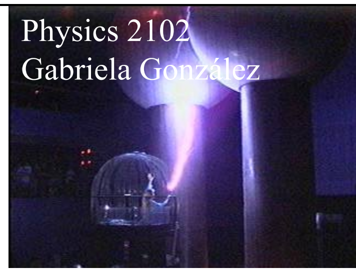
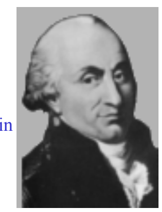


Physics 2102

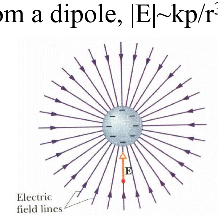
Introduction to Electricity, Magnetism and Optics

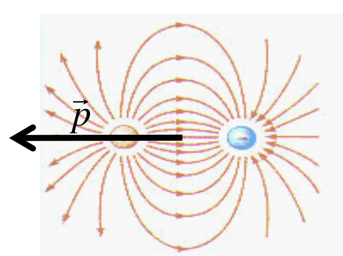
Charles-Augustin de Coulomb (1736-1806)



Summary of last lecture

- Electric field is a vector calculated the electric force on an imaginary 1C positive charge.
- Electric field lines start or end in electric charges.
- When fields are strong, electric field lines get closer.
- Electric field of a single charge is $|E|=kq/r^2$
- The "dipole moment" vector **p** has magnitude qa and direction from –ve to +ve charge.
- Far from a dipole, $|E|\sim kp/r^3$



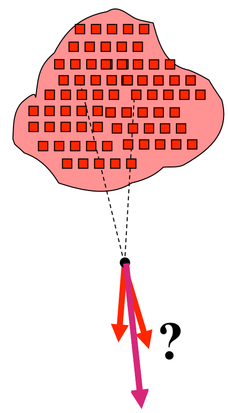


http://phet.colorado.edu/en/simulation/electric-hockey

Computing E of a continuous charge distribution

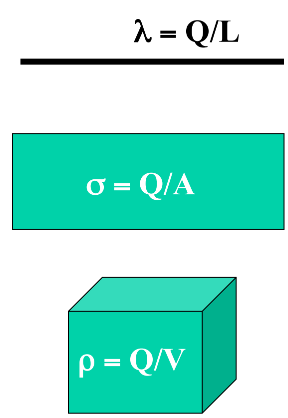
Thus far, we have only dealt with discrete, point charges. In general, a charged object has a "continuous" charge distributed on a line, a surface or a volume. How do we calculate the electric field produced by such an object?

- Approach: divide the continuous charge distribution into infinitesimally small elements
- Treat each element as a POINT charge & compute its electric field
- Sum (integrate) over all elements
- Always look for symmetry to simplify life!



Charge Density

- Useful idea: charge density
- Line of charge:
  charge per unit length $= \lambda$
- Sheet of charge:
  charge per unit area $= \sigma$
- Volume of charge:
  charge per unit volume $= \rho$



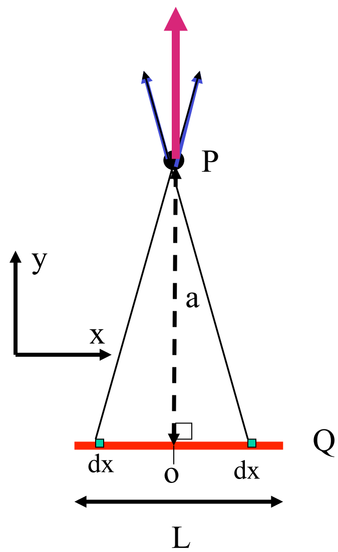
Example: Field on Bisector of Charged Rod

- Uniform line of charge +Q spread over length L
- What is the direction of the electric field at a point P on the perpendicular bisector?

(a) Field is 0.

(b) Along +y ✓

(c) Along +x



- Choose symmetrically located elements of length dx
- x components of E cancel

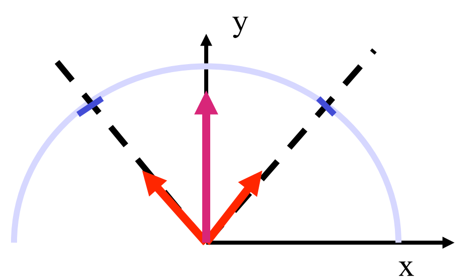
Example : Arc of Charge

- Figure shows a uniformly charged rod of charge **-Q** bent into a circular arc of radius R, centered at (0,0).
- What is the direction of the electric field at the origin?

(a) Field is 0.

(b) Along +y ✓

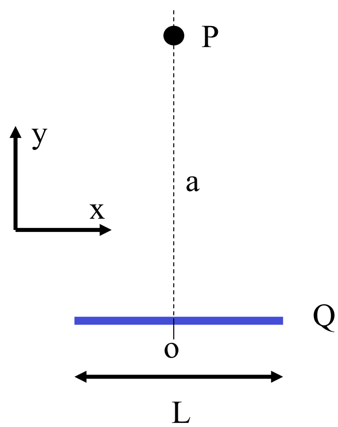
(c) Along -y



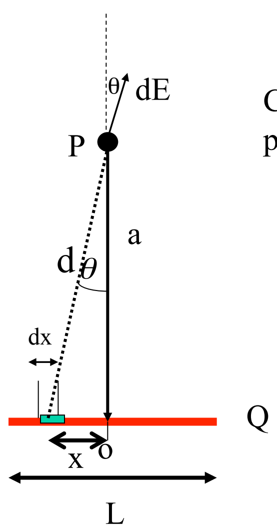
- Choose symmetric elements
- *x*-components cancel

Example --Line of Charge: Quantitative

- Uniform line of charge, length L, total charge Q
- Compute explicitly the magnitude of E at point P on perpendicular bisector
- Showed earlier that the net field at P is in the y direction -- let's now compute this!



Line Of Charge: Field on bisector



Distance $d = \sqrt{a^2 + x^2}$

Charge per unit length $\lambda = \frac{q}{L}$

$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$

$$dE = \frac{k(dq)}{d^2} = \frac{k(\lambda dx)}{a^2 + x^2}$$

$$dE_y = dE\cos\theta = \frac{k(\lambda\, dx)a}{(a^2 + x^2)^{3/2}}$$

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{(a^2 + x^2)^{3/2}}$$

Line Of Charge: Field on bisector

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{(a^2+x^2)^{3/2}} = k\lambda\, a \left[\frac{x}{a^2\sqrt{x^2+a^2}} \right]_{-L/2}^{L/2} = \frac{2k\lambda L}{a\sqrt{4a^2+L^2}}$$

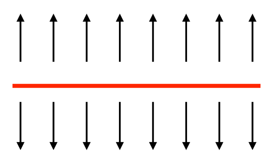
What is the field E very far away from the line (L<<a)?

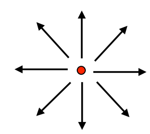
$$E_y \approx \frac{2k\lambda L}{a\sqrt{4a^2}} = \frac{2k\lambda L}{2a^2} = \frac{kQ}{a^2}$$

Far away, any "localized" charge looks like a point charge

What is field E if the line is infinitely long (L >> a)?

$$E_y = \frac{2k\lambda L}{a\sqrt{L^2}} = \frac{2k\lambda}{a}$$

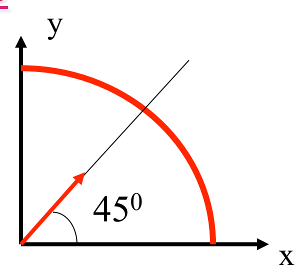




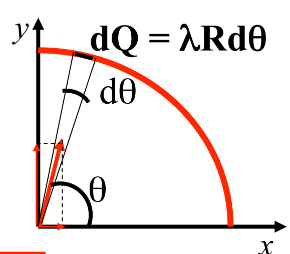
Q=λL is infinite! This is not a physical situation, it is an approximation for when we are very close to the charged line (and then it "looks" infinite!).

Arc of Charge

- Figure shows a uniformly charged rod of charge **−Q** bent into a circular arc of radius R, centered at (0,0).
- Compute the direction & magnitude of E at the origin.



$$dE_x = dE\cos\theta = \frac{kdQ}{R^2}\cos\theta$$



$$E_x = \int_0^{\pi/2} \frac{k(\lambda R d\theta)\cos\theta}{R^2} = \frac{k\lambda}{R}\int_0^{\pi/2}\cos\theta\, d\theta$$

$$\boxed{E_x = \frac{k\lambda}{R}} \quad \boxed{E_y = \frac{k\lambda}{R}} \quad \boxed{E = \sqrt{2}\,\frac{k\lambda}{R}}$$

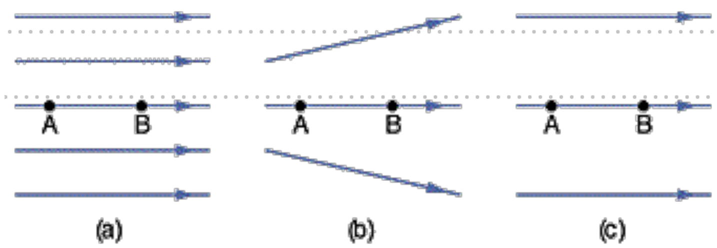
λ = 2Q/(πR)

Electric field lines and forces

The drawings below represent electric field lines.

a) Draw vectors representing the electric force on an electron at point A, and on a proton at point B.

b) If the magnitude of the force on an electron at A in (a) is 1.5μN, what is the electric field at point B in each case?

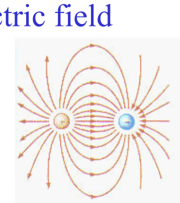


Electric charges and fields

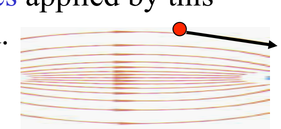
We work with two different kinds of problems, easily confused:

• **Given certain electric charges**, we calculate the electric field produced by those charges.

Example: we calculated the electric field produced by the two charges in a dipole :



• **Given an electric field**, we calculate the forces applied by this electric field on charges that come into the field.



Example: forces on a single charge when immersed in the field of a dipole:
(another example: force on a dipole when immersed in a uniform field)

Electric Dipole in a Uniform Field

- **Net force on dipole = 0; center of mass stays where it is.**
- **Net TORQUE τ: INTO page. Dipole rotates to line up in direction of E.**
- **$|\tau| = 2(QE)(a/2)(\sin\theta)$**
  $= (Qa)(E)\sin\theta$
  $= |p|\, E \sin\theta$
  $= |p \times E|$
- **The dipole tends to "align" itself with the field lines.**
- **What happens if the field is NOT UNIFORM??**

Distance between charges = a