Exam 3
Physics 2102 Fall 2009

November 19, 2009

Name: Solutions ID # ____________

Answer all questions (7).

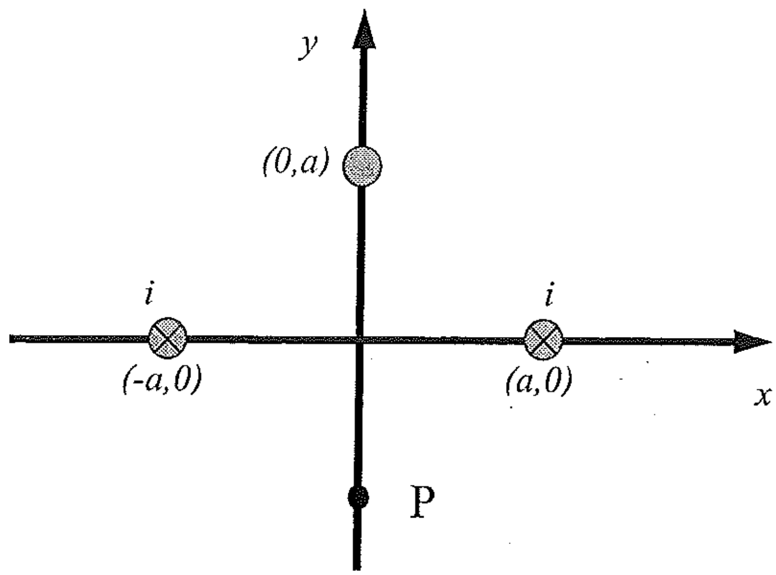
Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct.

On problems that are not multiple choice, be sure to show all of your work since no credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.

Please use complete sentences where explanations are asked for.
Please be sure that *all* numerical quantities include appropriate units.

The only electronic devices to be used during the exam are standard or graphing calculators.
All cell phones should be turned off and put away. Cell phones are not to be used as calculators.

1. (10 points)



In the picture above three thin long wires are carrying current perpendicular to the *xy*-plane. The wires passing through the points *(-a,0)* and *(a,0)* each carry current *i* into the page. What should be the direction and the magnitude of the current in the wire passing through the point *(0,a)* so that the net magnetic field at point P with the coordinates *(0, -a)* is zero?

i) (3 points) direction

(a) into the page;
(b) out of the page;
(c) left;
(d) right;
(e) does not matter: any direction will work;
(f) not enough information.

$B_1 = \frac{\mu_0 i}{2\pi\sqrt{2}a}$ ↙

$B_2 = \frac{\mu_0 i}{2\pi\sqrt{2}a}$ ↖

ii) (7 points) magnitude:

a) $2i$
b) $i/2$
c) i
d) $i\sqrt{2}$
e) $i/\sqrt{2}$
f) i/a
g) $i/2a$

$B_{12} = 2\left(\frac{\mu_0 i}{2\pi\sqrt{2}a}\right)\frac{1}{\sqrt{2}}$ ←

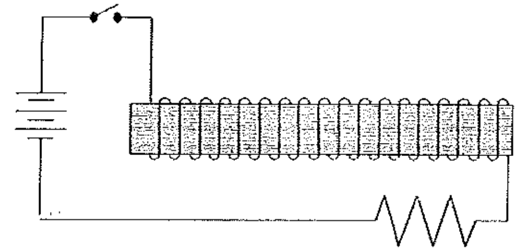
$= \frac{\mu_0 i}{2\pi a}$.

$B_3 = \frac{\mu_0 (x)}{2\pi (2a)}$ $(\rightarrow)$.

$B_3 = B_{12} \Rightarrow \underline{x = 2i}$

2. (18 points)

A solenoid of radius 1 cm and length 30.0 cm is made of 3000 turns of wire with negligible resistance. It is then connected in series with a battery of emf 3.00 V and a resistor with resistance of 3.00 Ohm.



i) (4 points) After a very long time, what is the current in the solenoid?

$$i_m = \frac{\mathcal{E}}{R} = \frac{3.00\ V}{3.00\ \Omega} = \underline{1.00\ [A]}.$$

ii) (4 points) After a very long time, what is the magnetic field inside the solenoid?

$$B = n\mu_0 i = \left(\frac{3000}{0.3}\right)(1.26\times10^{-6})\cdot(1.00) = \underline{1.26\times10^{-2}\ [T]}$$

iii) (5 points) Using the general definition of inductance, and the expression for the magnetic field inside a solenoid, calculate the inductance of this solenoid.

$$L = \frac{N\Phi_B}{i} = \frac{N}{i}B\cdot A = \frac{N}{i}\left(\frac{N}{\ell}\cdot\mu_0 i\right)\cdot\pi r^2$$

$$= \frac{(3000)^2}{0.3}(1.26\times10^{-6})\cdot\pi(0.01)^2 = \underline{1.19\times10^{-2}\ [H]}.$$

iv) (5 points) After the solenoid is first connected to the battery, how long does it take for the magnetic field to reach 1/3 of its final value?

$$B = \frac{1}{3}B_{max}$$

$$n\mu_0 i = \frac{1}{3}n\mu_0 i_{max}$$

$$i = i_{max}(1-e^{-t/\tau}) \quad , \quad \tau = \frac{L}{R} = \frac{1.19\times10^{-2}}{3.00}$$

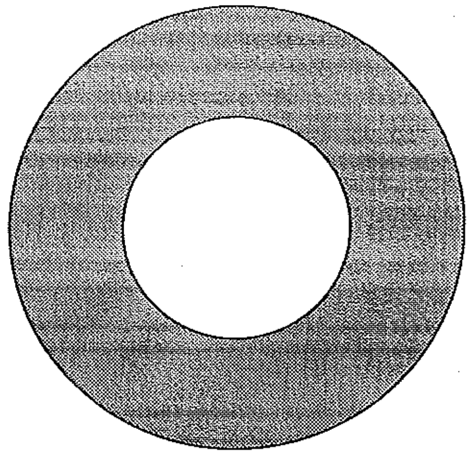
$$\Rightarrow i_{max}(1-e^{-t/\tau}) = \frac{1}{3}i_{max} \qquad = 4.0\times10^{-3}\ [S]$$

$$1 - e^{-t/\tau} = \frac{1}{3}$$

$$-t/\tau = \ln\left(\frac{2}{3}\right)$$

$$t = \tau\ln\left(\frac{3}{2}\right) \qquad \Rightarrow t = \underline{1.62\times10^{-3}\ [S]}.$$

3. (15 points)



The figure above shows a cross-section of a long hollow cylindrical conductor with inner radius *R* and outer radius *2R*. The conductor carries a uniformly distributed net current *i* parallel to the axis of the cylinder. Using Ampere's law find the magnitude of the current's magnetic field at the following radial distances:

i) (5 points) *R/2*

No current enclosed.

$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

$B \cdot 2\pi \left(\frac{R}{2}\right) = \mu_0 \cdot 0$ $\quad\quad$ $\underline{B = 0}$.

ii) (5 points) *3R/2*

$B \cdot 2\pi \left(\frac{3R}{2}\right) = \mu_0 i_{enc}$

$i_{enc} = i \dfrac{\pi \left(\frac{3R}{2}\right)^2 - \pi R^2}{\pi (2R)^2 - \pi R^2} = i\left(\frac{5}{12}\right)$.

$B = \dfrac{\mu_0 \left(\frac{5}{12} i\right)}{3\pi R} = \underline{\dfrac{5\mu_0 i}{36\pi R}}$.

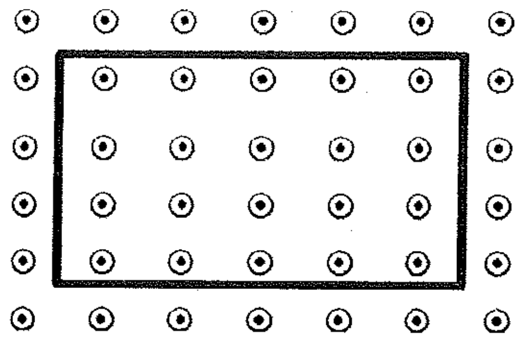
iii) (5 points) *5R/2*

$B \cdot 2\pi \left(\frac{5R}{2}\right) = \mu_0 i_{enc} = \mu_0 i$

$\underline{B = \dfrac{\mu_0 i}{5\pi R}}$.

4. (13 points)

The figure shows a retangular loop of height h and width w, which is perpendicular to a uniform magnetic field B directed out of the page. The magnitude of the magnetic field changes according to $B(t) = a\,t^2 + b\,t$, where a and b are time-independent constants.



i) (5 points) What is the direction of the induced current in the loop (Circle one).

Clockwise   Counterclockwise   No Current

ii) (8 points) What is the magnitude of the emf produced in the loop at time $t = 2$ sec.

(a) $(a+b)hw$
(b) $(2a+b)hw$
(c) $(a+2b)hw$
(d) $(4a+2b)hw$
(e) $(4a+b)hw$
(f) 0
(g) none of the above

$$\mathcal{E}_{mf} = -\frac{d\Phi_B}{dt}$$

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt}\cdot A.$$

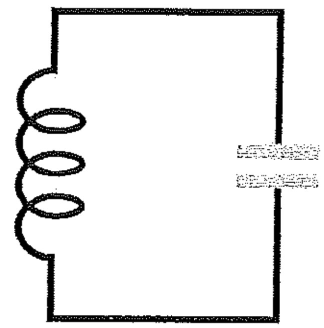
$$= (2at+b)\cdot hw$$

at $t=2$ sec.

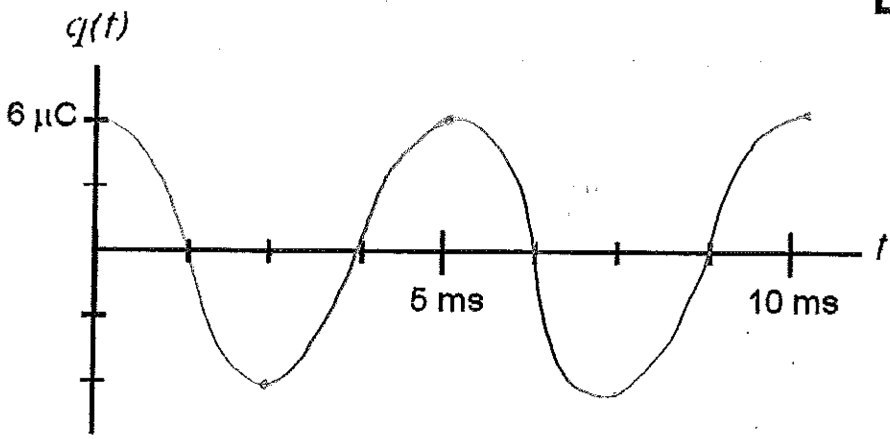
$$\frac{d\Phi_B}{dt} = (4a+b)hw$$

5. (18 points)

An LC circuit consists of a 10.0 μF capacitor and a 63.3 mH inductor. At time t = 0, the charge on the capacitor is 6 μC and the current is zero.



i) (5 points) Using the given axes below, sketch a graph that represents the charge on the capacitor as a function of time?



$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{1}{LC}}} = 5.0 \times 10^{-3} [S].$$

ii) (6 points) Calculate the maximum value of the current in this circuit.

$$\frac{1}{2} L i_{max}^2 = \frac{1}{2} \frac{q_{max}^2}{C}$$

$$i_{max}^2 = \frac{1}{LC} q_{max}^2 = \frac{(6\times10^{-6})^2}{(63.3\times10^{-3})(10\times10^{-6})}$$

$$i_{max} = 7.5 \times 10^{-3} [A].$$

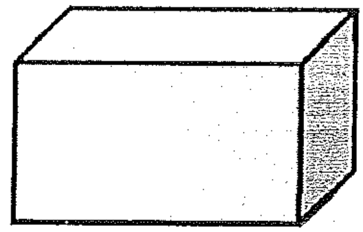
iii) (7 points) Calculate the total energy stored in the LC oscillator system (the capacitor and the inductor).

Either $\frac{1}{2} \frac{q_{max}^2}{C}$ or $\frac{1}{2} L i_{max}^2$

$$\frac{1}{2} \frac{q_{max}^2}{C} = \frac{1}{2} (6\times10^{-6})^2 \cdot \frac{1}{(10\times10^{-6})}$$

$$= 1.8 \times 10^{-6} [J].$$

6. (8 points).
A closed surface is shown in the figure. Through the top surface, there is a magnetic flux of +6 mWb, where the flux directed outwards is taken positive. No magnetic flux penetrates the four side faces.



i) (4 points) Through the bottom face, the magnetic flux is: (choose one correct answer)

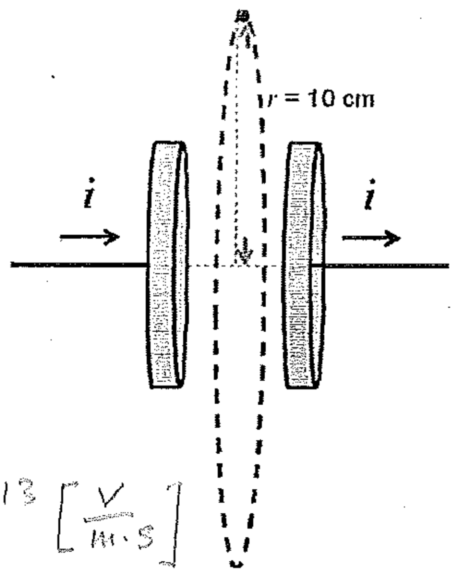
(a) Positive
(b) Negative
(c) Zero
(d) Not enough information

ii) (4 points) Along the bottom face, the direction of the magnetic field is: (choose one correct answer)

(a) The same as the direction of the magnetic field along the top face
(b) Opposite to the direction of the magnetic field along the top face
(c) Zero, regardless the direction of the magnetic field along the top face
(d) Not enough information

7. (18 points).

The figure shows a parallel plate capacitor with radius $R = 5$ cm and distance between the plates d = 3 mm being charged by a current of 2.5 A.



i) (4 points) Calculate the rate change of the magnitude of the electric field, dE/dt, in the capacitor.

$$E = \frac{q}{\epsilon_0 A}$$

$$\Rightarrow \frac{dE}{dt} = \frac{1}{\epsilon_0 A} \cdot i$$

$$= \frac{2.5}{\epsilon_0 \cdot \pi (0.05)^2} = 3.6 \times 10^{13} \left[\frac{V}{m \cdot s}\right]$$

ii) (7 points) Calculate the displacement current penetrating an imaginary circle of radius $r = 10$ cm centered at the axis of the current.

$$i_d = \epsilon_0 \cdot \frac{d\Phi_E}{dt} = \epsilon_0 \frac{dE}{dt} \cdot A.$$

$$= \epsilon_0 \frac{dE}{dt} \cdot \pi (0.05)^2 = 2.5 \,[A]$$

iii) (3 points) What is the direction of the displacement current found in (iii)?

Rightward Leftward Zero

iv) (4 points) The displacement current penetrating an imaginary circle of radius r = 2.5 cm centered at the axis of the current is ________: (Choose one correct answer).

(a) Larger than the one found in (iii)
(b) Smaller than the one found in (iii)
(c) Same as the one found in (iii)
(d) Not be determined with given information

$$i_d = \epsilon_0 \frac{dE}{dt} \cdot A$$

$$= \epsilon_0 \frac{dE}{dt} \pi \cdot (0.025)^2$$