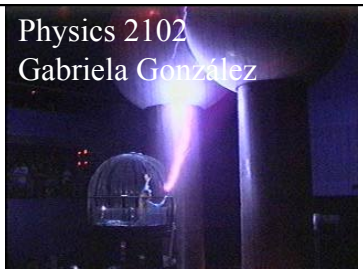
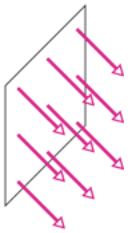


Physics 2102

Gauss' law

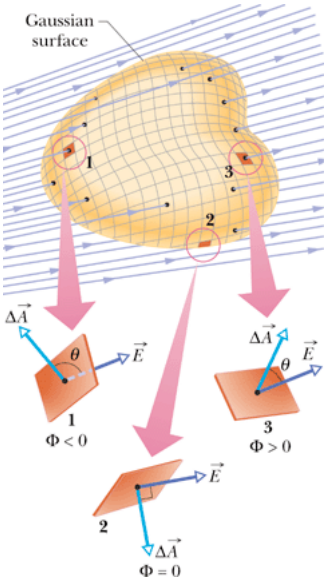
Carl Friedrich Gauss
1777-1855

Electric Flux

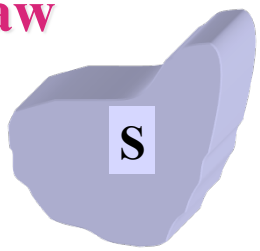


$$\Phi = \int \vec{E} \bullet d\vec{A}$$

- Electric Flux
  A *surface* integral!
- **CLOSED** surfaces:
  - define the vector dA as pointing **OUTWARDS**
  - **Inward E gives negative Φ**
  - **Outward E gives positive Φ**



Gauss' Law



- Consider any ARBITRARY CLOSED surface S -- NOTE: this does NOT have to be a "real" physical object!
- The TOTAL ELECTRIC FLUX through S is proportional to the TOTAL CHARGE ENCLOSED!
- The results of a complicated integral is a very simple formula: it avoids long calculations!

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

(One of Maxwell's 4 equations)

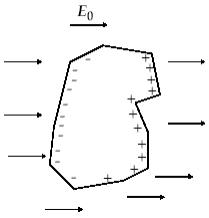
Properties of conductors

Inside a conductor in electrostatic equilibrium, the electric field is ZERO. Why?

Because if the field is not zero, then charges inside the conductor would feel forces and would be moving: the system is not in equilibrium yet.

SO: the electric field pushes charges in a conductor to redistribute themselves until they make the field inside the conductor ZERO.

Excess charges are always on the **surface** of the conductors (but never inside).



Example : "Faraday's Cage"

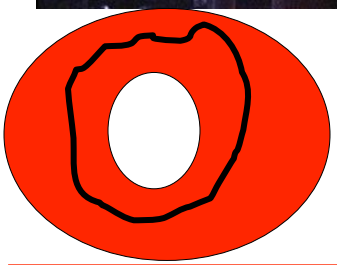
Given a hollow conductor of arbitrary shape. Suppose an excess charge Q is placed on this conductor. Suppose the conductor is placed in an external electric field. How does the charge distribute itself on outer and inner surfaces?

(a) Inner: Q/2; outer: Q/2
(b) Inner: 0; outer: Q
(c) Inner: Q; outer: 0

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- Choose any arbitrary surface inside the metal
- Gauss' law: since E = 0, flux = 0
- Hence total charge enclosed = 0
- All charge goes on outer surface!

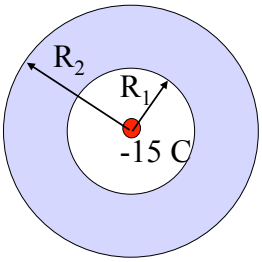
http://www.mos.org/sln/toe/



Inside cavity is "shielded" from all external electric fields: "Faraday Cage effect"

Gauss' Law: Example

- A spherical conducting shell has an excess charge of +10 C.
- A point charge of −15 C is located at center of the sphere.
- Use Gauss' Law to calculate the charge on inner and outer surface of sphere



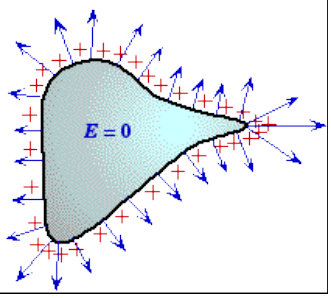
(a) Inner: +15 C; outer: 0
(b) Inner: 0; outer: +10 C
(c) Inner: +15 C; outer: -5 C

More Properties of conductors

We know the field inside the conductor is zero, and the excess charges are all on the surface. The charges produce an electric field outside the conductor.

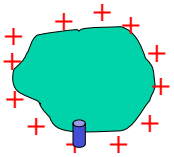
On the surface of conductors in electrostatic equilibrium, the electric field is always perpendicular to the surface.

Why?
Because if not, charges on the surface of the conductors would move with the electric field.

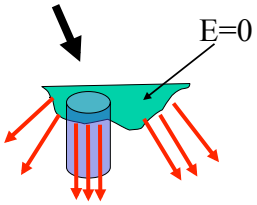


Gauss' Law: Example

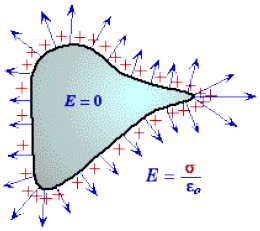
- Charged conductor of arbitrary shape: no symmetry; non-uniform charge density
- What is the electric field near the surface where the local charge density is σ?



Applying Gauss' law, we have, $\frac{A\sigma}{\varepsilon_0} = AE$



Solving for the electric field, we get $E = \frac{\sigma}{\varepsilon_0}$

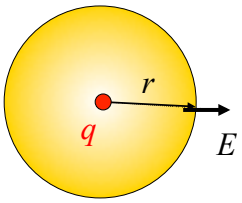


THIS IS A GENERAL RESULT FOR CONDUCTORS!

Gauss' Law: Example Spherical symmetry

- Consider a POINT charge q & pretend that you don't know Coulomb's Law
- Use Gauss' Law to compute the electric field at a distance *r* from the charge
- Use symmetry:
  - draw a spherical surface of radius R centered around the charge q
  - E has same magnitude anywhere on surface
  - E normal to surface

$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

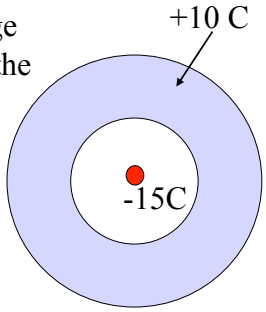


$$\Phi = |E| A = |E| 4\pi r^2$$

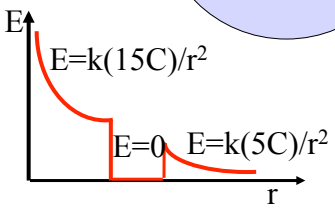
$$\Phi = \frac{q}{\varepsilon_0} \quad \Rightarrow \quad |E| = \frac{\Phi}{A} = \frac{q/\varepsilon_0}{4\pi r^2} = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{kq}{r^2}$$

Electric fields with spherical symmetry: shell theorem

The field outside a spherical distribution of charge is the same as if all the charge is at the center of the sphere.



A conducting spherical shell has a charge of +10C and a point charge of –15C at the center.
What is the electric field produced?

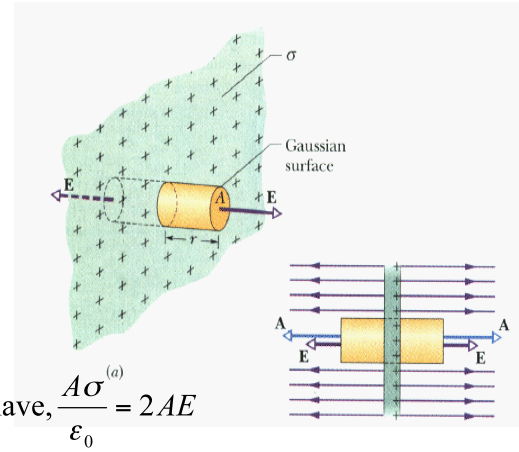


If the shell is conducting:

What is the electric field if the shell is a uniformly charged sphere?

Gauss' Law: Planar symmetry

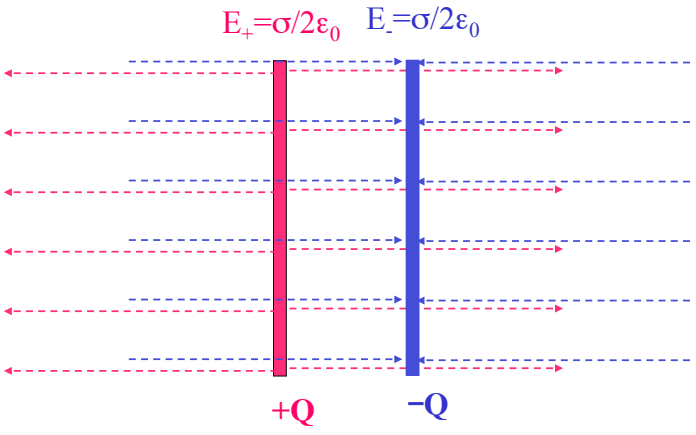
- Infinite plane with uniform charge density σ
- E is NORMAL to plane
- Construct Gaussian box as shown



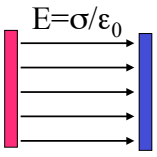
Applying Gauss' law $\frac{q}{\varepsilon_0} = \Phi$, we have, $\frac{A\sigma}{\varepsilon_0} \overset{(a)}{=} 2AE$

Solving for the electric field, we get $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

Two infinite planes

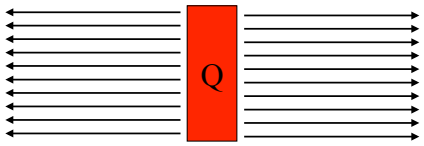


A uniform field!



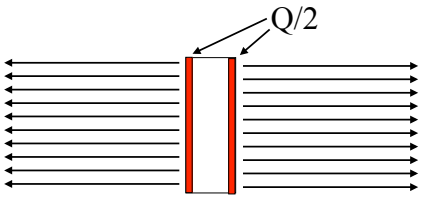
E=0 E=0

Insulating and conducting planes



$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Insulating plate: charge distributed homogeneously.

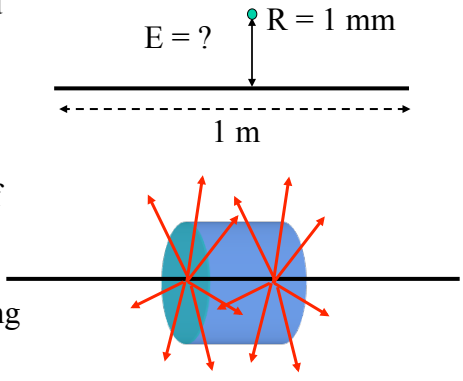


$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

Conducting plate: charge distributed on the outer surfaces.

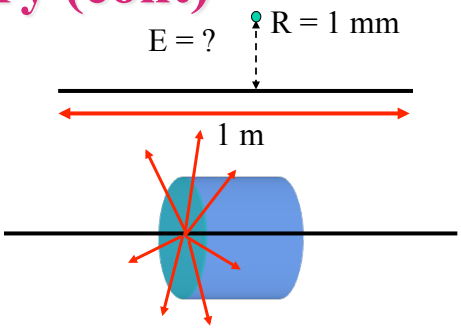
Gauss' Law: Cylindrical symmetry

- Charge of 10 C is uniformly spread over a line of length L = 1 m.
- Use Gauss' Law to compute magnitude of E at a perpendicular distance of 1 mm from the center of the line.
  - Approximate as infinitely long line -- E radiates outwards.
  - Choose cylindrical surface of radius R, length L co-axial with line of charge.



Gauss' Law: cylindrical symmetry (cont)

- Approximate as infinitely long line -- E radiates outwards.
- Choose cylindrical surface of radius R, length L co-axial with line of charge.



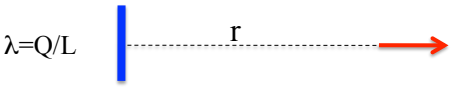
$$\Phi = |E|\,A = |E|\,2\pi RL$$

$$\Phi = \frac{q}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$|E| = \frac{\lambda L}{2\pi\varepsilon_0 RL} = \frac{\lambda}{2\pi\varepsilon_0 R} = 2k\frac{\lambda}{R}$$

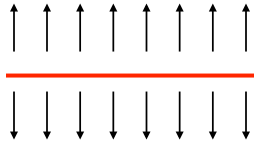
Compare with exact calculation!

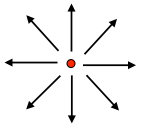
$$E_y = k\lambda\, r \int_{-L/2}^{L/2} \frac{dx}{(r^2+x^2)^{3/2}} = k\lambda\, r\left[\frac{x}{r^2\sqrt{x^2+r^2}}\right]_{-L/2}^{L/2} = \frac{k\lambda L}{r\sqrt{r^2+(L/2)^2}}$$



if the line is infinitely long (L >> r)…

$$E_y \approx \frac{k\lambda L}{r\sqrt{(L/2)^2}} = \frac{2k\lambda}{r}$$





Summary:

- Gauss' law provides a very direct way to compute the electric flux.
- In situations with symmetry, knowing the flux allows to compute the fields reasonably easily.
- Spherical field of a spherical uniform charge: kq_{ins}/r^2
- Uniform field of an insulating plate: $\sigma/2\varepsilon_{0,}$; of a conducting plate: $\sigma/\varepsilon_{0.}$.
- Cylindrical field of a long wire: $2k\lambda/r$
- Properties of conductors: field inside is zero; excess charges are always on the surface; field on the surface is perpendicular and $E=\sigma/\varepsilon_{0.}$