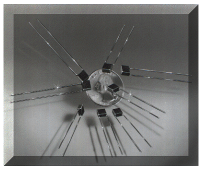
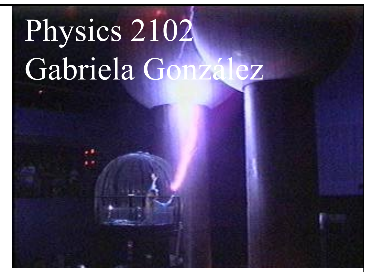


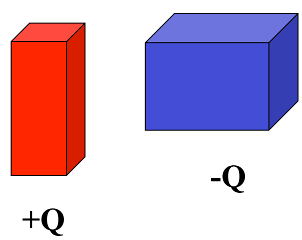


Physics 2102

Capacitors

Capacitors and Capacitance

Capacitor: any two conductors, one with charge +Q, other with charge -Q



Potential DIFFERENCE between conductors = V

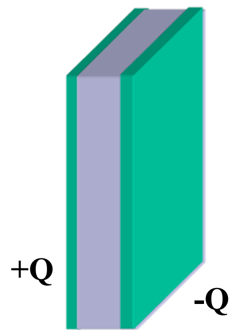
$Q = \mathbf{C}V$ -- C = capacitance

Units of capacitance:
Farad (F) = Coulomb/Volt

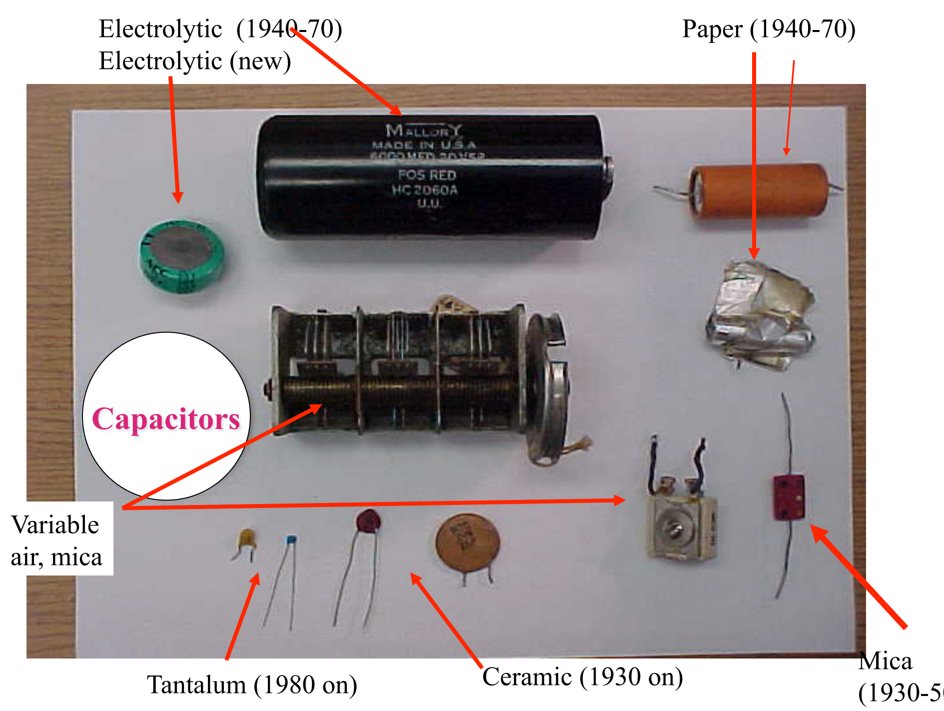
Uses: storing and releasing electric charge/energy.
Most electronic capacitors: micro-Farads (μF), pico-Farads (pF) -- 10^{-12} F
New technology: compact 1 F capacitors

Capacitance

- Capacitance depends only on GEOMETRICAL factors and on the MATERIAL that separates the two conductors
- e.g. Area of conductors, separation, whether the space in between is filled with air, plastic, etc.



(We first focus on capacitors where gap is filled by AIR!)



Capacitors and Capacitance

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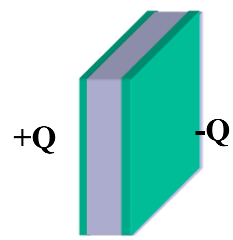


Uses: storing and releasing electric charge/energy.
Most electronic capacitors: micro-Farads (μF), pico-Farads (pF) -- 10^{-12} F
New technology: compact 1 F capacitors

Parallel Plate Capacitor

We want ***capacitance***: C=Q/V

E field between the plates: (Gauss' Law)

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

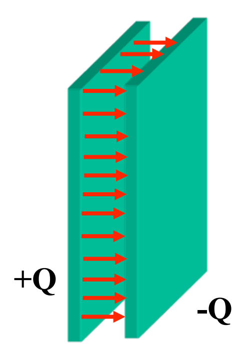
Relate E to potential difference V:

$$V = \int_0^d \vec{E}\cdot d\vec{x} = \int_0^d \frac{Q}{\varepsilon_0 A}dx = \frac{Qd}{\varepsilon_0 A}$$

What is the capacitance C?

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

Area of each plate = A
Separation = d
charge/area = σ= Q/A



Parallel Plate Capacitor -- example

- A huge parallel plate capacitor consists of two square metal plates of side 50 cm, separated by an air gap of 1 mm
- What is the capacitance?
- $C = \varepsilon_0 A/d =$

$(8.85 \times 10^{-12}\ F/m)(0.25\ m^2)/(0.001\ m)$

$= 2.21 \times 10^{-9}\ F$

(small!!)

Lesson: difficult to get large values of capacitance without special tricks!

Isolated Parallel Plate Capacitor

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon_0 A}{d}$$

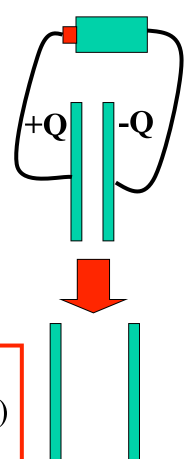
- A parallel plate capacitor of capacitance C is charged using a battery.
- Charge = Q, potential difference = V.
- Battery is then disconnected.
- If the plate separation is INCREASED, does potential difference V:

(a) Increase?

(b) Remain the same?

(c) Decrease?

- Q is fixed!
- C decreases ($=\varepsilon_0 A/d$)
- Q=CV; V increases.



Parallel Plate Capacitor & Battery

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon_0 A}{d}$$

- A parallel plate capacitor of capacitance C is charged using a battery.
- Charge = Q, potential difference = V.
- Plate separation is INCREASED while battery remains connected.

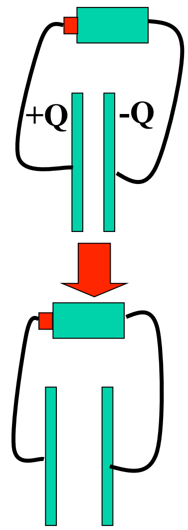
Does the electric field inside:

(a) Increase?

(b) Remain the same?

(c) Decrease?

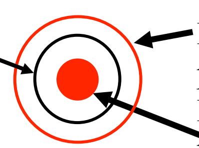
- V is fixed by battery!
- C decreases ($=\varepsilon_0 A/d$)
- Q=CV; Q decreases
- $E = Q/\varepsilon_0 A$ decreases



Spherical Capacitor

What is the electric field inside the capacitor? (Gauss' Law)

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$



Radius of outer plate = b

Radius of inner plate = a

Concentric spherical shells: Charge +Q on inner shell, -Q on outer shell

Relate E to potential difference between the plates:

$$V = \int_a^b \vec{E}\cdot d\vec{r} = \int_a^b \frac{kQ}{r^2}dr = \left[-\frac{kQ}{r}\right]_a^b = kQ\left[\frac{1}{a} - \frac{1}{b}\right]$$

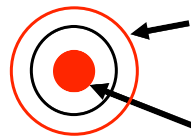
Spherical Capacitor

What is the capacitance?

C = Q/V =

$$= \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]}$$

$$= \frac{4\pi\varepsilon_0 ab}{(b-a)}$$



Concentric spherical shells:
Charge +Q on inner shell,
-Q on outer shell

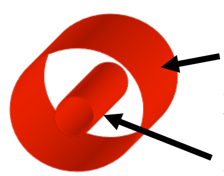
Isolated sphere: let b >> a,

$$C = 4\pi\varepsilon_0 a$$

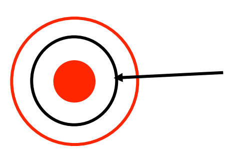
Cylindrical Capacitor

What is the electric field in between the plates?

$$E = \frac{Q}{2\pi\varepsilon_0 rL}$$



Length of capacitor = L
+Q on inner rod, -Q on outer shell



Relate E to potential difference between the plates:

$$V = \int_a^b \vec{E}\cdot d\vec{r}$$

$$= \int_a^b \frac{Q}{2\pi\varepsilon_0 rL}\,dr = \left[\frac{Q\ln r}{2\pi\varepsilon_0 L}\right]_a^b = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

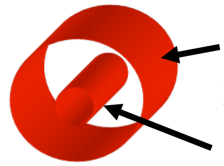
Cylindrical Capacitor

What is the capacitance C?

C = Q/V =

$$= \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$



Radius of outer plate = b

Radius of inner plate = a

Length of capacitor = L

Charge +Q on inner rod,

-Q on outer shell

Example: co-axial cable.

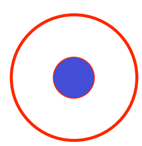
Summary

- Any two charged conductors form a capacitor.
- Capacitance : C= Q/V
- Simple Capacitors:

  Parallel plates: $C = \varepsilon_0 A/d$

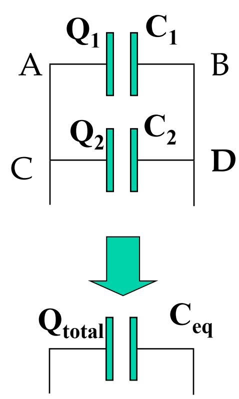
  Spherical : $C = 4\pi \varepsilon_0 ab/(b-a)$

  Cylindrical: $C = 2\pi \varepsilon_0 L/\ln(b/a)$



Capacitors in Parallel

- A wire is a conductor, so it is an equipotential.
- Capacitors in parallel have SAME potential difference but NOT ALWAYS same charge.
- $V_{AB} = V_{CD} = V$
- $Q_{total} = Q_1 + Q_2$
- $C_{eq}V = C_1V + C_2V$
- $\mathbf{C_{eq} = C_1 + C_2}$
- **Equivalent parallel capacitance = sum of capacitances**



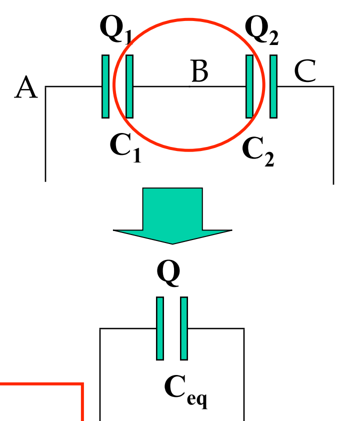
PARALLEL:
- V is same for all capacitors
- Total charge in C_{eq} = sum of charges

Capacitors in series

- $Q_1 = Q_2 = Q$ (WHY??)
- $V_{AC} = V_{AB} + V_{BC}$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

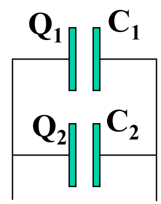
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

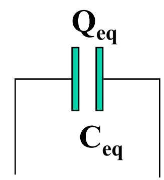


SERIES:
- Q is same for all capacitors
- Total potential difference in C_{eq} = sum of V

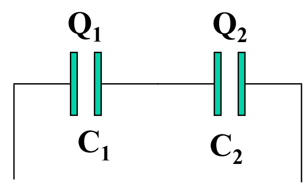
Capacitors in parallel and in series

- **In parallel :**
  - $C_{eq} = C_1 + C_2$
  - $V_{eq}=V_1=V_2$
  - $Q_{eq}=Q_1+Q_2$





- **In series :**
  - $1/C_{eq} = 1/C_1 + 1/C_2$
  - $V_{eq}=V_{1+}V_2$
  - $Q_{eq}=Q_{1=}Q_2$

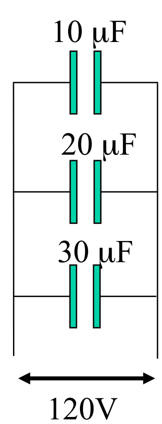


Example 1

What is the charge on each capacitor?

- $Q = CV$; $V = 120$ V
- $Q_1 = (10\ \mu F)(120V) = 1200\ \mu C$
- $Q_2 = (20\ \mu F)(120V) = 2400\ \mu C$
- $Q_3 = (30\ \mu F)(120V) = 3600\ \mu C$

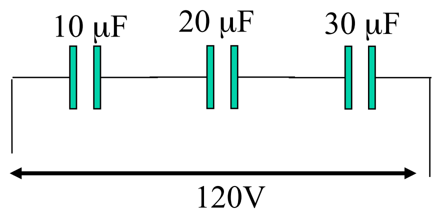
Note that:
- Total charge (7200 μC) is shared between the 3 capacitors in the ratio $C_1{:}C_2{:}C_3$ -- i.e. 1:2:3



Example 2

What is the potential difference across each capacitor?

- Q = CV; Q is same for all capacitors
- Combined C is given by:



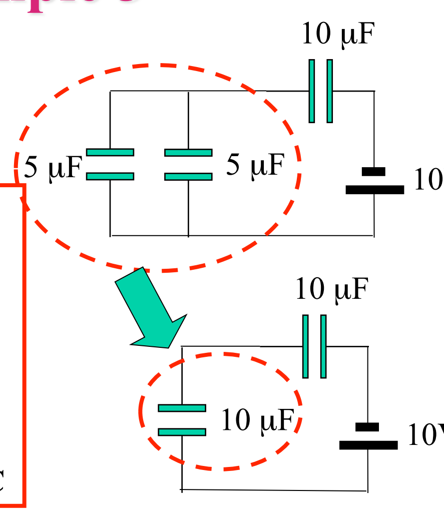
$$\frac{1}{C_{eq}} = \frac{1}{(10\mu F)} + \frac{1}{(20\mu F)} + \frac{1}{(30\mu F)}$$

- C_{eq} = 5.46 μF
- Q = CV = (5.46 μF)(120V) = 655 μC
- $V_1 = Q/C_1$ = (655 μC)/(10 μF) = 65.5 V
- $V_2 = Q/C_2$ = (655 μC)/(20 μF) = 32.75 V
- $V_3 = Q/C_3$ = (655 μC)/(30 μF) = 21.8 V

Note: 120V is shared in the ratio of INVERSE capacitances i.e.1:(1/2):(1/3)
(largest C gets smallest V)

Example 3

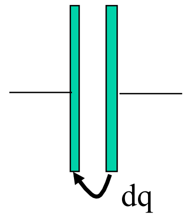
In the circuit shown, what is the charge on the 10μF capacitor?



- The two **5μF** capacitors are in parallel
- Replace by **10μF**
- Then, we have two **10μF** capacitors in series
- So, there is 5V across the **10μF** capacitor of interest
- Hence, Q = (**10μF**)(5V) = **50μC**

Energy Stored in a Capacitor

- Start out with uncharged capacitor
- Transfer small amount of charge dq from one plate to the other until charge on each plate has magnitude Q
- How much work was needed?



$$U = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}\,dq = \boxed{\frac{Q^2}{2C} = \frac{CV^2}{2}}$$

Energy Stored in Electric Field

- Energy stored in capacitor: $U = Q^2/(2C) = CV^2/2$
- View the energy as stored in ELECTRIC FIELD
- For example, parallel plate capacitor:
  Energy DENSITY = energy/volume = u =

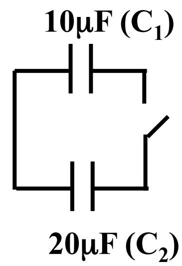
$$U = \frac{Q^2}{2CAd} = \frac{Q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)Ad} = \frac{Q^2}{2\varepsilon_0 A^2} = \frac{\varepsilon_0}{2}\left(\frac{Q}{\varepsilon_0 A}\right)^2 = \boxed{\frac{\varepsilon_0 E^2}{2}}$$

volume = Ad

General expression for any region with vacuum (or air)

Example

- **10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.**
- **Switch is closed, equilibrium is reached.**
- **How much energy is dissipated in the process?**



Initial charge on 10μF = (10μF)(120V)= 1200μC

After switch is closed, let charges = Q_1 and Q_2.

Charge is conserved: $Q_1 + Q_2 = 1200\mu C$

Also, V_{final} is same: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ⇨ $Q_1 = \frac{Q_2}{2}$ ⇨

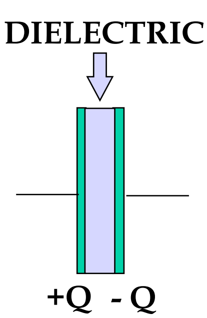
- $Q_1 = 400\mu C$
- $Q_2 = 800\mu C$
- $V_{final} = Q_1/C_1 = 40$ V

Initial energy stored = $(1/2)C_1V_{initial}^2 = (0.5)(10\mu F)(120)^2 = 72mJ$

Final energy stored = $(1/2)(C_1 + C_2)V_{final}^2 = (0.5)(30\mu F)(40)^2 = 24mJ$

Energy lost (dissipated) = 48mJ

Dielectric Constant

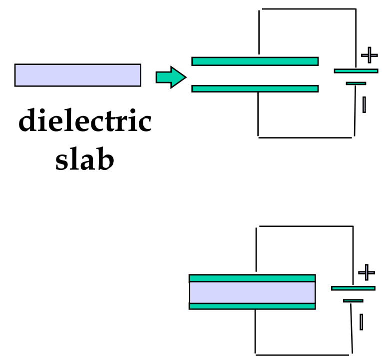


$C = \kappa\varepsilon_0 A/d$

- If the space between capacitor plates is filled by a dielectric, the capacitance **INCREASES** by a factor κ
- This is a useful, working definition for dielectric constant.
- Typical values of κ: 10 - 200

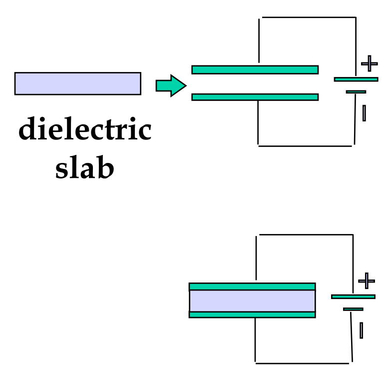
Example

- Capacitor has charge Q, voltage V
- Battery remains connected while dielectric slab is inserted.
- Do the following increase, decrease or stay the same:
  - Potential difference?
  - Capacitance?
  - Charge?
  - Electric field?



Example (soln)

- Initial values:
  capacitance = **C**; charge = **Q**;
  potential difference = **V**;
  electric field = **E**;
- Battery remains connected
- V is FIXED; $\mathbf{V_{new} = V}$ (**same**)
- $\mathbf{C_{new} = \kappa C}$ (**increases**)
- $\mathbf{Q_{new} = (\kappa C)V = \kappa Q}$ (**increases**).
- Since $\mathbf{V_{new} = V}$, $\mathbf{E_{new} = E}$ (**same**)



Energy stored? $u=\varepsilon_0 E^2/2$ => $u=\kappa\varepsilon_0 E^2/2 = \varepsilon E^2/2$

Summary

- Capacitors in series and in parallel:
  - in series: charge is the same, potential adds, equivalent capacitance is given by $1/C=1/C_1+1/C_2$
  - in parallel: charge adds, potential is the same, equivalent capaciatnce is given by $C=C_1+C_2$.
- Energy in a capacitor: $U=Q^2/2C=CV^2/2$; energy density $u=\varepsilon_0 E^2/2$
- Capacitor with a dielectric: capacitance increases $C'=\kappa C$