Exam 1
Physics 2102 Fall 2009

September 24, 2009

Name: Solutions ID # ________________

Answer all questions (7).

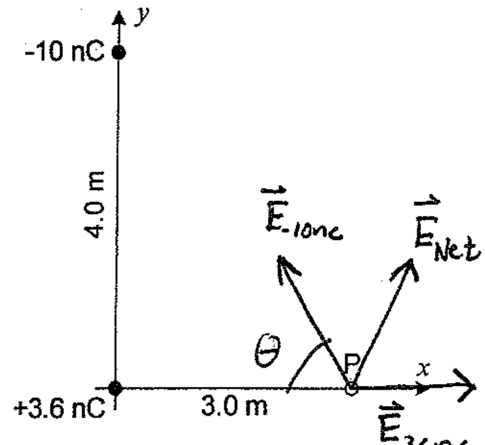
Some questions are multiple choice. You should work these problems starting with the basic equation listed on the formula sheet and write down all the steps. Although the work will not be graded, this will help you make the correct choice and be able to determine if your thinking is correct.

On problems that are not multiple choice, be sure to show all of your work since no credit will be given for an answer without explanation or work. These will be graded in full, and you are expected to show all relevant steps that lead to your answer.

Please use complete sentences where explanations are asked for.
Please be sure that *all* numerical quantities include appropriate units.

The only electronic devices to be used during the exam are standard or graphing calculators.
All cell phones should be turned off and put away. Cell phones are not to be used as calculators.

1.) (20 points) Two point charges, of -10 nC and +3.6 nC, respectively, are fixed in place on an *xy* coordinate plane as shown in the figure. At a point on the plane, *P*, no charge exists, but there is an electric field due to the presence of the two point charges.



a) Calculate the electric potential at point *P*, taking the potential at an infinite distance to be zero.

$$V_p = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$r_1 = 5.0\,m$

$r_2 = 3.0\,m$

$$V_p = \frac{1}{4\pi\epsilon_0}\left[\frac{-10\times10^{-9}\,C}{5.0\,m} + \frac{3.6\times10^{-9}\,C}{3.0\,m}\right] = -7.19\,V$$

$$\boxed{V_p = -7.2\,V}$$

b) At point *P* on the figure, sketch (and clearly label) three vectors
the electric field contribution due to the -10 nC charge
the electric field contribution due to the +3.6 nC charge
the net electric field

c) Calculate the electric field vector at *P* in unit vector notation.

$\vec{E}_N = \vec{E}_1 + \vec{E}_2$ by principle of superposition. $\rightarrow$ Vector addition

Break into components

$\theta = \tan^{-1}\frac{4}{3}$

$\theta = 53.1^\circ$

$$E_x = E_{1x} + E_{2x} = \frac{1}{4\pi\epsilon_0}\left[\frac{-|q_1|\cos\theta}{r_1^2} + \frac{|q_2|}{r_2^2}\right]$$

$$E_x = \frac{1}{4\pi\epsilon_0}\left[\frac{-10\times10^{-9}\,C\cos(53.1^\circ)}{(5\,m)^2} + \frac{3.6\times10^{-9}\,C}{(3.0\,m)^2}\right]$$

$E_x = 1.44\,N/C$

$$E_y = E_{1y} + \cancelto{0}{E_{2y}} = \frac{1}{4\pi\epsilon_0}\frac{|q_1|\sin\theta}{r_1^2}$$

$$E_y = \frac{1}{4\pi\epsilon_0}\left[\frac{10\times10^{-9}\,C\sin 53.1^\circ}{(5.0\,m)^2}\right] = 2.88\,\frac{N}{C}$$

$$\boxed{\vec{E}_{Net} = E_x\hat{i} + E_y\hat{j} = 1.4\,\frac{N}{C}\hat{i} + 2.9\,\frac{N}{C}\hat{j}}$$

2.) (10 points) The electric potential in a region of space is given by the expression

$V = 150xy^2z$ where x, y, and z are in meters, and V is in volts. Obtain an expression for the electric field vector in this region of space.

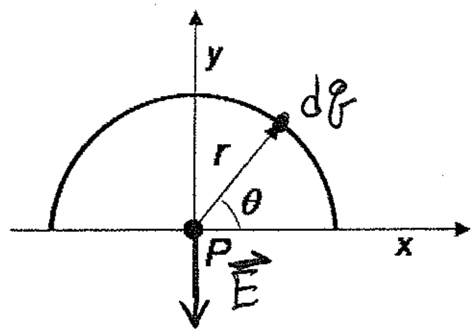
$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$\frac{\partial V}{\partial x} = 150y^2z \text{ V/m} \qquad \frac{\partial V}{\partial y} = 300xyz \text{ V/m}$$

$$\frac{\partial V}{\partial z} = 150xy^2 \text{ V/m}$$

$$\boxed{\therefore \vec{E} = -150y^2z\,\hat{i} - 300xyz\,\hat{j} - 150xy^2\,\hat{k} \quad \text{with } \vec{E} \text{ in V/m (or N/C)}}$$

3.) (20 points) A uniformly charged rod is bent into a semicircle radius r and has a positive charge density, charge per unit length, λ. Point P lies at the center of the circular arc formed by the charged rod.



a.) Without calculation, write down the magnitude of the x-component of the electric field at point P due to the charged semicircle. Indicate the direction of the electric field at point P on the figure above.

$E_x = 0$ by symmetry

b.) Write down an expression for the **y-component** of the differential field, d$\boldsymbol{E}$, at point P due to a small charge dq from a segment of length dS somewhere along the rod. Leave your answer in differential form, there is no need to integrate the expression for this part of the problem. [Hint: don't even think about using Gauss' law for this problem.]

$$dE_y = \frac{-1}{4\pi\epsilon_0}\frac{dq}{r^2}\sin\theta \quad \left[\text{from } \left[d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}\right]\cdot\hat{y}\right]$$

$$dq = \lambda\, dS$$

$$dE_y = \frac{-1}{4\pi\epsilon_0}\frac{\lambda\, dS}{r^2}\sin\theta$$

$$dS = r\, d\theta$$

$$\boxed{dE_y = \frac{-\lambda}{4\pi\epsilon_0}\frac{\sin\theta}{r}d\theta}$$

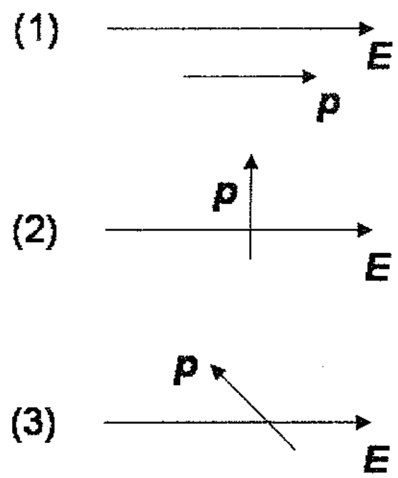
c.) Integrate this expression to obtain the y-component of the electric field at point P. Be sure to relate the variables S and θ as well as to explicitly write the limits of integration.

$$E_y = \int dE_y = \int_0^{\pi} \frac{-\lambda}{4\pi\epsilon_0}\frac{\sin\theta}{r}d\theta = \frac{-\lambda}{4\pi\epsilon_0 r}\int_0^{\pi}\sin\theta\, d\theta$$

$$E_y = \frac{+\lambda}{4\pi\epsilon_0 r}\cos\theta\Big|_0^{\pi} = \frac{\lambda}{4\pi\epsilon_0 r}\left[-1-1\right]$$

$$\boxed{E_y = \frac{-\lambda}{2\pi\epsilon_0 r}}$$

4.) (10 points) The figure below shows three different orientations of an electric dipole (with dipole moment $\boldsymbol{p}$) in a uniform electric field $\boldsymbol{E}$, which points in the positive x-direction.



(a) Rank the scenarios according to the potential energy of the dipole in the electric field, greatest first (circle the correct answer):

a) $U_1 > U_2 > U_3$
b) $U_2 > U_1 > U_3$
c) $U_3 > U_2 > U_1$
d) $U_1 > U_2 = U_3$
e) All tie

$U = -\vec{p} \cdot \vec{E}$

#1, $\vec{p} \parallel \vec{E}$, $U < 0$
#2 $\vec{p} \perp E$, $U = 0$
#3 $U > 0$

(b) Rank the scenarios according to the magnitude of the torque on the dipole in the electric field, greatest first (circle the correct answer):

a) $\tau_1 > \tau_2 > \tau_3$
b) $\tau_2 > \tau_3 > \tau_1$
c) $\tau_2 > \tau_1 = \tau_3$
d) $\tau_1 > \tau_2 = \tau_3$

$\vec{\tau} = \vec{p} \times \vec{E}$

$|\vec{\tau}| = |\vec{p}||\vec{E}| \sin\theta$

#1 $\vec{p} \parallel \vec{E}$, $\vec{\tau} = 0$
#2 $\vec{p} \perp E$ $|\vec{\tau}| = |\vec{p}||\vec{E}|$
#3 $|\vec{\tau}| = |\vec{p}||\vec{E}| \sin\theta$

$\sin\theta < 1$

5.) (10 points) Circle the correct statements about conductors. Note, there can be more than one true statement.

a) Electric field lines always point perpendicular to a conducting surface.

b) Conductors make poor shields for electric fields.

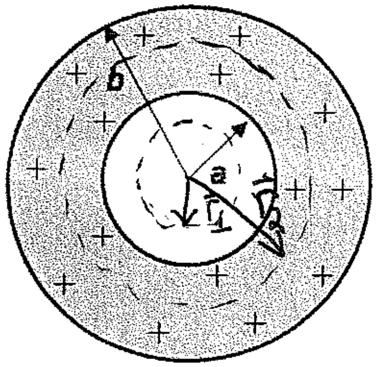
c) The static electric field within the volume of a conductor is always zero.

d) Lightning never strikes a good conductor.

e) Conductors are equipotential volumes.

f) Excess charge on a conductor is uniformly distributed throughout the volume of the conductor.

6.) (15 points.) The figure below shows a (nonconducting) spherical shell with uniform volume charge density, ρ, inner radius, *a*, and outer radius, *b*.



a) Using Gauss' law find an expression for the electric field at positions where r < a.

$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ Gauss' Law. For $r<a$ draw a gaussian surface - sphere radius r_1, centered at center of spherical shell, as shown. Note $q_{enc}=0$. Since for this gaussian surface $\vec{E} \parallel d\vec{A}$ by symmetry (if there were a non-zero $\vec{E}$) and $|\vec{E}|=$ constant by symmetry $EA=0$; $\boxed{E=0}$

b) Using Gauss' law find an expression for the electric field at positions where *a*<*r*<*b*.

Gauss' Law $\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$; Draw a gaussian surface - sphere radius r_2 centered at the center of the spherical shell. First calculate the charge enclosed by our gaussian surface

$$q_{enc} = \rho \cdot \text{Vol enclosed} = \rho \cdot \tfrac{4}{3}\pi\left(r_2^3 - a^3\right)$$ [Vol sphere - Vol empty center]

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint |\vec{E}|\,|d\vec{A}| = |\vec{E}|\oint |d\vec{A}| = |\vec{E}|A = \frac{\rho\frac{4}{3}\pi\left(r_2^3-a^3\right)}{\epsilon_0}$$

$$|\vec{E}| = \frac{1}{A}\cdot\frac{\rho\frac{4}{3}\pi\left(r_2^3-a^3\right)}{\epsilon_0} = \frac{\rho}{\epsilon_0}\cdot\frac{1}{4\pi r_2^2}\cdot\frac{4}{3}\pi\left(r_2^3-a^3\right)$$

$$|\vec{E}| = \frac{\rho}{3\epsilon_0}\left[r_2 - \frac{a^3}{r_2^2}\right]$$ or for general r $a<r<b$

$$\boxed{|\vec{E}| = \frac{\rho}{3\epsilon_0}\left[r - \frac{a^3}{r^2}\right]}$$

7.) (15 points) Short answers. Answer with one or two grammatically correct English sentences.

a.) The potential is a constant throughout a region of space. Is the electric field zero or non-zero in this region? Explain.

If the potential is constant throughout a region of space, the electric field must be zero in this region. The easiest way to see this is through the relation $\vec{E} = \vec{\nabla} V$, where the gradient of a constant is identically zero.

b.) In a region of space where the electric field is constant everywhere, but not zero, is the potential constant everywhere?

No, since $\Delta V = -\int \vec{E} \cdot d\vec{s}$ any path, $d\vec{s}$, that is not perpendicular to $\vec{E}$ yields a changing potential. $\therefore$ V is <u>not</u> constant in this region.

c.) A positive test charge is placed in an electric field. In what direction should the charge be moved relative to the field, such that the charge experiences no change in the electric potential?

To experience no change in electric potential the charge must be moved <u>perpendicular to the electric field</u>. Again, since $\Delta V = -\int \vec{E} \cdot d\vec{s}$, a path $\perp$ to $\vec{E}$ gives a zero dot product, and thus zero change in potential.