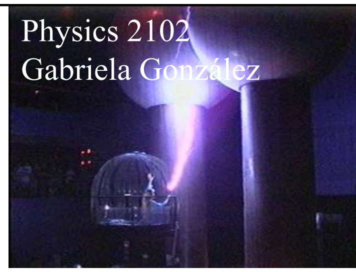
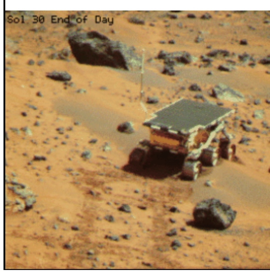


Physics 2102



Electric Potential

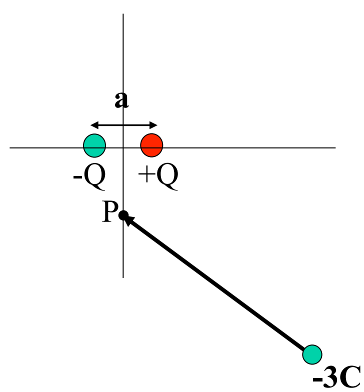
Electric Potential on Perpendicular Bisector of Dipole

You bring a charge of -3C from infinity to a point P on the perpendicular bisector of a dipole as shown. Is the work that <u>you</u> do:

a)Positive?

b)Negative?

c)Zero?



Electric Potential of Many Point Charges

What is the electric potential at the center of each circle?

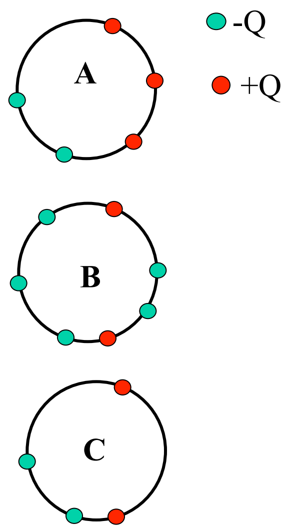
- Potential is a SCALAR
- All charges are equidistant from each center, hence contribution from each charge has same magnitude: V
- +Q has positive contribution
- -Q has negative contribution

A: -2V+3V = +V

B: -5V+2V = -3V

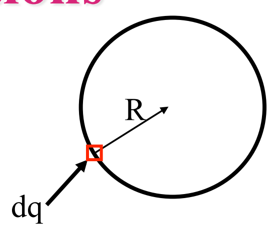
C: -2V+2V = 0

Note that the **electric field** at the center is a vector, and is NOT zero for C!



Continuous Charge Distributions

- Divide the charge distribution into differential elements
- Write down an expression for potential from a typical element -- treat as point charge
- Integrate!
- Simple example: circular rod of radius R, total charge Q; find V at center.



$$V = \int \frac{dq}{4\pi\varepsilon_0 R}$$

$$= \frac{1}{4\pi\varepsilon_0 R}\int dq = \frac{Q}{4\pi\varepsilon_0 R}$$

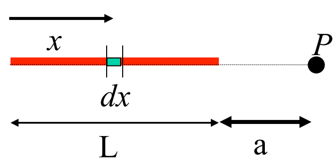
Potential of Continuous Charge Distribution: Example

- Uniformly charged rod
- Total charge q
- Length L
- What is V at position P shown?

$$\lambda = q/L \qquad dq = \lambda\, dx$$

$$V = \int \frac{k\,dq}{r} = \int_0^L \frac{k\lambda dx}{(L+a-x)}$$

$$= k\lambda\left[-\ln(L+a-x)\right]_0^L$$



$$V = k\lambda \ln\left[\frac{L+a}{a}\right]$$

Summary so far:

- **Electric potential**: work needed to bring +1C from infinity; units = V
- Work needed to bring a charge from infinity is W=qV
- Electric potential is a **scalar** -- add contributions from individual point charges
- We calculated the electric potential produced:
  - by a single charge: V=*kq*/*r*,
  - by several charges using superposition, and
  - by a continuous distribution using integrals.

Electric Field & Potential: A Neat Relationship!

Notice the following:

- Point charge:
  - $E = kQ/r^2$
  - $V = kQ/r$
- Dipole (far away):
  - $E \sim kp/r^3$
  - $V \sim kp/r^2$
- E is given by a DERIVATIVE of V!

Focus only on a simple case: electric field that points along +x axis but whose magnitude varies with x.

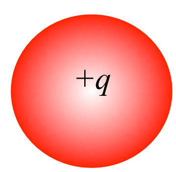
$$E_x = -\frac{dV}{dx}$$

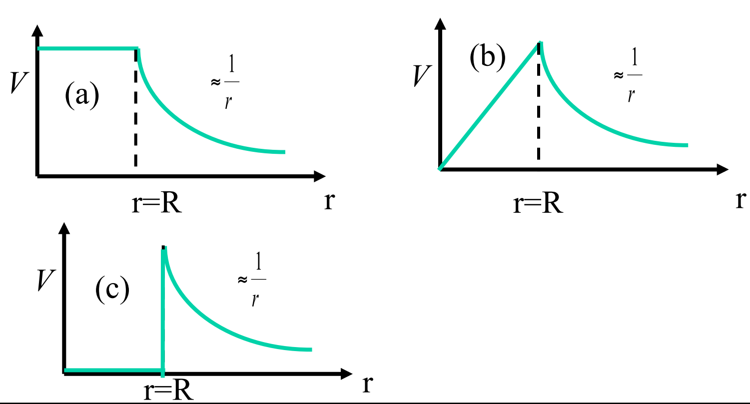
Note:

- **MINUS sign!**
- **Units for E -- VOLTS/ METER (V/m)**

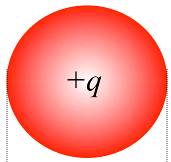
Electric Field & Potential: Example

- Hollow **metal** sphere of radius R has a charge +q
- Which of the following is the electric potential V as a function of distance r from center of sphere?





Electric Field & Potential: Example



Outside the sphere:

- Replace by point charge!

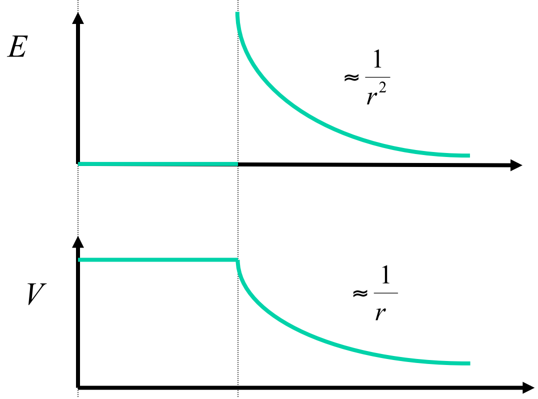
Inside the sphere:

- E =0 (Gauss' Law)
- → V=constant

$$E = -\frac{dV}{dr} = \frac{kQ}{r^2}$$

$$V = -\int E\,dr = -\int \frac{kQ}{r^2}\,dr$$

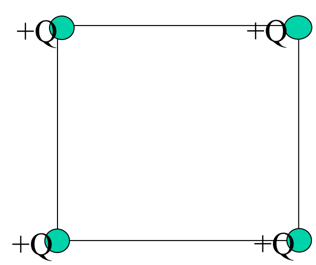
$$= \frac{kQ}{r}$$



Potential inside?
At r = R, V = kQ/R
For r < R, V = kQ/R.

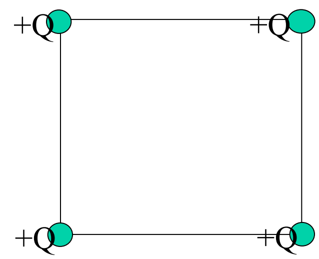
Potential Energy of A System of Charges

- 4 point charges (each +Q) are connected by strings, forming a square of side L
- If all four strings suddenly snap, what is the kinetic energy of each charge when they are very far apart?
- Use conservation of energy:
  - Final kinetic energy of all four charges = initial potential energy stored = energy required to assemble the system of charges



Do this from scratch! Understand, not memorize the formula in the book!

Potential Energy of A System of Charges: Solution



- No energy needed to bring in first charge: $U_1=0$
- Energy needed to bring in 2nd charge: $U_2 = QV_1 = \frac{kQ^2}{L}$
- Energy needed to bring in 3rd charge =

$$U_3 = QV = Q(V_1 + V_2) = \frac{kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

- Energy needed to bring in 4th charge =

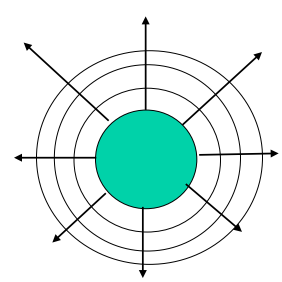
$$U_4 = QV = Q(V_1 + V_2 + V_3) = \frac{2kQ^2}{L} + \frac{kQ^2}{\sqrt{2}L}$$

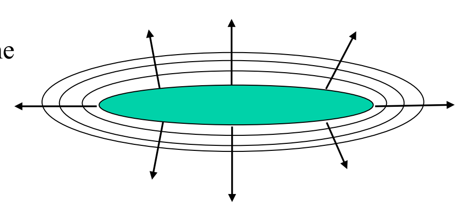
Total potential energy is sum of all the individual terms shown on left hand side = $\frac{kQ^2}{L}\left(4+\sqrt{2}\right)$

So, final kinetic energy of each charge = $\frac{kQ^2}{4L}\left(4+\sqrt{2}\right)$

Equipotentials and Conductors

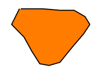
- Conducting surfaces are EQUIPOTENTIALs
- At surface of conductor, E is normal to surface
- Hence, no work needed to move a charge from one point on a conductor surface to another
- Therefore, electric potential is constant on the surface of conductors.
- Equipotentials are normal to E, so they follow the shape of the conductor near the surface.
- Inside the conductor, E=0: therefore, potential is constant. Potential is not necessarily zero! It is equal to the potential on the surface.





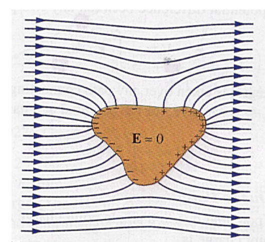
Conductors change the field around them!

An uncharged conductor:

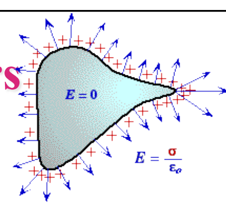


A uniform electric field:

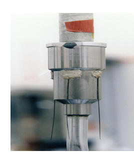
An uncharged conductor in the initially uniform electric field:

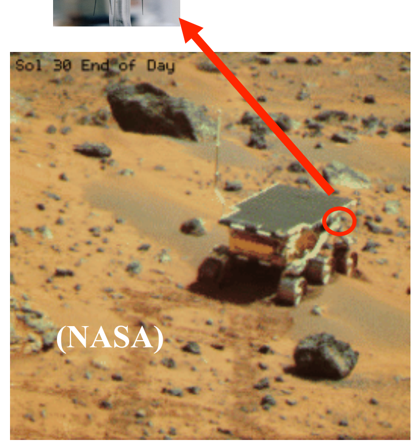


"Sharp" conductors



- Charge density is higher at conductor surfaces that have small radius of curvature
- $E = \sigma/\varepsilon_0$ for a conductor, hence STRONGER electric fields at sharply curved surfaces!
- Used for attracting or getting rid of charge:
  - lightning rods
  - Van de Graaf -- metal brush transfers charge from rubber belt
  - Mars pathfinder mission -- tungsten points used to get rid of accumulated charge on rover (electric breakdown on Mars occurs at ~100 V/m)





Summary:

- Electric field and electric potential: $E = -dV/dx$
- **Electric potential energy**: work used to build the system, charge by charge. Use $W=qV$ for each charge.
- **Conductors**: the charges move to make their surface equipotentials.
- Charge density and electric field are higher on sharp points of conductors.