shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page.



(a) [4 pts] What is the direction of the net force on wire a, due to the other two wires:

a) Toward the right

b) Upward

c) Toward the left

d) Downward

e) net force is zero

Parallel attract

Opposite repel

$$\frac{F_{12}}{L} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

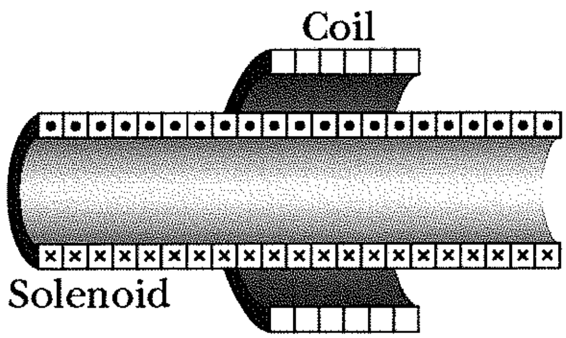
(b) [6 pts] Rank the wires according to the magnitude of the net force on each of them, due to the other two wires

$F_b > F_c > F_a$ $\qquad$ $F_c > F_a > F_b$ $\qquad$ $F_b > F_a = F_c$

$F_c > F_b > F_a$ $\qquad$ $F_a = F_b = F_c$

Problem 2 [16 points]

The figure shows a 120 turn coil of radius 1.8 cm and resistance 7.0 Ω which is coaxial with a solenoid with 210 turns/cm and radius 1.6 cm. The current in the solenoid drops from 1.5 A to zero at in the time interval from t = 0 to t = 25 ms.



(a) [5 pts] Calculate the magnitude of the magnetic field inside the **solenoid** at time t = 0:

$$B_{sol} = \mu_0 i n_{sol}$$

$$= 4\pi \cdot 10^{-7} \frac{Tm}{A} \cdot 1.5A \cdot 210 \frac{turns}{cm} \cdot 100 \frac{cm}{m}$$

$$= \underline{0.0396}\ T$$

(b) [11 pts] Calculate the magnitude of the current which is induced in the **coil** during the time between t = 0 and t = 25 ms.

$$i = \frac{\mathcal{E}}{R} = \frac{1}{R}\left| N \frac{d\Phi_B}{dt} \right| \qquad \text{where } N = N_{coil}$$

$$= \frac{N_c}{R}\left| \frac{d}{dt} \int \vec{B} \cdot d\vec{A} \right| = \frac{N_c}{R}\left| \frac{d}{dt}(BA_s) \right|$$

where $A_s = A_{solenoid}$ since $B = 0$ outside solenoid.

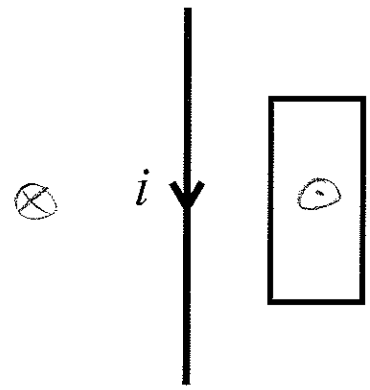
$$i = \frac{N_c}{R} A_s \left| \frac{dB}{dt} \right| = \frac{N_c \pi r_s^2}{R} \mu_0 n_{sol} \left| \frac{di}{dt} \right|$$

$$= \frac{120\ \pi \cdot (1.6 \cdot 10^{-2} m)^2}{7\ \Omega} \cdot 4\pi \cdot 10^{-7} \frac{Tm}{A} \cdot 21000 \frac{t}{m} \cdot \frac{1.5A - 0A}{25 \cdot 10^{-3} s}$$

$$= \underline{0.0218}\ A$$

Question 2 [10 points]

The figure shows a long straight wire placed next to (in the plane of) a rectangular conducting loop. The straight wire carries a constant current i.



(i) [4 pts] What is the direction of the magnetic field from the long straight wire, at the position of the loop?

a) Toward the right

b) Toward the left

c) Into the page

d) Out of the page

e) The magnetic field is zero

(ii) [6 pts] Now the wire is moved toward the loop. While the wire is being moved toward the loop, what is the direction of the induced current in the loop:

a) Counterclockwise

b) Clockwise

c) Counterclockwise on the left side and clockwise on the right side

d) Clockwise on the left side and counterclockwise on the right side

e) The induced current is zero

Φ_B from $\vec{B}$ $\odot$ incr $\Rightarrow$ $\vec{B}_{ind}$ $\otimes$

$\Rightarrow$ i_{ind} CW

Problem 3 [16 points]

A coil with an inductance of 1.6 H and a resistance of 10 Ω is suddenly connected to an ideal battery with an emf $\mathcal{E} = 100$ V.

(a) [11 pts] At a time of 0.5 s after the connection is made, calculate the rate at which energy is being stored in the magnetic field:

$$u_B(t) = \frac{1}{2} L i^2(t) \qquad \frac{du_B}{dt} = \frac{1}{2} L \, 2 i(t) \frac{di}{dt} = L i \frac{di}{dt}$$

$$\left(\text{or rate} = P_L = i(t) \cdot V_L(t) = i(t) \cdot L \frac{di}{dt}\right)$$

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \qquad \tau_L = \frac{L}{R}$$

$$\frac{di}{dt} = \frac{\mathcal{E}}{R}\left(-\left(-\frac{1}{\tau_L}\right) \cdot e^{-t/\tau_L}\right) = + \frac{\mathcal{E}}{R\tau_L} e^{-t/\tau_L}$$

$$= \frac{\mathcal{E}}{L} e^{-t/\tau_L} \qquad L \frac{di}{dt} = \mathcal{E} e^{-t/\tau_L}$$

$$P_L = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \cdot \mathcal{E} e^{-t/\tau_L}$$

$$= \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right) e^{-t/\tau_L}$$

$$= \underline{\underline{42 \text{ W}}}$$

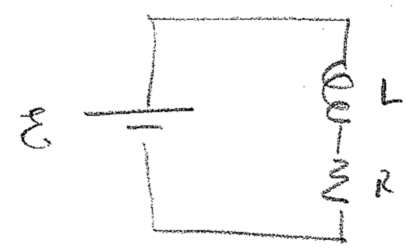
$\mathcal{E} = 100$ V

$R = 10\ \Omega$

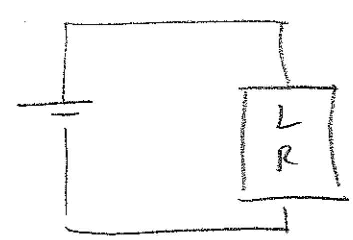
$\tau_L = \frac{1.6 \text{ H}}{10\ \Omega} = 0.16$ s

(b) [5 pts] A long time after the connection is made, what is the potential difference over the coil:

① After long time $\frac{di}{dt} = 0$.
So if R & L are separated,
then $V_L = L \frac{di}{dt} = \underline{\underline{0}}$

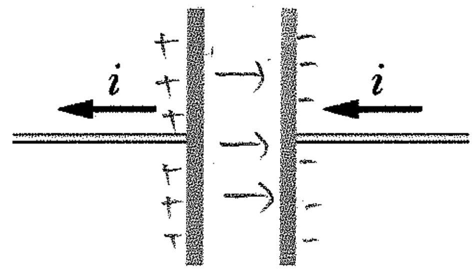


② Here though, L & R are together
so potential difference over coil
is always same as emf,
$V_{coil} = \mathcal{E} = 100$ V



Question 3 [10 points]

The figure shows a parallel plate capacitor and the current in the connecting wires that is *discharging* the capacitor.



(i) [5 pts] What is the direction of the electric field E between the plates?

a) Toward the right

b) Toward the left

c) Into the page

d) Out of the page

e) The electric field is zero

(ii) [5 pts] What is the direction of the displacement current i_D between the two plates?

a) Toward the right

b) Toward the left

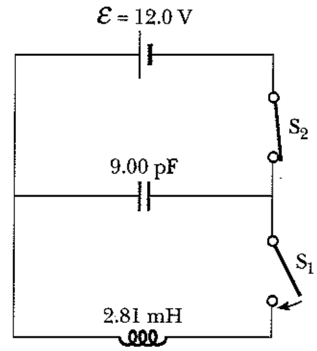
c) Into the page

d) Out of the page

e) The displacement current is zero

Problem 4 [18 points]

The figure shows a circuit in which switch S_2 has been closed for a long time. Now we open switch S_2 and at the same time close switch S_1, to make an LC circuit.



(a) [6 pts] Calculate the frequency f of the oscillation in the circuit

$$f = \frac{\omega}{2\pi}$$

$$= \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{\sqrt{2.81 \cdot 10^{-3} H \cdot 9 \cdot 10^{-12} F}} = \underline{1.0 \cdot 10^{6}} \ Hz \quad (= 1 \ MHz)$$

(b) [6 pts] Calculate the maximum current in the circuit:

$$I_{max} = \omega Q_{max}$$

$$= \omega C V_{max} = \omega C \mathcal{E}$$

$$= 2\pi \cdot 1.0 \cdot 10^{6} s^{-1} \cdot 9 \cdot 10^{-12} F \cdot 12 V$$

$$= \underline{6.79 \cdot 10^{-4} A}$$

(c) [6 pts] Calculate the total energy stored in the circuit:

$$U_{TOT} = U_{Bmax} = \frac{1}{2} L I_{max}^{2}$$

$$= \frac{1}{2} \cdot 2.81 \cdot 10^{-3} H \cdot (6.79 \cdot 10^{-4} A)^{2}$$

$$= \underline{6.48 \cdot 10^{-10} J}$$