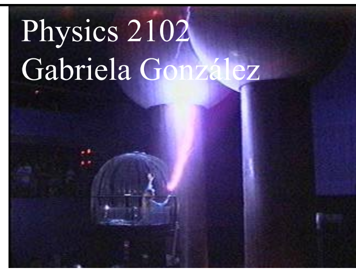
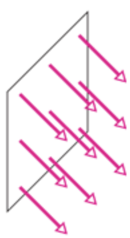


Physics 2102

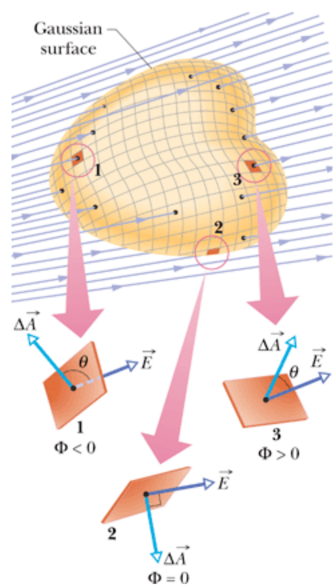
Gauss' law

Carl Friedrich Gauss
1777-1855

Electric Flux

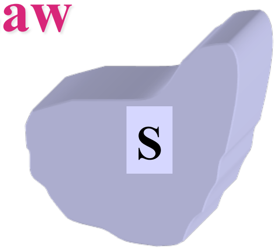


$$\Phi = \int \vec{E} \bullet d\vec{A}$$



- Electric Flux
  A *surface* integral!
- **CLOSED** surfaces:
  - define the vector dA as pointing **OUTWARDS**
  - **Inward E gives negative Φ**
  - **Outward E gives positive Φ**

Gauss' Law



- Consider any ARBITRARY CLOSED surface S -- NOTE: this does NOT have to be a "real" physical object!
- The TOTAL ELECTRIC FLUX through S is proportional to the TOTAL CHARGE ENCLOSED!
- The results of a complicated integral is a very simple formula: it avoids long calculations!

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

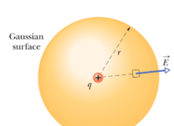
(One of Maxwell's 4 equations)

Gauss' law, using symmetry

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(electric flux through a Gaussian surface).}$$

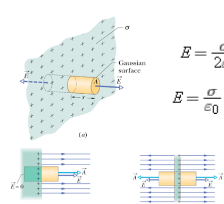
$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss ' law),}$$

Spherical symmetry



$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} .$$

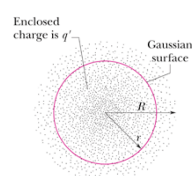
Planar symmetry



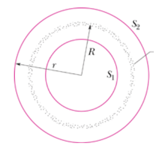
$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(sheet of charge) .}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface) .}$$

Shell theorem



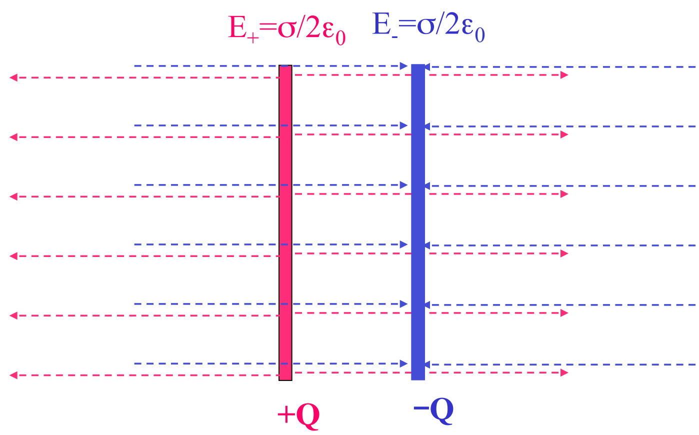
$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \quad \text{(uniform charge, field at } r \le R) .$$



$$E = 0 \quad \text{(spherical shell, field at } r < R),$$

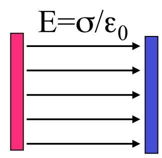
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \quad \text{(spherical shell, field at } r \ge R).$$

Two infinite planes



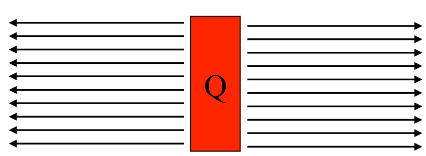
A uniform field!

E=0



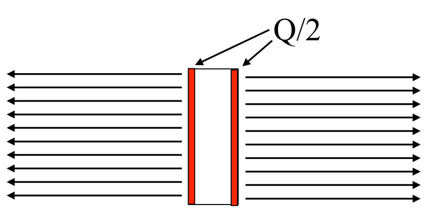
E=0

Insulating and conducting planes



$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Insulating plate: charge distributed homogeneously.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Conducting plate: charge distributed on the outer surfaces.

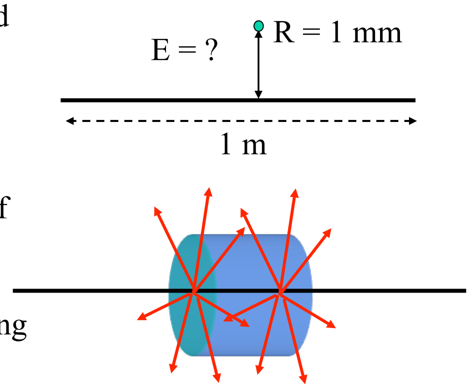
Gauss' Law: Cylindrical symmetry

- Charge of 10 C is uniformly spread over a line of length L = 1 m.

- Use Gauss' Law to compute magnitude of E at a perpendicular distance of 1 mm from the center of the line.

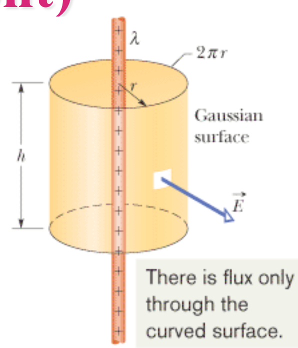
    - Approximate as infinitely long line -- E radiates outwards.

    - Choose cylindrical surface of radius R, length L co-axial with line of charge.



Gauss' Law: cylindrical symmetry (cont)

- Approximate as infinitely long line -- E radiates outwards.
- Choose cylindrical surface of radius r, length h co-axial with line of charge.



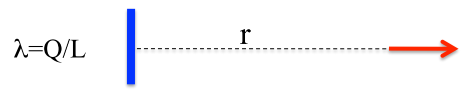
$$\Phi = |E| A = |E| 2\pi r h$$

$$\Phi = \frac{q}{\varepsilon_0} = \frac{\lambda h}{\varepsilon_0}$$

$$|E| = \frac{\lambda h}{2\pi\varepsilon_0 r h} = \frac{\lambda}{2\pi\varepsilon_0 r} = 2k\frac{\lambda}{R}$$

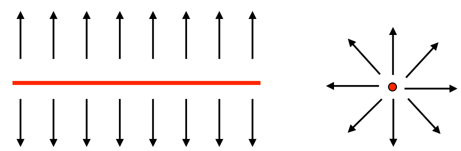
Compare with exact calculation!

$$E_y = k\lambda\, r \int_{-L/2}^{L/2} \frac{dx}{(r^2+x^2)^{3/2}} = k\lambda\, r\left[\frac{x}{r^2\sqrt{x^2+r^2}}\right]_{-L/2}^{L/2} = \frac{k\lambda L}{r\sqrt{r^2+(L/2)^2}}$$



if the line is infinitely long (L >> r)…

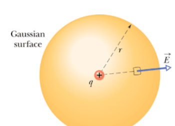
$$E_y \approx \frac{k\lambda L}{r\sqrt{(L/2)^2}} = \frac{2k\lambda}{r}$$



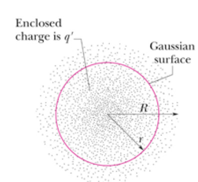
Gauss' law, using symmetry

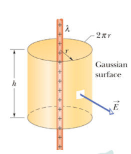
$\Phi = \oint \vec{E}\cdot d\vec{A}$ (electric flux through a Gaussian surface).

$\varepsilon_0 \Phi = q_{enc}$ (Gauss' law),



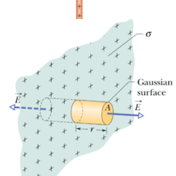
$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$.





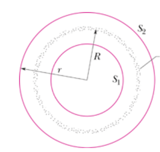
$E = \frac{\lambda}{2\pi\varepsilon_0 r}$ (line of charge).

$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$ (uniform charge, field at $r \le R$).

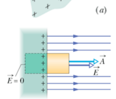


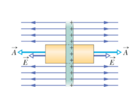
$E = \frac{\sigma}{2\varepsilon_0}$ (sheet of charge).

$E = \frac{\sigma}{\varepsilon_0}$ (conducting surface).



$E = 0$ (spherical shell, field at $r < R$),

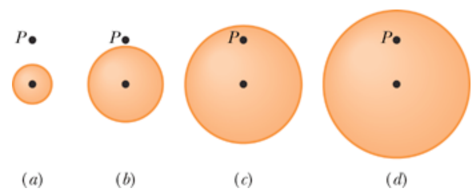




Question

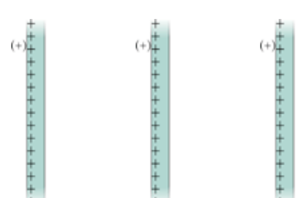
The figure shows four solid spheres, each with charge *Q* uniformly distributed through its volume.
(a) Rank the spheres according to their volume charge density, greatest first.

The figure also shows a point *P* for each sphere, all at the same distance from the center of the sphere.
(b) Rank the spheres according to the magnitude of the electric field they produce at point *P*, greatest first.



Question

Three infinite nonconducting sheets, with uniform positive surface charge densities σ, 2σ, and 3σ, are arranged to be parallel. What is their order, from left to right, if the electric field produced by them is zero in one region and has magnitude $E = 2\sigma/\varepsilon_0$ in another region?

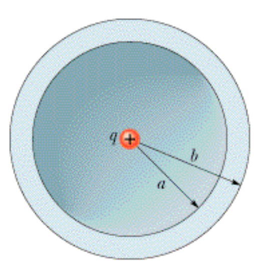


Ch 23 Summary:

- We define electric flux through a surface: $\Phi = \int \vec{E} \bullet d\vec{A}$
- Gauss' law provides a very direct way to compute the electric flux : $\Phi = q_{ins} / \varepsilon_0$
- In situations with symmetry, knowing the flux allows to compute the fields reasonably easily:
  - Spherical field of a spherical uniform charge: kq_{ins}/r^2
  - Uniform field of an insulating plate: $\sigma/2\varepsilon_{0,}$; of a conducting plate: $\sigma/\varepsilon_{0.}$.
  - Cylindrical field of a long wire: $2k\lambda/r$
- Properties of conductors: field inside is zero; excess charges are always on the surface; field on the surface is perpendicular and $E=\sigma/\varepsilon_{0.}$

Problem

In the figure below, a nonconducting spherical shell of inner radius a and outer radius b has (within its thickness) a positive volume charge density ρ = A/r, where A is a constant and r is the distance from the center of the shell. In addition, a small ball of charge q is located at the center. What constant A produces a uniform electric field in the shell a <r <b ?



Electric potential energy, electric potential

Electric potential energy of a system =
= − work (against electrostatic forces) needed to needed to build the system

$$U = -W$$

Electric potential difference between two points = work per unit charge needed to move a charge between the two points:

$$\Delta V = V_f - V_i = -W/q$$

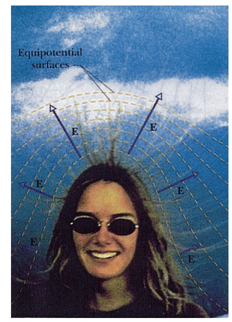
Electric potential energy, electric potential

Units : [U] = [W]=Joules;
[V]=[W/q] = Joules/C= Nm/C= Volts
[E]= N/C = Vm

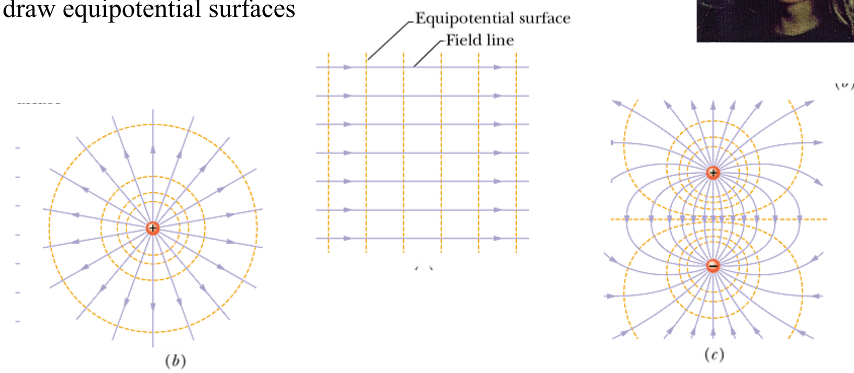
1eV = work needed to move an electron through a potential difference of 1V:

$W = q\Delta V = e \times 1V$
$= 1.60\ 10^{-19}$ C x 1J/C = $1.60\ 10^{-19}$ J

Electric field lines and equipotential surfaces



Given a charged system, we can:
- calculate the electric field everywhere in space
- calculate the potential difference between every point and a point where V=0
- draw electric field lines
- draw equipotential surfaces



http://phet.colorado.edu/en/simulation/charges-and-fields