Exam 2
Physics 2102 Fall 2009

October 22, 2009

Name:________________________

Answer all questions (7).

Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct.

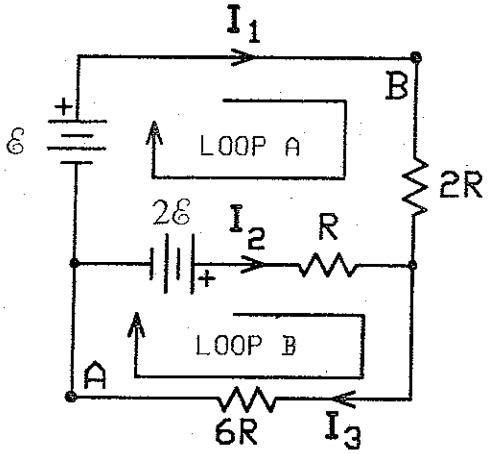
On problems that are not multiple choice, be sure to show all of your work since no credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.

Please use complete sentences where explanations are asked for.
Please be sure that *all* numerical quantities include appropriate units.

The only electronic devices to be used during the exam are standard or graphing calculators.
All cell phones should be turned off and put away. Cell phones are not to be used as calculators.

solution

1. (20 points)



a) (5 points) For the circuit at right, using the indicated current directions, write the loop equations (Kirchoff's Voltage Law) for the loops A and B.

loop A : $-i_1 \cdot 2R + i_2 R - 2\mathcal{E} + \mathcal{E} =$

$= -i_1 2R + i_2 R - \mathcal{E} = 0$

loop B : $2\mathcal{E} - i_2 R - i_3 \cdot 6R = 0$

b) (3 points) Write the equation (Kirchoff's Current Law) for the node at the junction of *R*, *2R*, and *6R*.

$i_1 + i_2 = i_3$

c) (8 points) Find an expression, in terms of ℰ and/or *R* **only**, for the current through the resistor *2R*; i.e. solve the equations above for the current $\mathbf{I}_1$ in terms of ℰ and *R*.

Solve :
$$\begin{cases} -i_1 \cdot 2R + i_2 R - \mathcal{E} = 0 \\ 2\mathcal{E} - i_2 R - i_3 \cdot 6R = 0 \\ i_1 + i_2 = i_3 \end{cases} \longrightarrow \begin{cases} -i_1 \cdot 2R + i_2 R - \mathcal{E} = 0 \\ -2i_1 \cdot 2R + i_2 R - i_3 6R = 0 \\ i_1 + i_2 = i_3 \end{cases}$$

$$\longrightarrow \begin{cases} -i_1 \cdot 2R + i_2 R - \mathcal{E} = 0 \\ -10 i_1 - 5 i_2 = 0 \\ i_1 + i_2 = i_3 \end{cases} \Rightarrow \begin{cases} -i_1 \cdot 2R + i_2 R - \mathcal{E} = 0 \\ i_1 = -\frac{i_2}{2} \\ i_3 = \frac{i_2}{2} \end{cases}$$

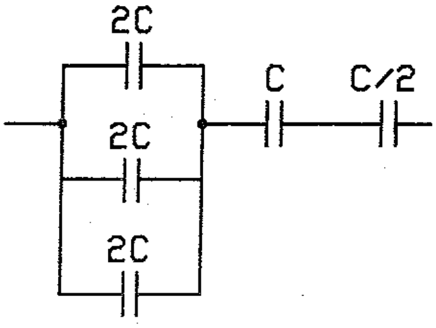
$$[-2R - 2R] i_1 = \mathcal{E} \qquad \boxed{i_1 = -\frac{\mathcal{E}}{4R}}$$

d) (4 points) Find the potential difference between points A and B. Indicate which point is at a higher potential.

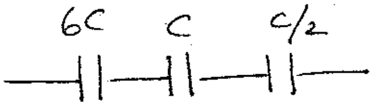
$\mathcal{E}$ with B at higher potential

A & B connected directly to the battery terminals.

2. (10 points)

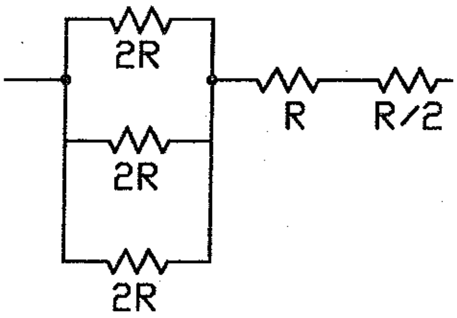


a) In terms of C, what is the equivalent capacitance of the capacitor arrangement at right? *Give your answer as a fraction,* e.g *17C/12, and not as decimal numbers*

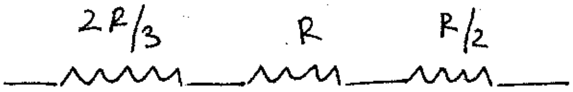


$$\frac{1}{C_{eq}} = \frac{1}{6C} + \frac{1}{C} + \frac{2}{C} = \frac{1+6+12}{6C} = \frac{19}{6C}$$

$$\boxed{C_{eq} = \frac{6C}{19}}$$



b) In terms of *R*, what is the equivalent resistance of the resistor arrangement at right? *Give your answer as a fraction, e.g 17R/12, and not as decimal numbers.*



$$R_{eq} = \frac{2R}{3} + R + \frac{R}{2} = \frac{13R}{6}$$

3. (15 points). Multiple choice questions. Each correct answer 3 points.

A parallel plate capacitor has capacitance *C* when its plates are separated by air. A dielectric with the dielectric constant κ is inserted between the plates of this capacitor.

i) ***After the insertion of the dielectric*** the capacitance of the capacitor has

→ [✓ a) increased;] $\varkappa > 1$ for all dielectrics
b) decreased;
c) remained the same;
d) not enough information.

ii) If the capacitor were ***kept connected*** to a battery with emf $\mathcal{E}$ while the dielectric was being inserted. After the insertion of the dielectric the potential difference across the capacitor would have

a) increased;
b) decreased;
→ [✓ c) remained the same;] it is still connected to the same battery
d) not enough information.

iii) If the capacitor were initially charged from a battery with emf $\mathcal{E}$, and then ***disconnected from the battery before*** the dielectric was inserted, the potential difference across the capacitor after the insertion of the dielectric would have

a) increased;
[✓ b) decreased;]
c) remained the same;
d) not enough information.

$V = \frac{Q}{C}$ Q : same (disconnected)
C : increased

iv) If the capacitor were ***kept connected*** to a battery with emf $\mathcal{E}$ while the dielectric was being inserted, after the insertion of the dielectric the charge on the capacitor would have

[✓ a) increased;]
b) decreased;
c) remained the same;
d) not enough information.

$Q = CV$ ∴ V : same (connected)
C : increased

v) For the case in iii) where the battery is ***disconnected***, would you, person inserting the dielectric, do positive of negative work while inserting the dielectric.

a) positive;
[✓ b) negative;]
c) zero work done;
d) not enough information.

stored energy : $\frac{Q^2}{2C}$

$C_{final} = \varkappa C > C_{initial} = C$

final stored energy lower

=> some work done by the field in the capacitor

4. (20 points) Parts a) – d) 3 points each; part iii) 8 points.

A capacitor with capacitance $C = 12$ pF is charged for a very long time from a battery with whose emf is $\mathcal{E} = 12$ V. The capacitor is then disconnected from the battery and connected to a resistor of $R = 12\ \Omega$.

i) *Immediately after the capacitor is connected to the resistor,* what are (explain your reasoning for each)

a) the charge on the capacitor

$$Q = C\mathcal{E} = 144\,pC$$

b) the current through the resistor

$$i = \frac{\mathcal{E}}{R} = 1A$$

ii) *A very long time after the capacitor is connected to the resistor,* what are (explain your reasoning for each)

c) the charge on the capacitor

$$Q = 0$$

d) the potential difference across the resistor.

$$V = iR = 0$$

iii) At what time after the capacitor is connected to the resistor is the current through the resistor 0.6 of its maximal value?

$$Q = Q_0\, e^{-t/\tau} \qquad i = i_0\, e^{-t/\tau} \qquad \tau = RC$$

$$0.6 = \frac{i}{i_0} = e^{-t/\tau}$$

$$t/\tau = -\ln\frac{i}{i_0}$$

$$t = -RC\ln\frac{i}{i_0} = 74\,ps$$

5. (15 points)

A wire with a circular cross-section of radius R carries the current density whose magnitude as a function of radius is given by

$$J(r) = A\left(1 - \frac{r^2}{R^2}\right)$$

where A is a constant, and r is the distance to the center of the wire.

a) (3 points) At what distance from the center of the wire is the current density maximum?

$r = 0 \qquad J(r) = A$

or $\quad \frac{\partial J}{\partial r} = -A \cdot \frac{2r}{R^2} = 0 \Rightarrow \boxed{r = 0}$

b) (4 points) Write the current *di* carried by the part of wire of thickness *dr* at radius *r* from the center of the wire.

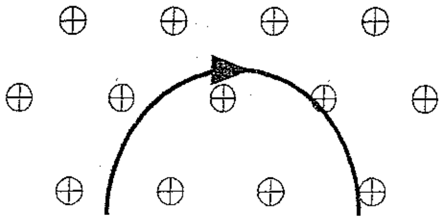
$$di = J(r)\,dA = J(r)\cdot 2\pi r\,dr = A\left[1 - \frac{r^2}{R^2}\right] 2\pi r\,dr$$

c) (8 points) In terms of *A* and *R*, what is the total current carried by the wire?

$$i = \int_0^R J(r)\cdot 2\pi r\,dr = A\int_0^R \left\{1 - \frac{r^2}{R^2}\right\} 2\pi r\,dr =$$

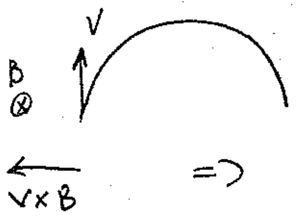
$$= 2\pi A\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \frac{\pi A R^2}{2}$$

6. (10 points)



A particle enters a region where a uniform magnetic field of $B = 5$ T is directed into the page.
The particle moves along the trajectory shown at right.

a) (3 points)
Is the particle positively or negatively charged? Explain.



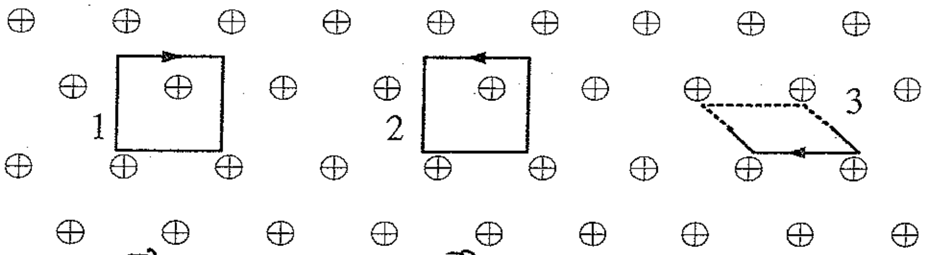
=> negatively charged

b) (7 points) If the velocity of the particle is 8000 m/s, the radius of its orbit 10 cm, and we know that the magnitude of its charge is the elementary charge, *e*, what is the mass of this particular particle?

$$m\frac{v^2}{r} = qvB \qquad m\frac{v}{R} = qB$$

$$m = \frac{qBr}{v} = \frac{1.6\cdot10^{-19}\cdot5\cdot10^{-1}}{8\cdot10^{3}} = 10^{-23}\ \text{kg}.$$

7. (10 points) The figure below shows three different orientations of the same loop of wire carrying a current, I, in a region where there is a uniform magnetic field, *B* Solid lines represent parts of the loop that are above the surface of the paper, and dashed lines represent parts that are below the surface of the paper. The arrow indicates the direction of current in the loop.



$\vec{\tau} = 0$ $\vec{\mu} \parallel \vec{B}$

$\vec{\tau} = 0$ $\vec{\mu} \parallel -\vec{B}$

$\vec{\tau} \neq 0$ $\vec{\mu} \perp \vec{B}$

i) Rank the scenarios according to the magnitude of the torque produced on the loop by the field

a) $\tau_1 = \tau_2 = \tau_3$

b) $\tau_1 = \tau_2 > \tau_3$

[✓ c) $\tau_1 = \tau_2 < \tau_3$]

d) $\tau_1 < \tau_2 < \tau_3$

e) $\tau_1 > \tau_2 > \tau_3$

f) $\tau_1 > \tau_3 > \tau_2$

Rank the scenarios according to the potential energy of the loop in the magnetic field

a) $U_1 > U_2 > U_3$

b) $U_3 > U_2 > U_1$

c) $U_1 = U_2 > U_3$

d) $U_1 = U_2 < U_3$

e) $U_1 > U_3 > U_2$

[✓ f) $U_2 > U_3 > U_1$]