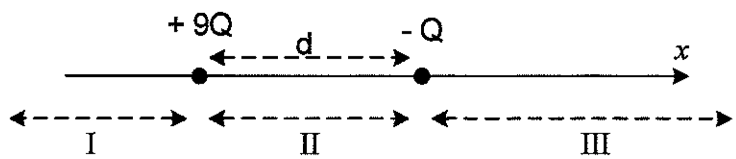
Question 1 [10 points]

Two point charges, a positive charge of +9Q and a negative charge of –Q, are fixed in place on an x-axis, separated by a distance d. The axis is divided into three regions: I, II, and III, as shown.



(a) [6 points] In which region (or regions) along the axis could you place a third point charge, in such a way that the third charge would be in equilibrium? (circle the right answer)

I　　　II　　　III　　　I and II　　　I and III

Must be closer to the smaller charge

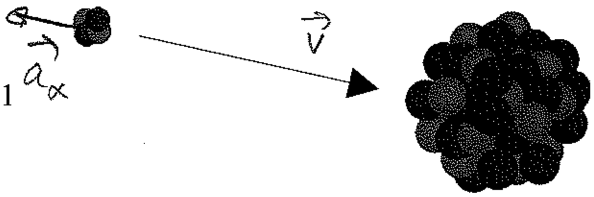
(b) [4 points] If equilibrium is possible, should this third charge be positive or negative, or does it matter at all?

positive　　　negative　　　doesn't matter

Since sign of third charge will enter in both $\vec{F}_{31}$ and $\vec{F}_{32}$.

Problem 1 [15 points]
An alpha particle is a helium nucleus, composed of 2 protons and 2 neutrons. Thus it possesses a mass of approximately 6.7×10^{-27} kg, and a charge of +2e. By comparison, the nucleus of an atom of gold has 79 protons and 118 neutrons, giving a mass of 3.3×10^{-25} kg, and a charge of +79e.

The figure shows an alpha particle moving towards a gold nucleus.



(a) [6 points] How close would the alpha particle have to approach the gold nucleus in order to experience a force of 1 N due to the presence of the gold nucleus?

$$|\vec{F}| = \frac{k|q_1||q_2|}{d^2}$$

$$\Rightarrow \quad d = \sqrt{\frac{k|q_1||q_2|}{|F|}}$$

$$= \sqrt{\frac{8.99\cdot10^{9}\ \frac{Nm^2}{C^2}\cdot 2\cdot1.6\cdot10^{-19}C\cdot 79\cdot1.6\cdot10^{-19}C}{1.0\ N}}$$

$$= \underline{1.9\cdot10^{-13}\ m}$$

(b) [5 points] What is the magnitude and direction of the acceleration experienced by the alpha particle when it is at the position found above in part (a)? Indicate the direction of **a** in the figure or with words.

$$\vec{F}_\alpha = m_\alpha\cdot\vec{a}_\alpha \quad , \quad |\vec{a}_\alpha| = \frac{|\vec{F}_\alpha|}{m} = \frac{1.0\ N}{6.7\cdot10^{-27}\ kg}$$

$$= 7.5\cdot10^{26}\ m/s^2$$

Repulsive force, away from gold nucleus
$\vec{a}_\alpha$ away from —"—

(c) [4 points] When the alpha particle is at the position found in (a), calculate the magnitude of the electrostatic force exerted **on the gold nucleus**?

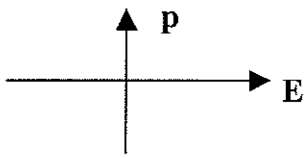
$$|\vec{F}_{gold}| = |\vec{F}_\alpha| \quad \text{(equal \& opposite, } 3^{rd} \text{ law)}$$

$$= \underline{1.0\ N}$$

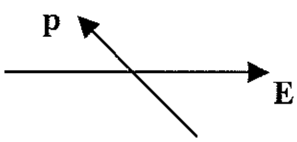
Question 2 [10 points]

The figure shows three different orientation of an electric dipole (with dipole moment **p**) in a uniform electric field **E**, which points in the positive x-direction.

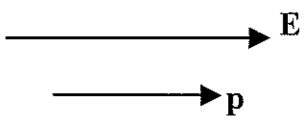
(1)



(2)



(3)



(a) [4 pts] Rank the scenarios according to the potential energy of the dipole in the electric field, greatest first (circle the right answer):

$U_1 > U_2 > U_3$ $\qquad$ $U_2 > U_1 > U_3$ $\qquad$ $U_2 > U_1 = U_3$

$U_1 > U_2 = U_3$ $\qquad$ All tie

$U = -\vec{p}\cdot\vec{E}$

(b) [3 pts] Rank the scenarios according to the magnitude of the torque on the dipole in the electric field, greatest first (circle the right answer):

$\tau_1 > \tau_2 > \tau_3$ $\qquad$ $\tau_2 > \tau_1 > \tau_3$ $\qquad$ $\tau_2 > \tau_1 = \tau_3$

$\tau_1 > \tau_2 = \tau_3$ $\qquad$ All tie

$\vec{\tau} = \vec{p} \times \vec{E}$

$|\tau| = |\vec{p}||\vec{E}||\sin\theta|$

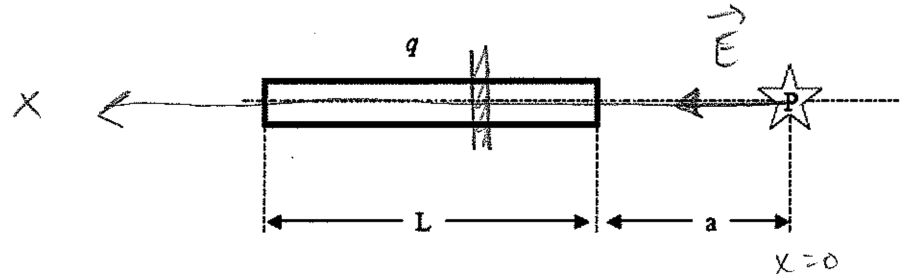
(c) [3 pts] In which scenario (or scenarios) is the dipole in equilibrium? Circle the right answer.

1 $\qquad$ 2 $\qquad$ 3 $\qquad$ 1 and 2 $\qquad$ 2 and 3

Only in (3) will the dipole not rotate.

Problem 2 [20 points]

The figure shows a one-dimensional insulating rod of length L = 8.0 cm. It has charge q = -2.0 nC deposited uniformly along its length, with a constant linear charge density. The point P is located distance a = 5.0 cm from the end of the rod.



(a) [4 pts] Calculate the linear charge density of the rod

$$\lambda = \frac{q}{L} = \frac{-2.0 \cdot 10^{-9}\ C}{8.0 \cdot 10^{-2}\ m} = \underline{-2.5 \cdot 10^{-8}\ \frac{C}{m}}$$

(b) [3 pts] What is the direction of the electric field at point P, due to the charged rod? Indicate your answer on the figure.

(c) [5 pts] Pick a charge element dq along the rod and write down an expression for the magnitude of the electric field dE at point P due to charge dq. Indicate the location of dq on the figure.

$$|dE| = k\frac{|dq|}{x^2} = \frac{k|\lambda|dx}{x^2}$$

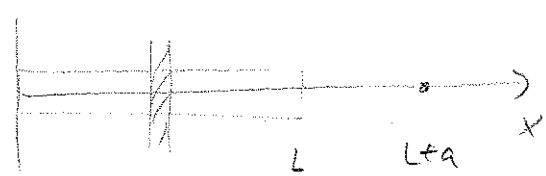
(d) [8 pts] Calculate the magnitude of the electric field at point P. Useful integral: $\int 1/x^2\, dx = [-1/x]$

$$|E| = \int |dE| = \int_a^{a+L} \frac{k|\lambda|dx}{x^2} = k|\lambda|\left[-\frac{1}{x}\right]_a^{a+L}$$

$$= -k|\lambda|\left(\frac{1}{a+L} - \frac{1}{a}\right) = k|\lambda|\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

$$= 8.99 \cdot 10^9 \frac{Nm^2}{C^2} \cdot 2.5 \cdot 10^{-8} \frac{C}{m}\left(\frac{1}{5.0 \cdot 10^{-2} m} - \frac{1}{13 \cdot 10^{-2} m}\right)$$

$$= \underline{2.77 \cdot 10^3\ \frac{N}{C}}$$



$$|dE| = \frac{k\,dq}{(L+a-x)^2}$$

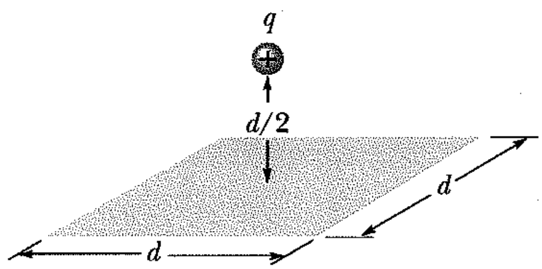
$u = L+a-x$

$du = -dx$

$$|E| = \int_0^L \frac{k\lambda\, dx}{(L+a-x)^2} = \int_{L+a}^{a} -\frac{k\lambda\, du}{u^2} = k\lambda\left[\frac{1}{u}\right]_{L+a}^{a}$$

Question 3 [10 points]

The figure shows a charge $+q$ which is located at a distance $d/2$ directly above the center of a surface which is a square with sides d.



(a) [7 pts] What is the magnitude of the electric flux through the square surface? (Hint 1: Think of the square as one face of a cube with edges d. Hint 2: no calculus involved). Circle the correct answer.

q/ε_0 $\quad$ $q/2\varepsilon_0$ $\quad$ $q/4\varepsilon_0$

$q/6\varepsilon_0$ $\quad$ $q/(4\pi\varepsilon_0 d^2)$ $\quad$ $q/(\pi\varepsilon_0 d^2)$

Gaussian surface cube, sides d, centered on q.

$\Phi_{TOT} = \frac{q_{encl}}{\varepsilon_0}$ by Gauss' law

$\Phi_{side} = \Phi_{TOT}/6 = \frac{q_{encl}}{6\varepsilon_0}$

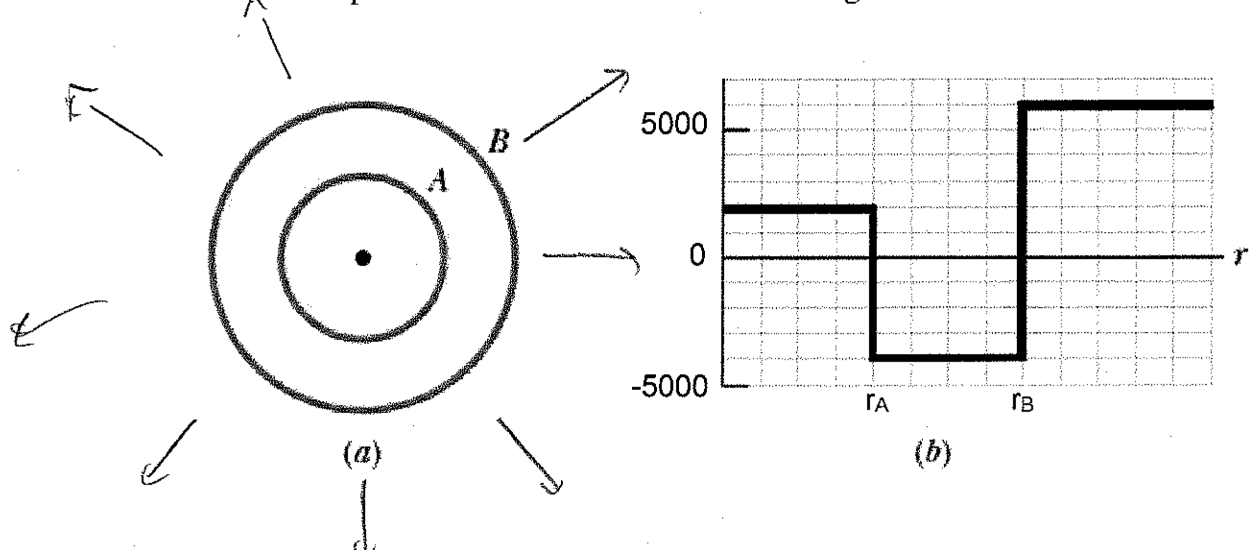
(b) [3 pts] If you were to move the charge closer to the square, so that it would be located at distance $d/4$ above the square, would the magnitude of the flux through the square be the same or different? Circle the correct answer:

Larger $\quad$ The same $\quad$ Smaller

Flux increases through this side of the cube because it is close to charge

(Φ_{TOT} is constant)

Problem 3 [20 points]
A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of non-conducting material. The figure (a) below shows a cross section. Figure (b) shows the net electric flux through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. The scale on the vertical axis is given in units of Nm^2/C



(a) [7 pts] Calculate the charge of the particle at the center of the shell (magnitude and sign):

$$q_{part} = q_{encl} = \varepsilon_0 \Phi \quad \text{for} \quad r < r_A$$

$$= 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \cdot 2000 \frac{Nm^2}{C}$$

$$= \underline{1.77 \; 10^{-8} \; C}$$

(b) [7 pts] Calculate the net charge on shell A (magnitude and sign):

$$q_{encl} = q_{part} + q_A \quad \text{for} \quad r_A < r < r_B$$

$$q_A = q_{encl} - q_{part} = \varepsilon_0 \Phi_{A \to B} - q_{part}$$

$$= 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \cdot \left(-4000 \frac{Nm^2}{C}\right) - q_{part}$$

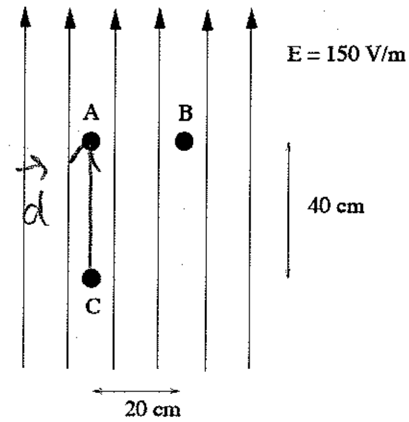
$$= -3.54 \cdot 10^{-8} C - 1.77 \cdot 10^{-8} C = \underline{-5.31 \cdot 10^{-8} C}$$

(d) [6 pts] Indicate on the figure the direction of the electric field lines outside of shell B. Explain your choice briefly:

Total flux beyond shell B is positive, so $\vec{E}$ is outward, and is spherically symmetric

Problem 4 [15 points]

The figure shows a uniform electric field with magnitude $E = 150$ V/m. Three different locations in the electric field are marked with A, B, and C.



(a) [4 pts] Circle the correct statement about the electric potential at points A, B, and C:

$V_C < V_A < V_B$ $\quad$ $V_C < V_A = V_B$ $\quad$ $V_C > V_A = V_B$ $\quad$ $V_C > V_A > V_B$

(b) [6 pts] Calculate the work needed by an external force to move a point charge q = -2.0 μC (at rest) from point C to point A. Specify both magnitude and sign of the work:

$$W_{ext} = +\Delta U = +q \cdot \Delta V$$
$$= +q\,(V_A - V_C) = +q\,(-\vec{E} \cdot \vec{d})$$
$$= -q\,|\vec{E}|\,|\vec{d}|$$
$$= -(-2.0 \cdot 10^{-6}\,C) \cdot 150\,\frac{V}{m} \cdot 40 \cdot 10^{-2}\,m = \underline{1.2 \cdot 10^{-4}\,J}$$

(c) [5 pts] Calculate the work (magnitude and sign) needed by an external force to move a point charge q = -2.0 μC (at rest)

(i) from point C to point B:

Same $V_A = V_B$, no work to go $A \rightarrow B$

(ii) from point C to point A, then from A to B, then from B to C:

$W = 0$, conservative force

ΔV path-independent

so ΔV over loop $= 0$.