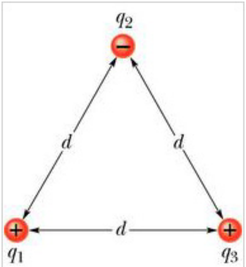
Physics 2102

Marathon review of the course:

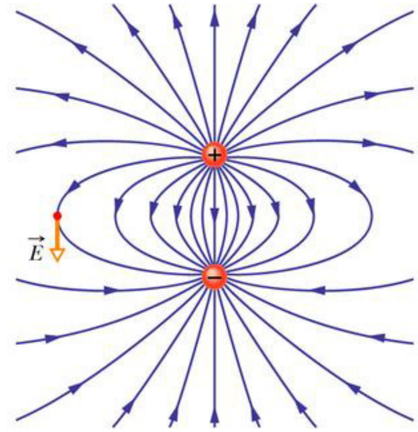
15 weeks in ~60 minutes!

Overview

- **Fields: electric & magnetic**
  - electric and magnetic forces on electric charges
  - potential energy, electric potential, work (for electric fields), electric and magnetic energy densities
  - fundamental laws on how fields are produced: **Maxwell's equations!**
- **Circuits & components:**
  - Capacitors, resistors, inductors, batteries
  - Circuits: circuits with R and batteries, RC, LR, LC.
- **Waves :**
  - Speed, frequency, wavelength, polarization, intensity
  - Wave optics: interference



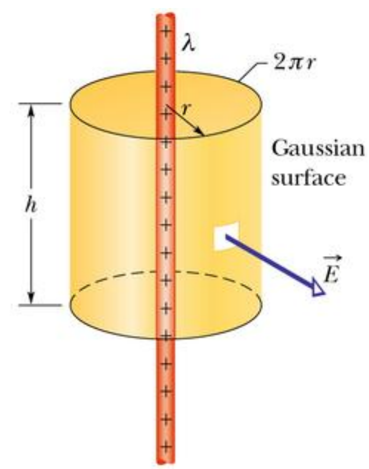
Electric Field



- **Forces** due to electric fields: $\mathbf{F}=q\mathbf{E}$, Coulomb's Law
- **Electric field** due to:
  - single point charge, several point charges
  - Electric dipoles: field produced by a dipole, electric torque on a dipole
  - charged lines: straight lines, arcs
  - surface charges: conducting and insulating planes, disks, surfaces of conductors
  - volume charges: insulating spheres, conductors (E=0 inside).

Gauss' law!

- **Electric flux**, Gauss' law,
  applied to spherical, cylindrical, plane symmetry
- **Electric potential** of a single charge, of several charges, of distributed charges.
- **Work, potential energy**



Electric fields

- Coulomb's law: $|\vec{F}| = k\frac{|q_1||q_2|}{r^2}$
- Force on a charge in an electric field: $\vec{F} = q\vec{E}$
- Electric field of a point charge: $|\vec{E}| = k\frac{|q|}{r^2}$
- Electric field of a dipole on axis, far away from dipole: $\vec{E} = \frac{2k\vec{p}}{z^3}$
- Electric field of an infinite line charge: $|\vec{E}| = \frac{2k\lambda}{r}$
- Torque on a dipole in an electric field: $\vec{\tau} = \vec{p} \times \vec{E}$, Potential energy of a dipole in $\vec{E}$ field: $U = -\vec{p} \cdot \vec{E}$
- Electric flux: $\Phi = \int \vec{E} \cdot d\vec{A}$
- Gauss' law: $\epsilon_o \oint \vec{E} \cdot d\vec{A} = q_{enc}$
- Electric field of an infinite non-conducting plane with a charge density σ: $E = \frac{\sigma}{2\epsilon_o}$
- Electric field of infinite conducting plane or close to the surface of a conductor: $E = \frac{\sigma}{\epsilon_o}$
- Electric potential, potential energy, and work:

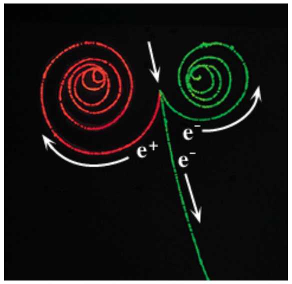
  $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$ In a uniform field: $\Delta V = -Ed\cos\theta$

  $\vec{E} = -\vec{\nabla}V$, $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

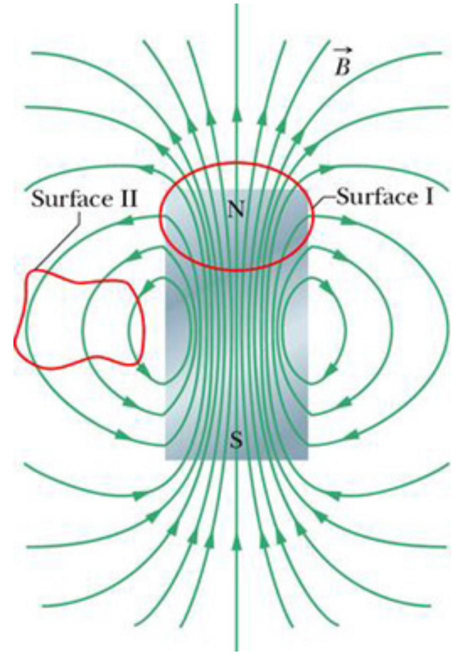
  Potential of a point charge q: $V = k\frac{q}{r}$ Potential of n point charges: $V = \sum_{i=1}^{n} V_i = k\sum_{i=1}^{n}\frac{q_i}{r_i}$

  Electric potential energy: $\Delta U = q\Delta V$ $\Delta U = -W_{\text{field}}$

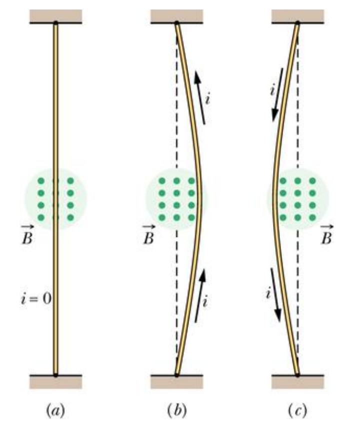
  Potential energy of two point charges: $U_{12} = W_{\text{ext}} = q_2V_1 = q_1V_2 = k\frac{q_1q_2}{r_{12}}$

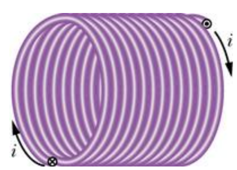


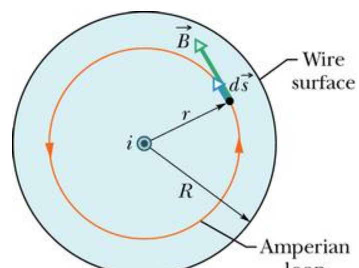
Magnetic Fields



- **Force** exerted by a magnetic field on a charge, on a current.
- Magnetic field lines: always closed!
- **Magnetic fields** created by currents:wires, loops, solenoids (Biot-Savart's law, Ampere's law)
- Magnetic dipoles: field produced by a dipole, torque on a dipole







Magnetic fields

- **Magnetic Fields:**

Magnetic force on a charge q: $\vec{F} = q\vec{v} \times \vec{B}$

Lorentz force: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Circular motion in a magnetic field: $qv_\perp B = \frac{mv_\perp^2}{r}$

with period: $T = \frac{2\pi m}{qB}$

Magnetic force on a length of wire: $\vec{F} = i\vec{L} \times \vec{B}$

Magnetic Dipole: $\vec{\mu} = Ni\vec{A}$

Torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential energy: $U = -\vec{\mu} \cdot \vec{B}$

- **Generating Magnetic Fields:** $(\mu_0 = 4\pi \times 10^{-7}\frac{\mathrm{T \cdot m}}{\mathrm{A}})$

Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s} \times \vec{r}}{r^3}$

Magnetic field of a long straight wire: $B = \frac{\mu_0}{4\pi}\frac{2i}{r}$

Magnetic field of a circular arc: $B = \frac{\mu_0}{4\pi}\frac{i}{r}\phi$

Force between parallel current carrying wires: $F_{ab} = \frac{\mu_0 i_a i_b}{2\pi d}L$

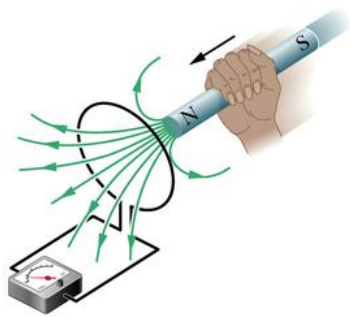
Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$

Magnetic field of a solenoid: $B = \mu_0 in$

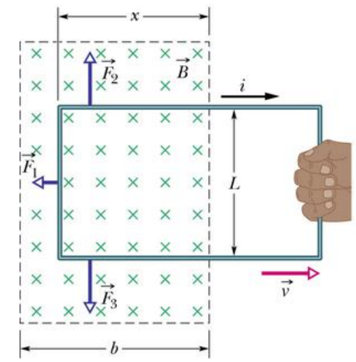
Magnetic field of a dipole on axis, far away: $\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$

Induced fields

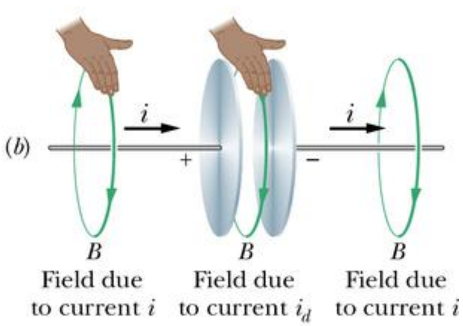
- Changing magnetic flux creates an induced electric field (and a current if there is a wire!): Faraday's law



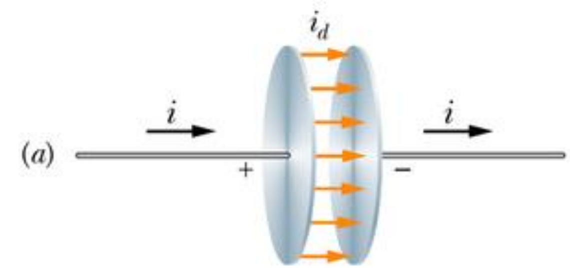
$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$



- Changing electric flux creates an induced magnetic field: Ampere-Maxwell's law, displacement current



$$\oint_C \vec{B} \bullet d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$



Induced fields

Magnetic Flux: $\Phi = \int \vec{B} \cdot d\vec{A}$

Faraday's law: $\mathcal{E} = -\frac{d\Phi}{dt}$ $\left(= -N\frac{d\Phi}{dt}\right.$ for a coil with N turns)

Induced Electric Field: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi}{dt}$

Motional emf: $\mathcal{E} = BLv$

Displacement current: $i_d = \epsilon_0 \frac{d\Phi_E}{dt}$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Maxwell's equations

$$\oint_S E \bullet dA = q_{enc} / \varepsilon_0 \qquad \oint_S B \bullet dA = 0$$

$$\oint_C B \bullet ds = \mu_0 i_{enc} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S E \bullet dA = \mu_0 i_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

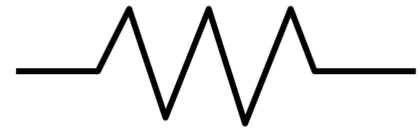
$$\oint_C E \bullet ds = -\frac{d}{dt} \int_S B \bullet dA = -\frac{d\Phi_B}{dt}$$

Plus: $$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

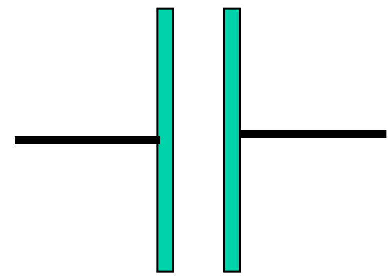
Resistors, Capacitors, Inductors

- $V=iR$, $Q=CV$, $E=-L\,di/dt$
- Resistivity: $E=J\rho$, $R=\rho L/A$
- Parallel plate, spherical, cylindrical capacitors
- Capacitors with dielectrics
- Resistors, capacitors in series and in parallel
- Power delivered by a battery: $P=iE$
- Energy dissipated by a resistor: $P=i^2R=V^2/R$
- Energy stored in capacitors, inductors (energy stored in electric, magnetic fields)
- Ideal and real batteries (internal resistance)

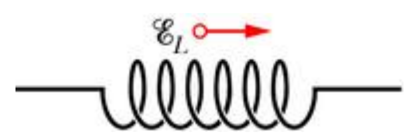
$$V=iR$$



$$Q = CV$$



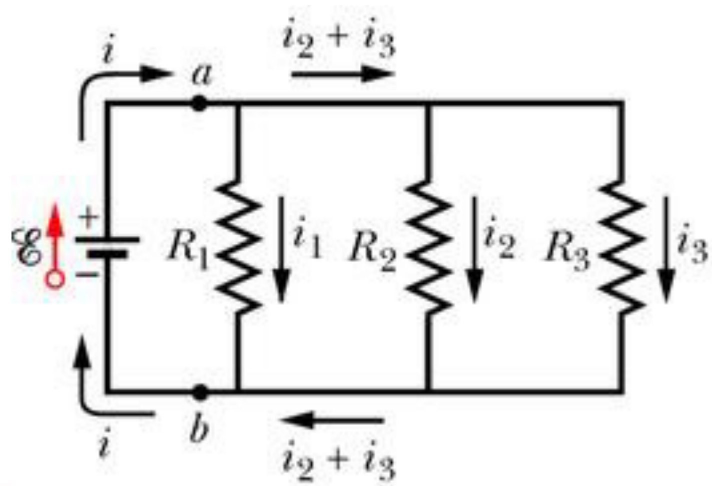
$$\mathbf{E} = -L\frac{di}{dt}$$

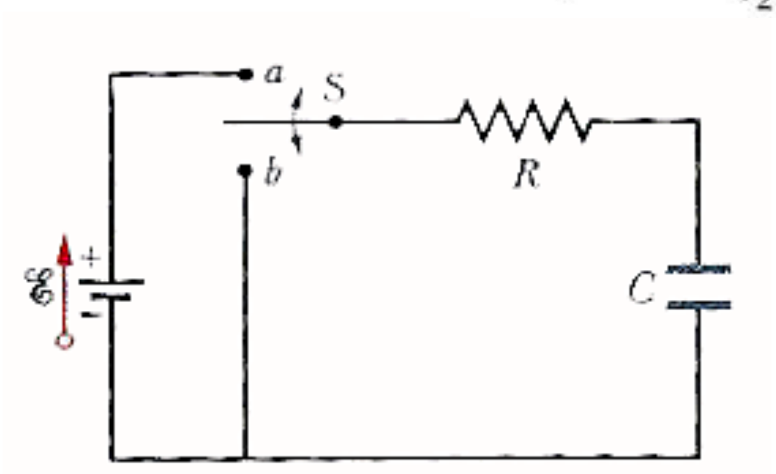


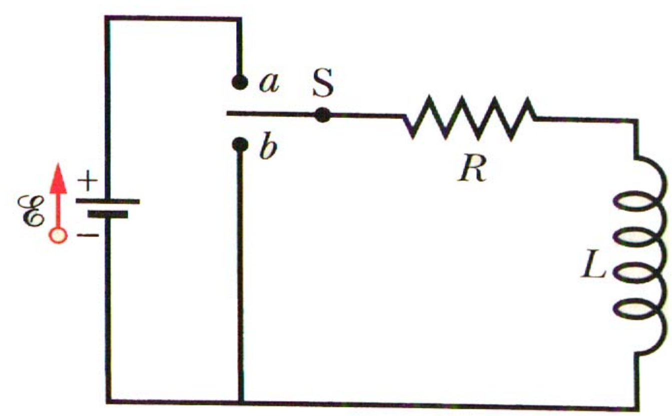
DC Circuits

- Single and multiloop circuits:
  - Junction rule for current
  - Loop rule for potential difference
- RC/RL circuits:
  - time constant: τ=RC, L/R
  - charging/discharging
  - POTENTIAL across capacitor CANNOT CHANGE SUDDENLY!
  - CURRENT in inductor CANNOT CHANGE SUDDENLY !

DC circuits





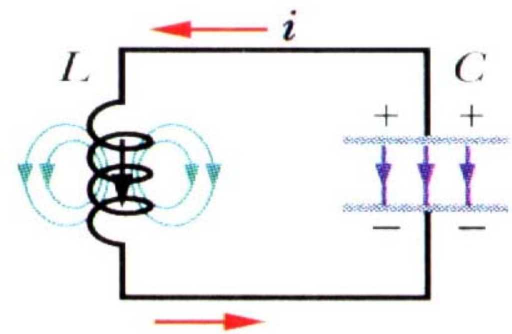


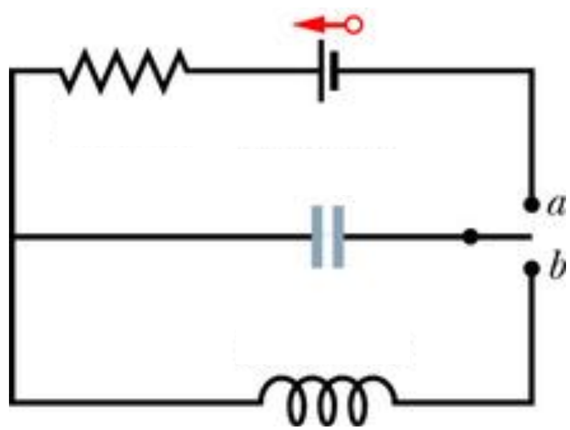
Plus: resistors and capacitors in series and in parallel

AC Circuits

- LC Oscillations:
  - careful about difference between frequency & angular frequency!
  - Physical understanding of stages in LC cycle
  - RLC circuits: energy is dissipated in the resistor.

$$\omega = \frac{1}{\sqrt{LC}}$$





Circuits, circuit elements

- **Capacitance: definition:** $q = CV$

  Capacitor with a dielectric: $C = \kappa C_{air}$ **Parallel plate:** $C = \varepsilon_0 \frac{A}{d}$

  Potential Energy in Cap: $U = \frac{q^2}{2C} = \frac{1}{2}qV = \frac{1}{2}CV^2$ **Energy density of electric field:** $u = \frac{1}{2}\kappa\varepsilon_o|\vec{E}|^2$

  Capacitors in parallel: $C_{eq} = \sum C_i$ **Capacitors in series:** $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$

- **Current:** $i = \frac{dq}{dt} = \int \vec{J} \cdot d\vec{A}$, **Constant current density:** $J = \frac{i}{A}$, **Charge carrier's drift speed:** $\vec{v}_d = \frac{\vec{J}}{ne}$

- **Definition of resistance:** $R = \frac{V}{i}$ **Definition of resistivity:** $\rho = \frac{|\vec{E}|}{|\vec{J}|}$

- **Resistance in a conducting wire:** $R = \rho \frac{L}{A}$ **Temperature dependence:** $\rho - \rho_\circ = \rho_\circ \alpha(T - T_\circ)$

- **Power in an electrical device:** $P = iV$ **Power dissipated in a resistor:** $P = i^2R = \frac{V^2}{R}$

- **Definition of** *emf* : $\mathcal{E} = \frac{dW}{dq}$

- **Resistors in series:** $R_{eq} = \sum R_i$ **Resistors in parallel:** $\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$

- **Loop rule in DC circuits:** the sum of changes in potential across any closed loop of a circuit must be zero.

- **Junction rule in DC circuits:** the sum of currents entering any junction must be equal to the sum of currents leaving that junction.

- **Charging a capacitor, series RC circuit:** $q(t) = C\mathcal{E}(1 - e^{-\frac{t}{\tau_c}})$, time constant $\tau_C = RC$

  Discharging: $q(t) = q_o e^{-\frac{t}{\tau_c}}$

  Definition of Self-Inductance: $L = \frac{N\Phi}{i}$ **Inductance of a solenoid:** $L = \mu_0 n^2 Al$

  EMF (Voltage) across an inductor: $\mathcal{E} = -L\frac{di}{dt}$

  RL Circuit: Rise of current: $i = \frac{\mathcal{E}}{R}(1 - e^{-\frac{tR}{L}})$ Time constant: $\tau_L = L/R$ **Decay** of current: $i = i_0 e^{-\frac{tR}{L}}$

  Magnetic Energy: $U_B = \frac{1}{2}Li^2$ **Magnetic energy density:** $u_B = \frac{B^2}{2\mu_0}$

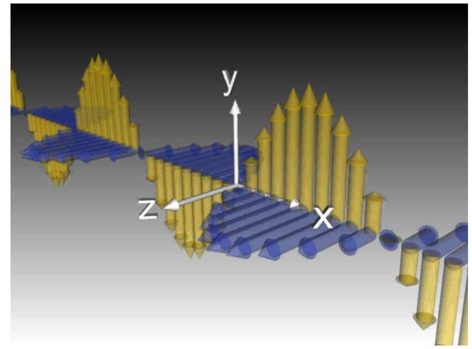
- **LC circuits:**

  Electric energy in a capacitor: $U_E = \frac{q^2}{2C} = \frac{CV^2}{2}$ **Magnetic energy in an inductor:** $U_B = \frac{Li^2}{2}$

  LC circuit oscillations: $q = Q\cos(\omega t + \phi)$ $(i = \frac{dq}{dt},\ \ q = Cv)$ $\omega = \frac{1}{\sqrt{LC}}$ $T = \frac{2\pi}{\omega}$ $f = \frac{1}{T}$

- **Series RLC circuit:** $q(t) = Qe^{-Rt/(2L)}\cos(\omega' t + \phi)$ where $\omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$

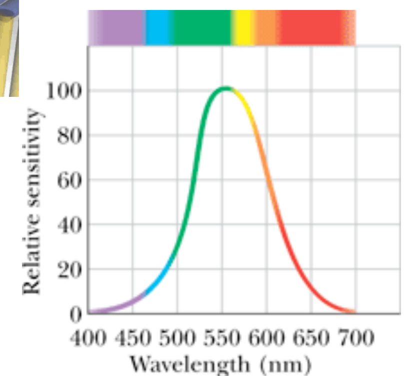
Electromagnetic waves



Wave propagating in *x* direction:

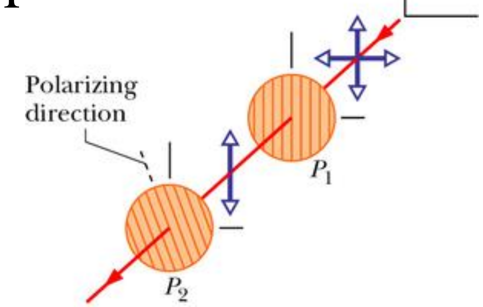
$\mathbf{E}=E_m \sin(kx-\omega t)\,\mathbf{j}$

$\mathbf{B}=B_m \sin(kx-\omega t)\,\mathbf{k}$



$c=E_m/B_m=(\mu_0\varepsilon_0)^{-1/2}$: speed of light in vacuum

$\mathbf{S}=\mathbf{E}\times\mathbf{B}/\mu_0=(E_m^2/\mu_0 c)\sin^2(kx-\omega t)\,\mathbf{i}$: Poynting vector



$I= E_m^2/2\mu_0 c$: intensity, or power per unit area

Spherical waves: $I=P_s/4\pi r^2$

Radiation pressure: $F/A=I/c$ (absorption), $F/A=2I/c$ (reflection)

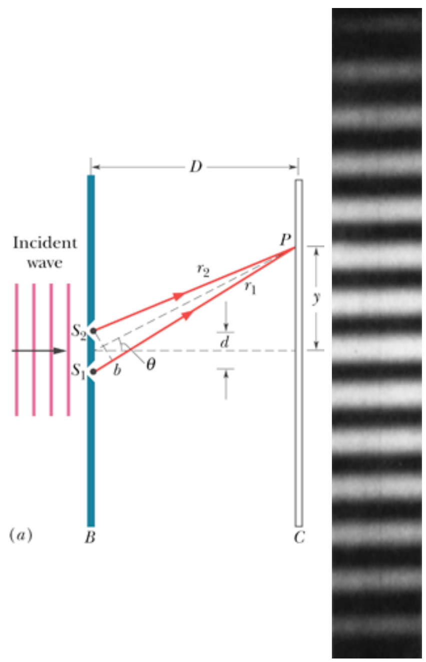
Polarizers: $I=I_0/2$ (unpolarized light), $I=I_0\cos^2\theta$ (polarized light)

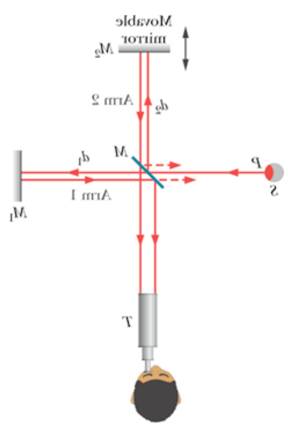
Wave Optics

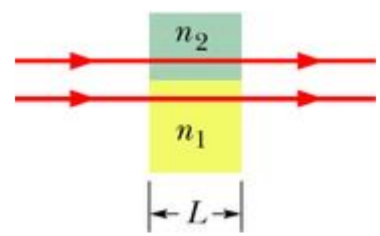
- **Refraction**: $\lambda = \lambda_0/n$, $v=c/n$, $n_2\sin\theta_2=n_1\sin\theta_1 \Rightarrow v_1\sin\theta_2=v_2\sin\theta_1$

- Two-beam **Interference** due to difference in phase : $\Delta\Phi/(2\pi)=\Delta L/\lambda$
  $\Delta L=m\lambda$ (constructive), $\Delta L=(m+1/2)\lambda$ (destructive)

- Coherent light through a **double slit** produces fringes:
  $d\sin\theta=m\lambda$ (bright), $d\sin\theta=(m+1/2)\lambda$ (dark), fringe spacing $\Delta x=L\lambda/d$







EM waves

- **Electromagnetic Waves:**
  Wave traveling in +x direction: $E = E_m \sin(kx - \omega t) \qquad B = B_m \sin(kx - \omega t)$
  where $\vec{E} \perp \vec{B}$, the direction of travel is $\vec{E} \times \vec{B}$, $E_m/B_m = c$, $f\lambda = c$, $\lambda = 2\pi/k$
  Velocity of light in vacuum $= c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}$
  Energy flow: $\vec{S} = \dfrac{1}{\mu_0}\vec{E} \times \vec{B} \qquad I = \dfrac{1}{2c\mu_0}E_m^2 \qquad E_{rms} = \dfrac{E_m}{\sqrt{2}} \qquad I = \dfrac{P}{Area}$
  Radiation force and pressure: total absorption: $F_r = \dfrac{IA}{c}$, $p_r = \dfrac{I}{c}$ total reflection: $F_r = \dfrac{2IA}{c}$, $p_r = \dfrac{2I}{c}$
- **Polarizing Sheets:**
  Unpolarized → polarized: $I = \frac{1}{2}I_0$ **Polarized → polarized:** $I = I_0 \cos^2\theta$
- **Reflection/refraction:**
  Law of reflection: $\theta_i = \theta_r$ **Law of refraction:** $n_2 \sin\theta_2 = n_1 \sin\theta_1$
  Total internal reflection (critical angle): $\theta_c = \sin^{-1}\dfrac{n_2}{n_1}$
- **Interference:**
  Constructive interference: Phase difference: $\Delta\Phi = (m)2\pi$ (path length difference $m\lambda$), $m = 0, 1, 2, \ldots$
  Destructive interference: Phase difference: $\Delta\Phi = (m + \dfrac{1}{2})2\pi$ (path length difference $(m + \dfrac{1}{2})\lambda$), $m = 0, 1, 2, \ldots$
  Index of refraction, $n : \lambda_n = \dfrac{\lambda}{n}$, $n = \dfrac{c}{v}$, v = velocity of light in a medium
  Phase difference (in wavelengths) for two different media of the same length L: $N_2 - N_1 = \dfrac{L}{\lambda}(n_2 - n_1)$
- **Two-slit interference:**
  Bright fringes: $d \sin\theta = (m)\lambda$, **Dark fringes:** $d \sin\theta = (m + \frac{1}{2})\lambda$, $m = 0, 1, 2, \ldots$
  Intensity in Two-Slit Interference: $I = 4I_0 \cos^2 \dfrac{1}{2}\phi$, $\qquad \phi = \dfrac{2\pi d}{\lambda}\sin\theta$

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  - fundamental laws on how fields are produced: **Maxwell's equations!**
- **Circuits & components:**
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  - Circuits: circuits with R and batteries, RC, LR, LC.
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  - Speed, frequency, wavelength, polarization, intensity
  - Wave optics: interference

Summary

- Think about and understand basic **concepts**!
- Look at your past exams and quizzes: why didn't you get 100%? Predict your problems in the final exam!
- Study the equation sheet, invent a problem for each formula.
- Read all lecture slides, review hwk problems and problems in class.
- Practice with a couple of past exams: timing is important!
- Save time to eat lunch and relax the hour before the exam.
- Enjoy the summer!