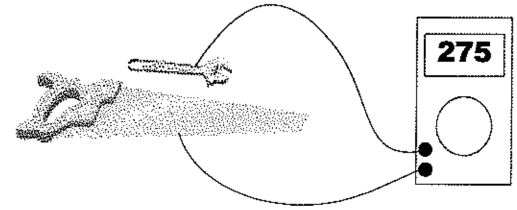
Problem 1 [15 points]

The saw in the figure has a net charge on it of +700 pC. The metric crescent wrench has a net charge of –700 pC. A voltmeter is connected from the wrench to the saw in order to measure the potential difference (275 V) between the two tools.



(a) [5 pts] Calculate the capacitance of this arrangement of the two tools.

$$q = CV \iff C = \frac{q}{V} = \frac{700 \cdot 10^{-12}\,C}{275\,V} = \underline{\underline{2.55 \cdot 10^{-12}\,F}}$$

(b) [7 pts] Suppose you used insulating gloves (so that no charge can escape) and moved the wrench to a new position so that the voltmeter now reads twice as great a voltage (550 volts). Calculate the work required by an external force (in this case you) to move the wrench in this way. Make sure to indicate whether the work is positive or negative.

q same, change $C_{new} = \frac{q}{V_{new}} = \frac{700 \cdot 10^{-12}\,C}{550\,V} = 1.27 \cdot 10^{-12}\,F$

Then $U_{new} = \frac{q^2}{2C_{new}} = \frac{q^2}{2 \cdot C_{old}/2} = 2 \cdot \frac{q^2}{2C} = 2U_{old}$

So $W_{ext} = +\Delta U = +(U_{new} - U_{old}) = 2U_{old} - U_{old}$

$= U_{old} = \frac{(700 \cdot 10^{-12}\,C)^2}{2 \cdot 2.55 \cdot 10^{-12}\,F} = \underline{\underline{9.61 \cdot 10^{-8}\,J}} \quad \underline{\underline{> 0}}$

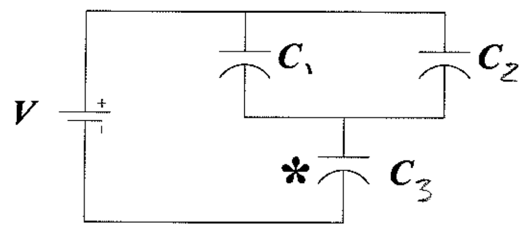
(c) [3 pts] Did you move the wrench closer to, or farther away from the saw? Explain your reasoning in one or two sentences.

Parallel plate $C = \varepsilon_0 \frac{A}{d}$, C decreases when d increases

So <u>further apart</u>.

Question 1 [10 points]

The circuit shows three identical capacitors of capacitance C, connected to an ideal battery of potential difference V, in the configuration illustrated.



(a) [5 pts] The equivalent capacitance for this arrangement of three capacitors is

a) C
b) $2C$
c) $C/2$
d) $2C/3$
e) $3C/2$
f) none of the above

$$C_{12} = C_1 + C_2 = 2C$$

$$C_{123} = \frac{1}{\frac{1}{C_{12}} + \frac{1}{C_3}} = \frac{1}{\frac{1}{2C} + \frac{1}{C}} = \underline{\underline{\frac{2}{3}C}}$$

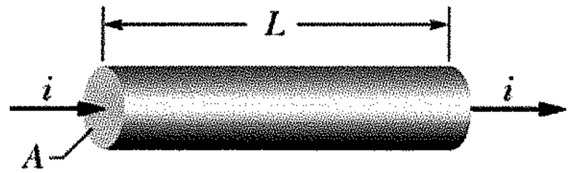
(b) [5 pts] The amount of charge stored on the capacitor marked "*" is

a) CV
b) C/V
c) V/C
d) $2CV/3$
e) $3CV/2$
f) none of the above

C_3 in series w. C_{12}

$$q_3 = q_{eq} = V \cdot C_{eq} = \underline{\underline{V \cdot \frac{2}{3}C}}$$

Problem 2 [15 points] The figure shows a copper wire of cross sectional area $A = 2.0 \times 10^{-6}\ \text{m}^2$ and length $L = 4.0$ m has a current of 2.0 A uniformly distributed across that area. The resistivity of copper is $1.69 \times 10^{-8}\ \Omega\text{m}$.



(a) [4 pts] Calculate the magnitude of the electric field inside the wire:

$$E = \rho J = \rho \frac{i}{A}$$

$$= 1.69 \cdot 10^{-8}\ \Omega m \ \frac{2.0\ A}{2.0 \cdot 10^{-6}\ m^2} = \underline{1.69 \cdot 10^{-2}\ \frac{V}{m}}$$

(b) [3 pts] Which end of the wire has the highest electric potential? Explain your answer briefly

Left end. Current runs high $\rightarrow$ low V.

Also $\vec{E} \parallel \vec{J} \parallel i$

and V decreases in direction of $\vec{E}$.

(c) [8 pts] Calculate how much electrical energy is dissipated in the wire in 30 minutes:

$$\Delta U = P \cdot \Delta t$$

$R = 0.0338\ \Omega$

$$= i^2 R \cdot \Delta t$$

$\Delta V = 0.0676\ V$

$$= i^2 \rho \frac{L}{A} \Delta t$$

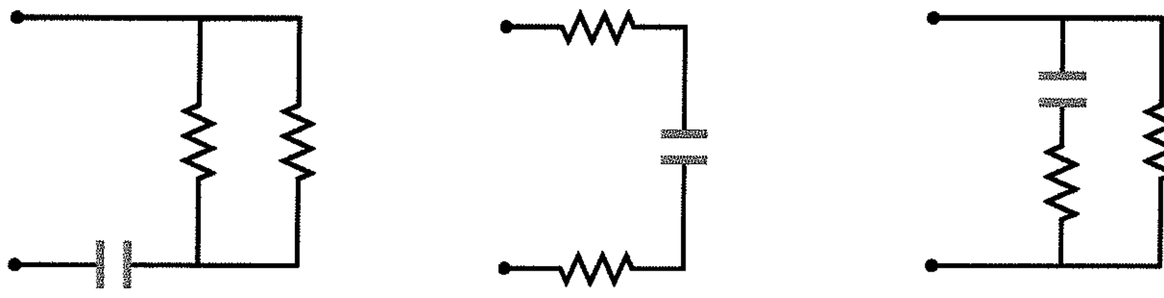
$$= (2.0 \cdot A)^2 \cdot 1.69 \cdot 10^{-8}\ \Omega m \cdot \frac{4.0\ m}{2.0 \cdot 10^{-6}\ m^2} \cdot 30 \cdot 30\ s$$

$$= \underline{244\ J}$$

or $P = iV = iEL$, $U = iEL\Delta t$

Question 2 [10 points]

The figures show three sections of an RC-circuit that are to be connected in turn to the same battery. The resistors are all identical, as are the capacitors.



(1) (2) (3)

(a) [5 pts] Rank the circuits according to the final (equilibrium) charge on the capacitor:

$Q_2 > Q_3 > Q_1$ $\quad$ $Q_1 > Q_3 > Q_2$ $\quad$ $Q_2 > Q_1 = Q_3$

$Q_1 > Q_2 > Q_3$ $\quad$ $Q_1 = Q_2 = Q_3$

Same battery, same final charge $Q = CV$

(depends on C and V only, not R)

(b) [5 pts] Rank the circuits according to the time required for the capacitor to reach 50% of its final charge:

$t_2 > t_3 > t_1$ $\quad$ $t_1 > t_3 > t_2$ $\quad$ $t_2 > t_1 = t_3$

$t_1 > t_2 > t_3$ $\quad$ $t_1 = t_2 = t_3$

t determined by $\tau = RC$

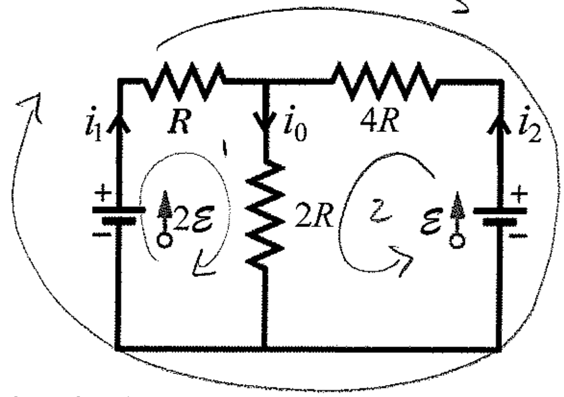
$R_1 = \frac{R}{2}$ $\quad$ $R_2 = 2R$ $\quad$ $R_3 = R$ (only charging through 1R)

so $t_2 > t_3 > t_1$

3

Problem 3 [20 points]

In the circuit shown in the figure $\mathcal{E} = 1.0$ V and resistance $R = 1.0\ \Omega$.
Use the indicated directions of the currents i_1, i_2, and i_0 in your equations below.



(a) [3 pts] Write down the junction equation for the junction located just above the i_0 arrow:

$$i_0 = i_1 + i_2$$

(b) [8 pts] Write down two different loop equations for this circuit. Indicate on the figure which loop is which:

① $-i_1R - i_0 2R + 2\mathcal{E} = 0$

② $-i_2 4R - i_0 2R + \mathcal{E} = 0$ $\quad -i_2 4R - (i_1+i_2)2R + \mathcal{E} = 0$

③ $-i_1 R + i_2 4R - \mathcal{E} + 2\mathcal{E} = 0$ $\quad \therefore\ i_2 = \frac{1}{6R}(\mathcal{E} - i_1 2R)$

(c) [4 pts] Write down an expression for current i_2 in terms of i_1, $\mathcal{E}$, and R

Use ③ $\quad -i_1R + i_2 4R + \mathcal{E} = 0$

$$\therefore\ i_2 = \frac{i_1 R - \mathcal{E}}{4R} \quad (*)$$

or $\quad -i_1R - (i_1+i_2)2R + 2\mathcal{E} = 0$

$$\therefore\ -i_2 \cdot 2R - i_1 \cdot 3R + 2\mathcal{E} = 0 \qquad i_2 = \frac{2\mathcal{E} - i_1 3R}{2R} \quad (**)$$

$= -0.0714$ A

(d) [5 pts] Calculate the power of the 2V battery (emf) on the left:

Equate $*$ and $**$

$$\frac{i_1R - \mathcal{E}}{4R} = \frac{2\mathcal{E} - i_1 3R}{2R}$$

$$\therefore\ i_1R - \mathcal{E} = 2(2\mathcal{E} - i_1 3R) = 4\mathcal{E} - 6Ri_1$$

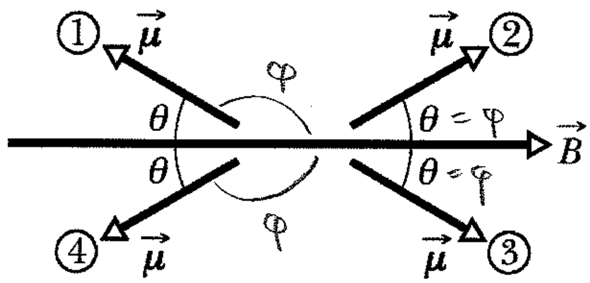
$$\therefore\ 7i_1R = 5\mathcal{E}$$

$$\therefore\ i_1 = \frac{5\mathcal{E}}{7R} = \frac{5V}{7\Omega} = \underline{0.71\ A}$$

$$P = i\mathcal{E} = 0.71A \cdot 2V = \underline{1.42\ W}$$

Question 3 [10 points]

The figures show four orientations of a magnetic dipole with moment $\boldsymbol{\mu}$ in a magnetic field $\mathbf{B}$.



(a) Rank the orientations with respect to the magnitude of the torque on the dipole:

$\tau_4 > \tau_1 > \tau_2 > \tau_3$ $\quad$ $\tau_1 = \tau_4 > \tau_2 = \tau_3$

$\tau_2 = \tau_3 > \tau_1 = \tau_4$ $\quad$ $\tau_1 = \tau_2 = \tau_3 = \tau_4$

$|\tau| = |\vec{\mu} \times \vec{B}| = \mu B |\sin\varphi|$

(b) Rank the orientations with respect to the potential energy of the dipole:

$U_4 > U_1 > U_2 > U_3$ $\quad$ $U_1 = U_4 > U_2 = U_3$

$U_2 = U_3 > U_1 = U_4$ $\quad$ $U_1 = U_2 = U_3 = U_4$

$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\varphi$

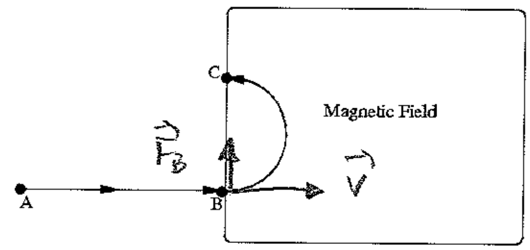
(c) If the dipole is rotated from orientation 1 to orientation 2 is the work done by the magnetic field:

positive $\quad$ negative $\quad$ zero

$W_{field} = -\Delta U$

$= U_{init} - U_{final}$

Problem 4 [20 points]
The figure shows a beam of electrons which is first accelerated from rest (from point A to point B) through a potential difference of 500 V. Then the electrons enter a rectangular region with a uniform magnetic field. The magnetic force causes the electrons to follow a curved path with a radius of 10 cm. You can find the electron and proton masses in the formula sheet.



(a) [4 pts] What is the direction of the magnetic field in the rectangular region:

into the page   out of the page   upward   downward

(b) [6 pts] Calculate the magnitude of the magnetic field in the rectangular region:

$$qvB = \frac{mv^2}{r} \Leftrightarrow B = \frac{mv}{qr} \quad \text{Find } v:$$

$$q\Delta V = \frac{1}{2}mv^2 \Leftrightarrow v = \sqrt{2q\Delta V/m} = 1.32 \cdot 10^7 \text{ m/s}$$

$$B = \frac{m}{qr}\sqrt{2q\Delta V/m} = \frac{9.11 \cdot 10^{-31}\text{ kg}}{1.602 \cdot 10^{-19}\text{C} \cdot 10 \cdot 10^{-2}\text{m}} \cdot 1.32 \cdot 10^7 \text{ m/s}$$

$$= \underline{7.54 \cdot 10^{-4}\text{ T}}$$

(c) [5 pts] Calculate the time it takes for the electrons to follow the curved path inside the magnetic field region, from point B to point C:

Circular motion, half a period to make a half circle

$$\frac{T}{2} = \frac{1}{2} \cdot \frac{2\pi m}{qB} = \frac{\pi m}{qB} = \underline{2.37 \cdot 10^{-8}\text{ s}}$$

(d) [5 pts] If instead a beam of protons entered the same magnetic field with the same initial velocity, what would be the radius of the proton's circular path?

$$r_p = \frac{m_p v}{qB} = \frac{m_p}{m_e}\frac{m_e v}{qB} = \frac{m_p}{m_e} \cdot r_e$$

$$= \frac{1.67 \cdot 10^{-27}\text{ kg}}{9.11 \cdot 10^{-31}\text{ kg}} \cdot 10\text{ cm} = \underline{183\text{ m}}$$