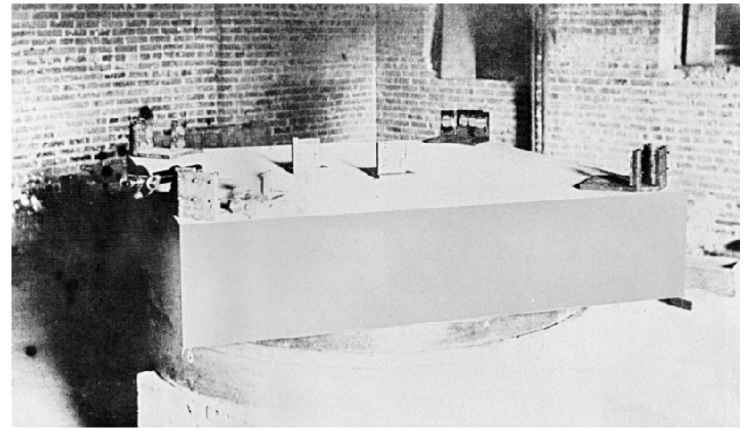
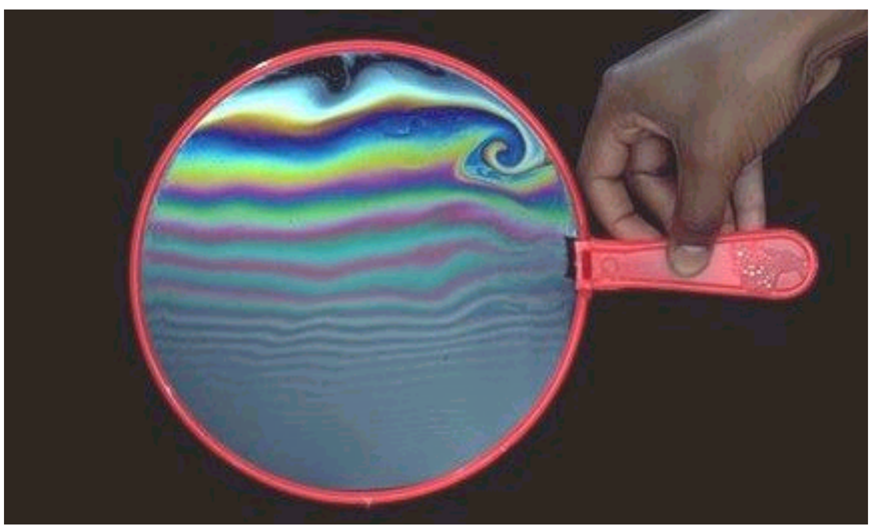
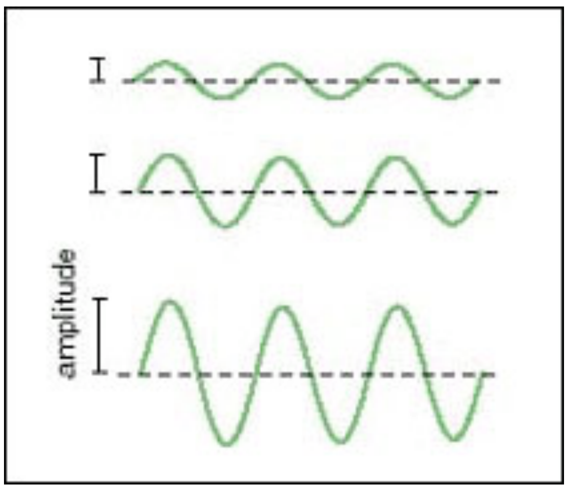


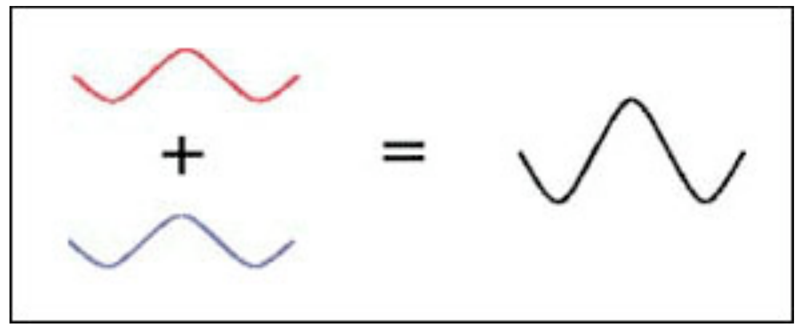
Interference

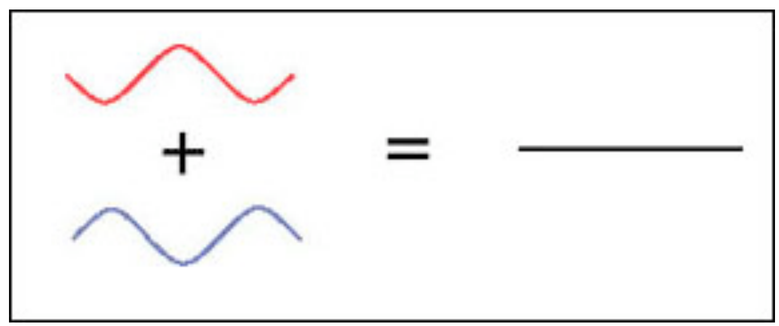


Light is a wave

When two beams of light combine, we can have constructive or destructive "interference."

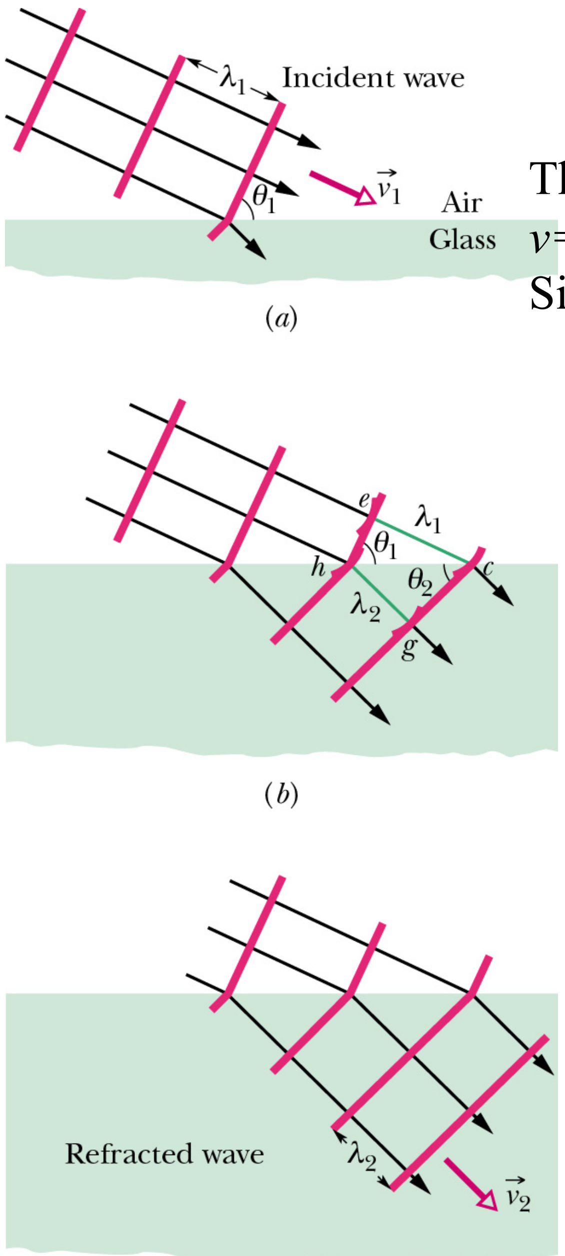






http://www.colorado.edu/physics/2000/applets/fourier.html

Reflection and refraction laws



(a) (b) (c)

The light travels more slowly in more dense media:
$v=c/n$ (n = index of refraction)
Since the period T is the same, the wavelength has to change ($v=\lambda/T$)

$$\sin\theta_1 = \frac{\lambda_1}{hc}, \quad \sin\theta_2 = \frac{\lambda_2}{hc} \quad \Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\boxed{\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}}$$

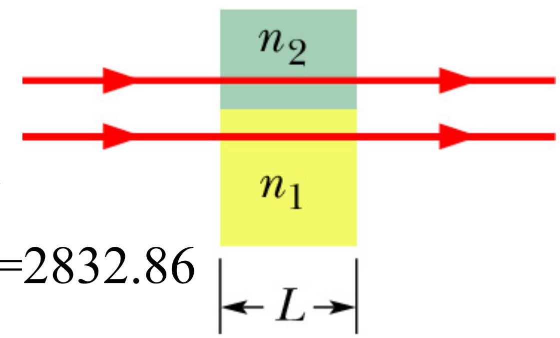
Snell's law!

Wavelength: $$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} \qquad \lambda_n = \lambda\frac{v_n}{c} = \frac{\lambda}{n}$$

Frequency: $$f_n = \frac{v_n}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f$$

Interference: example

A red light beam with wavelength λ=0.625μm travels through glass (n=1.46) a distance of 1mm. A second beam, parallel to the first one and originally in phase with it, travels the same distance through sapphire (n=1.77).



•How many wavelengths are there of each beam inside the material?

In glass, λ_g=0.625μm/1.46= 0.428 μm and $N_g=L/\lambda_g$=2336.45

In sapphire, λ_s=0.625μm/1.77= 0.353 μm (UV!) and $N_s=L/\lambda_s$=2832.86

•What is the phase difference in the beams when they come out?

The difference in wavelengths is N_s-N_g=496.41.

Each wavelength is 360^o, so ΔN=496.41 means $\Delta\phi=\Delta N\text{x}360^o=0.41\text{x}360^o=148^o$

•How thick should the glass be so that the beams are exactly out of phase at the exit? (destructive interference!)

$\Delta N=L/\lambda_s - L/\lambda_g = (L/\lambda)(n_2-n_1)=0.31\ (L/\lambda)=m+1/2$

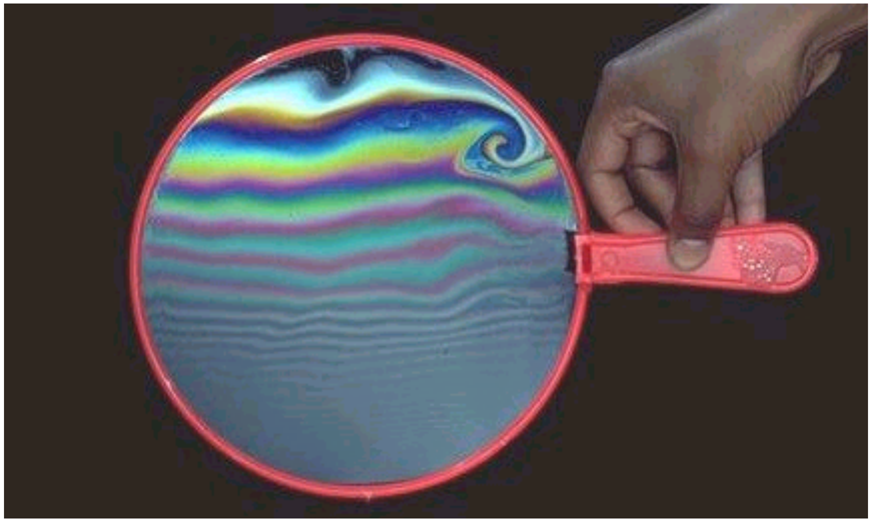
A thickness L=(m+0.5) 2.02 μm would make the waves OUT of phase.

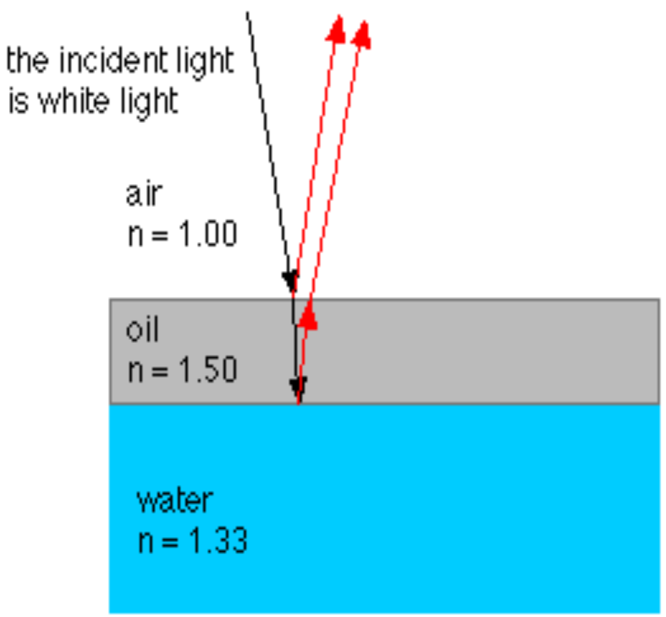
For example, 1.009 mm = 499.5 x 2.02 μm makes them come OUT of phase,

and 1.010 mm = 500.0 x 2.02 μm makes them IN phase.

Thin film interference:

The patterns of colors that one sees in oil slicks on water or in bubbles is produced by interference of the light bouncing off the two sides of the film.



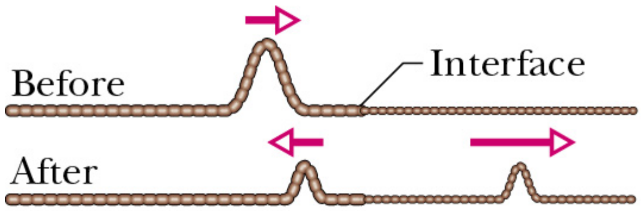


Light reflecting from the bottom surface of the oil film has no phase change

To understand this we need to discuss the phase changes that occur when a wave moves from one medium to the another where the speed is different. This can be understood with a mechanical analogy.

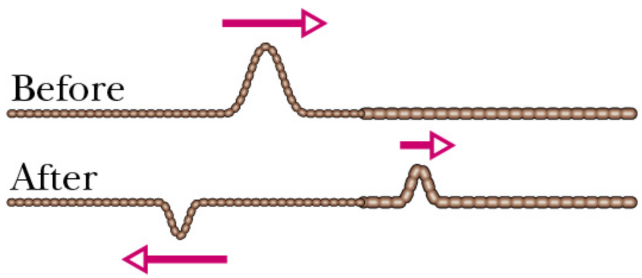
Reflection, refraction and changes of phase:

Consider a transverse pulse moving in a rope, that reaches a juncture with another rope of different density. A reflected pulse is generated.



(a)

The reflected pulse is on the same side of the rope as the incident one if the speed of propagation in the rope of the right is **faster** than on the left.



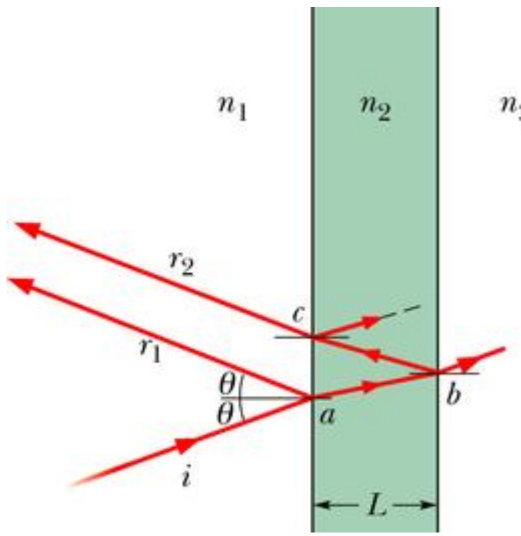
(b)

The reflected pulse is on the opposite side of the rope if the speed of propagation in the right is **slower** than on the left.

The extreme case of no speed on the right corresponds to a rope anchored to a wall.

If we have a wave instead of a pulse "being on the opposite side of the rope" means 180 degrees out of phase, or one-half wavelength out of phase.

Interference: thin films



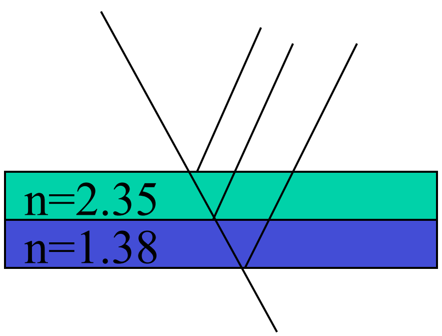
Phase with respect to incident beam:

r_1: $\phi_1 = 180^o$ (if $n_2>n_1$)
$\phi_1 = 0^o$ (if $n_2<n_1$)
r_2: $\phi_2 = 2n_2L/\lambda + 180^o$ (if $n_3>n_2$)
$\phi_1 = 2n_2L/\lambda$ (if $n_3<n_2$)

if we have air-oil-water (or air),
Constructive interference $\rightarrow 2n_2L/\lambda = (2m+1)\pi$
Destructive interference $\rightarrow 2n_2L/\lambda = 2m\,\pi$

Example: mirror coatings

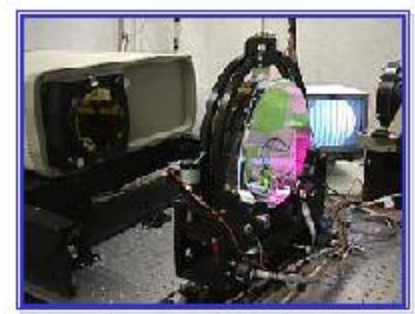
To make mirrors that reflects light of only a given wavelength, a coating of a specific thickness is used so that there is constructive interference of the given wavelength. Materials of different index of refraction are used, most commonly $MgFe_2$ (n=1.38) and CeO_2 (n=2.35), and are called "dielectric films". What thickness is necessary for reflecting IR light with λ=1064nm?



First ray: $\Delta\phi$=180deg=π

Second ray: $\Delta\phi=2L(2\pi/(\lambda/n))=\pi$
=> $L= \lambda_{Ceo2}/4=(\lambda/n)/4=113$nm

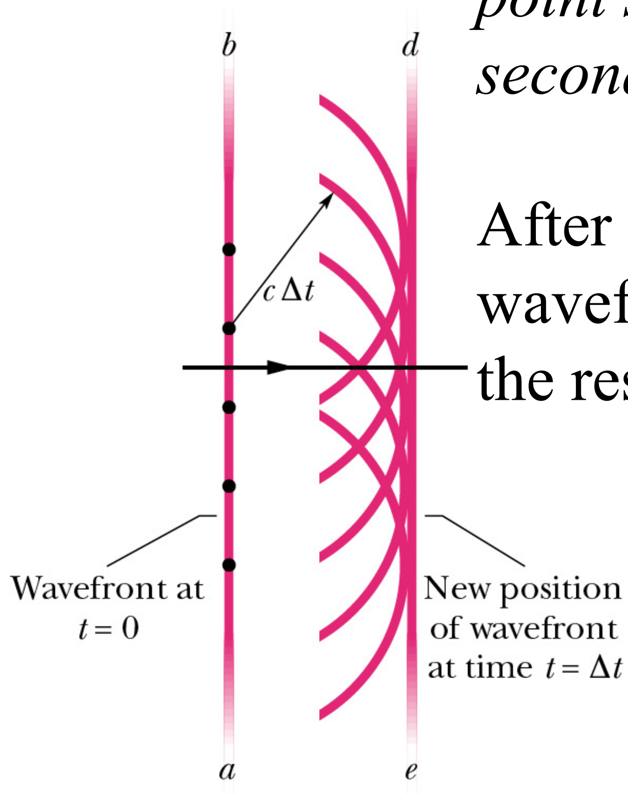
Third ray? If wafer has the same thickness (and is of the same material), $\Delta\phi=4L(2\pi/(\lambda/n)=2\pi$: destructive!



Choose MgFe2 wafer so that
$\Delta\phi=(2n_1L_1+2n_2L_2)\ (2\pi/\lambda)= \pi+ 2n_2L_2\ (2\pi/\lambda)=3\pi$
=> $L_2= \lambda/2n_2$ = 386 nm

We can add more layers to keep reflecting the light, until no light is transmitted: all the light is either absorbed or reflected.

Huygen’s principle

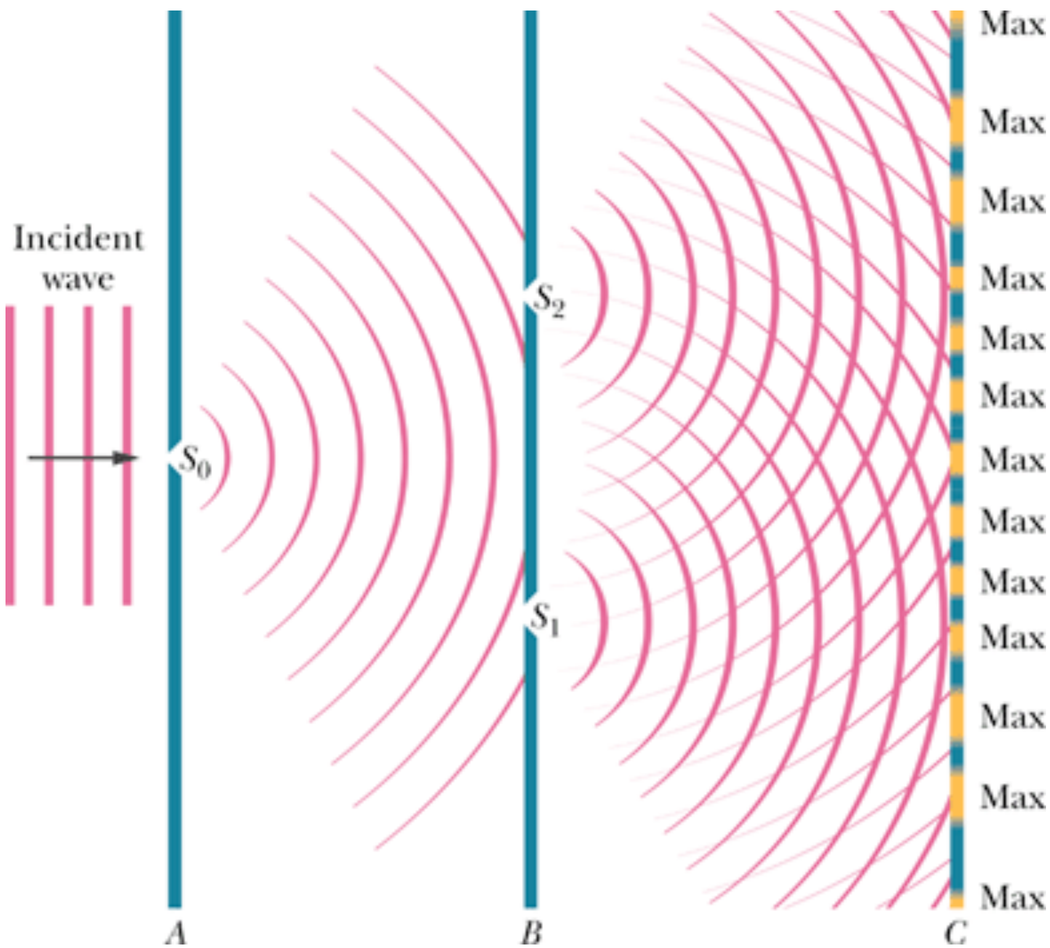


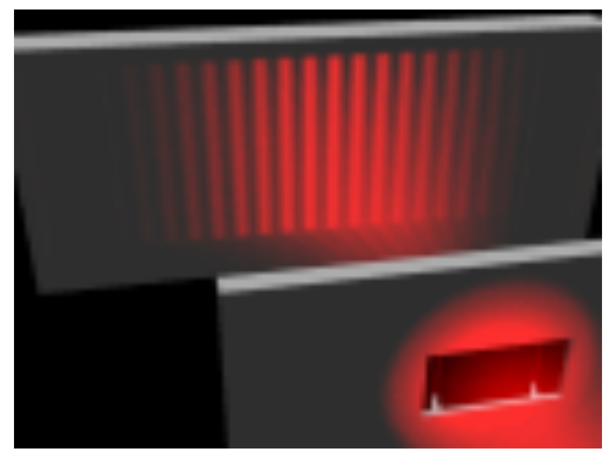
All points in a wavefront serve as point sources of spherical secondary waves.

After a time t, the new wavefront will be the tangent to all the resulting spherical waves.

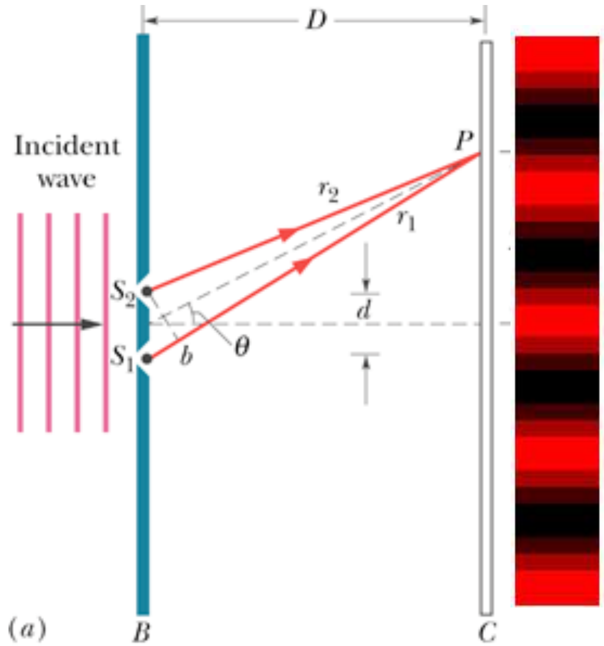
Christian Huygens
1629-1695

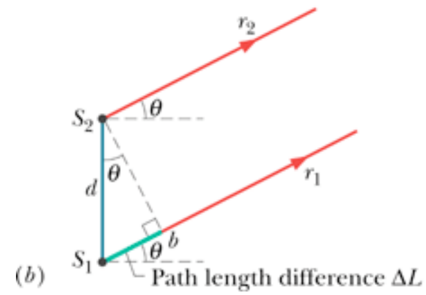
Young's double slit experiment





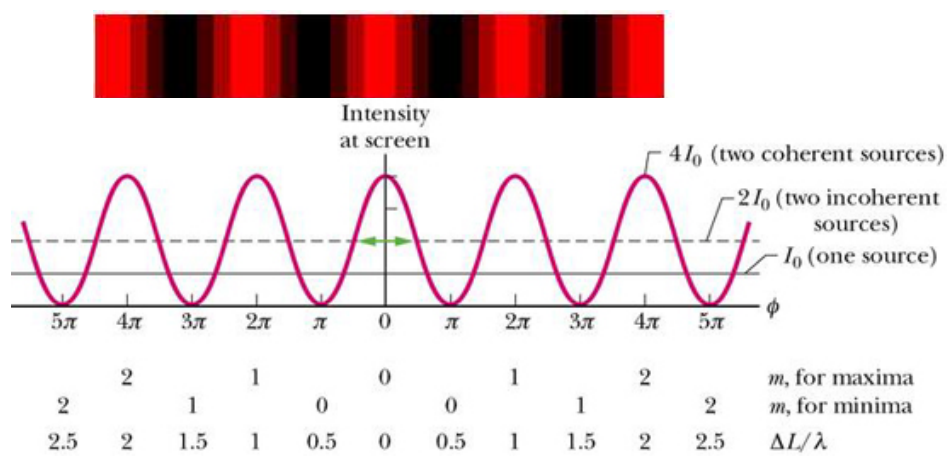
Young's double slit experiment





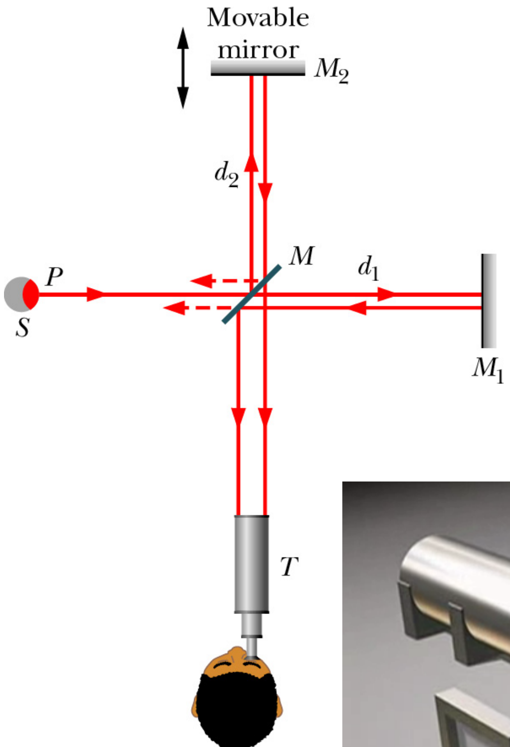
Path difference: $\Delta L = d \sin \theta$
Bright fringe: $\Delta L = m\lambda = d \sin \theta$
Dark fringe: $\Delta L = (m+1/2)\lambda = d \sin \theta$
The intensity on the screen is

$$I/I_0 = 4\cos^2\phi/2 \text{ with } \phi = (2\pi d/\lambda)\sin\theta$$

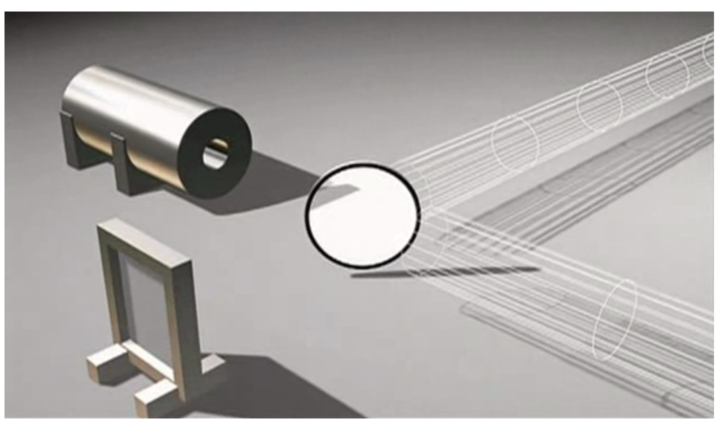


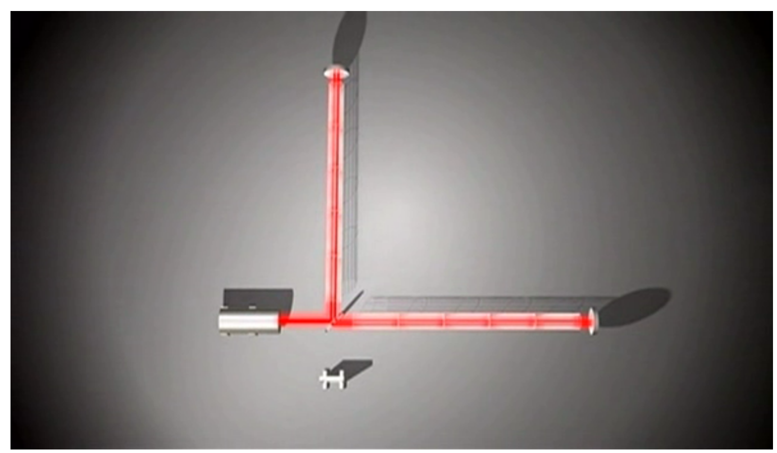
Michelson interferometers:

As we saw in the previous example, interference is a spectacular way of measuring small distances (like the thickness of a soap bubble), since we are able to resolve distances of the order of the wavelength of the light (for instance, for yellow light, we are talking about 0.5 of a millionth of a meter, 500nm). This has therefore technological applications.



In the Michelson interferometer, light from a source (at the left, in the picture) hits a semi-plated mirror. Half of it goes through to the right and half goes upwards. The two halves are bounced back towards the half plated mirror, interfere, and the interference can be seen by the observer at the bottom. The observer will see light if the two distances travelled d_1 and d_2 are equal, and will see darkness if they differ by half a wavelength.

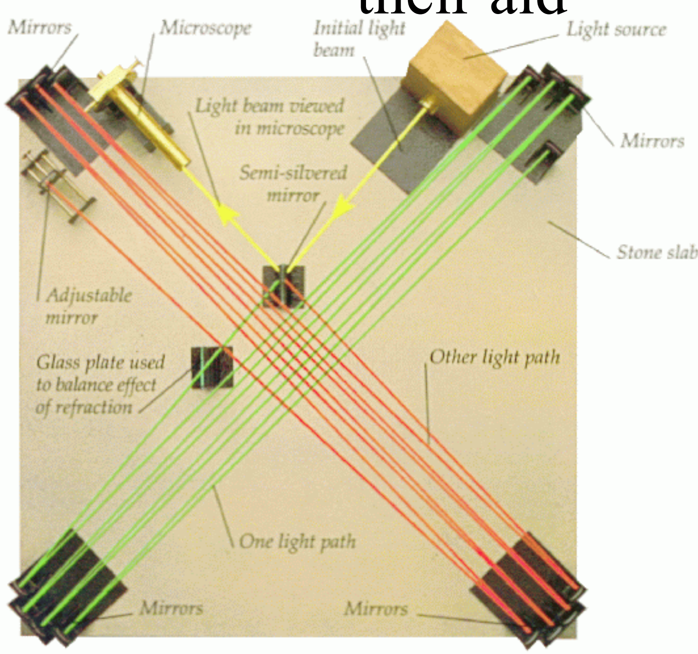


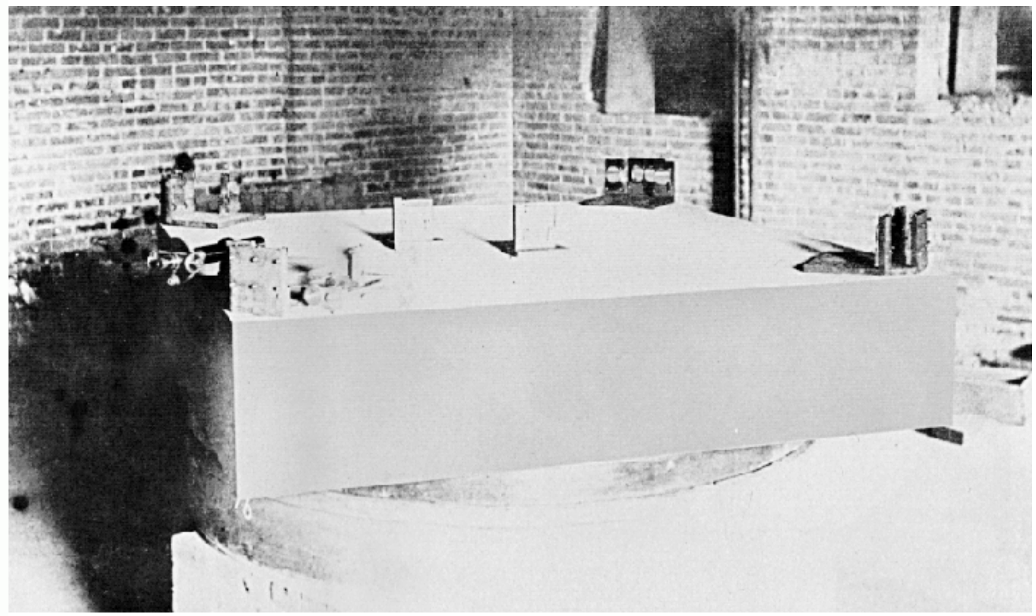


Einstein's messengers (einsteinsmessengers.org)

Michelson-Morley experiment

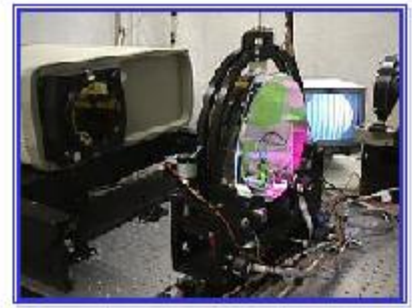
Michelson won the Nobel prize in 1907, "for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid"





"The interpretation of these results is that there is no displacement of the interference bands. ... The result of the hypothesis of a stationary ether is thus shown to be incorrect." (A. A. Michelson, Am. J. Sci, 122, 120 (1881))

The largest Michelson interferometer in the world is i
in LSU owned land (it is operated by a project funded
Science Foundation run by Caltech and MIT, and LSU
the project).

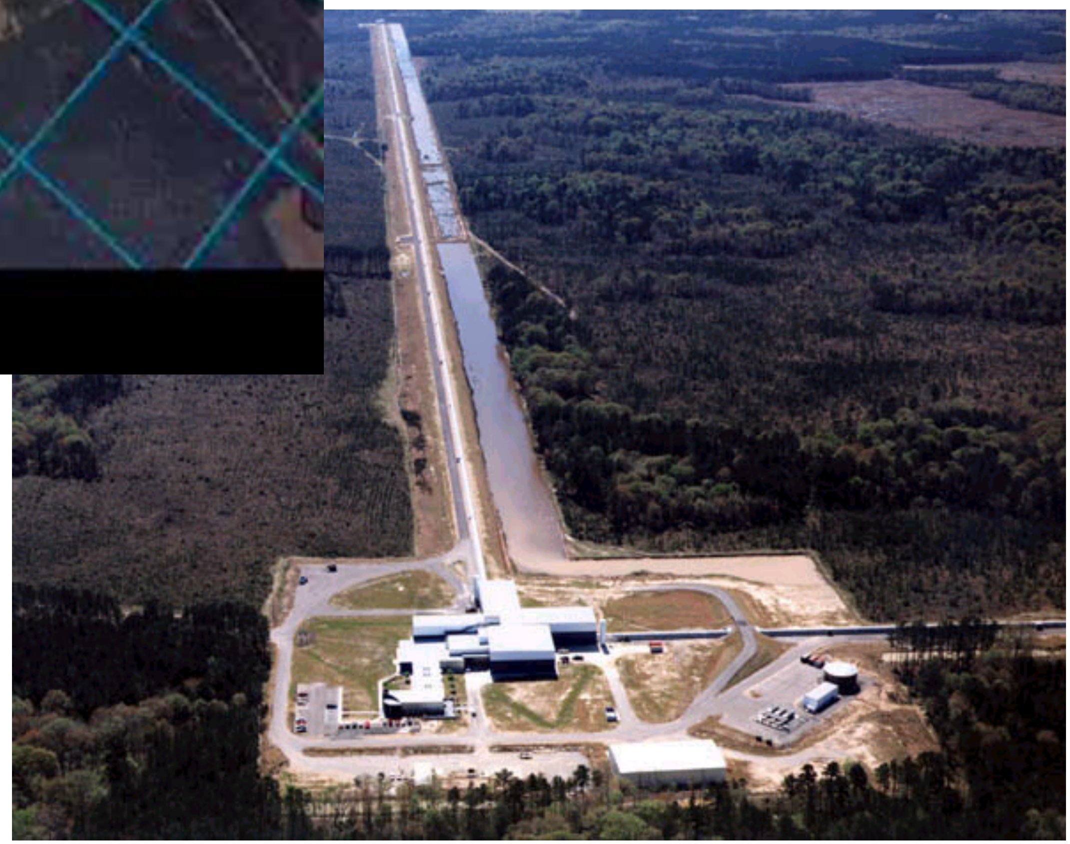


Mirrors are suspended
with wires and will move
detecting ripples in
the gravitational field due
to astronomical events.

http://www.ligo-la.caltech.edu

http://www.amnh.org/sciencebulletins/?sid=a.f.gravity.20041101&src=l

American Museum of
Natural History
Science Bulletins
Gravity: Making Waves



Examples

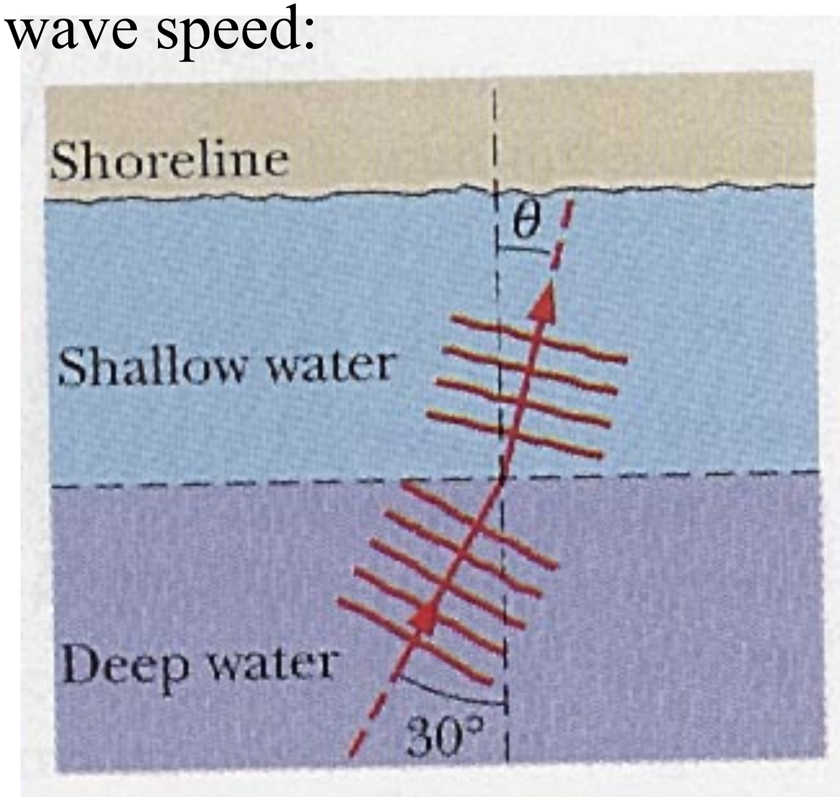
Example

Ocean waves moving at a speed of 4.0m/s are approaching a beach at an angle of 30o to the normal. The water depth changes abruptly near the shore, and the wave speed there drops to 3.0m/s. Close to the beach, what is the direction of wave motion?

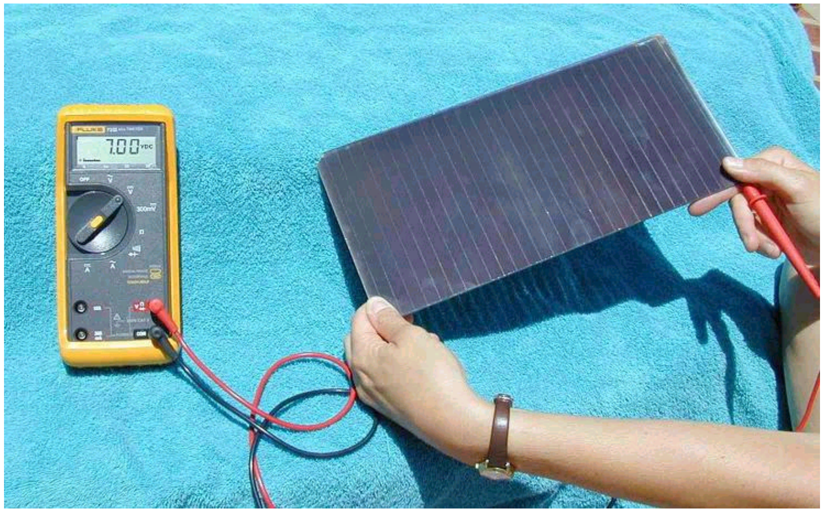
Refraction index is (inversely) related to wave speed:
$n_2/n_1=v_1/v_2=4/3$

Snell's law: $n_2\sin\theta_2=n_1\sin\theta_1$

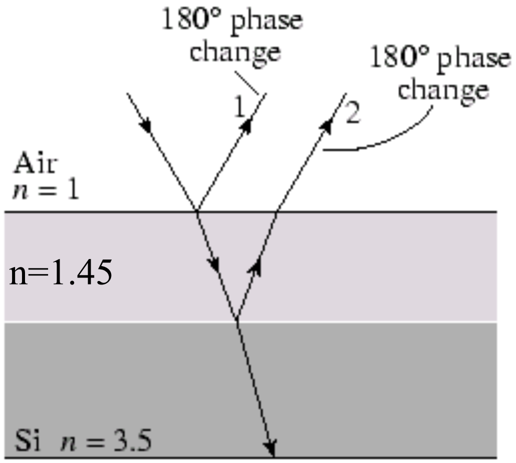
$\theta_2=\mathrm{asin}((n_1/n_2)\sin\theta_1)$
$=\mathrm{asin}(.75\sin(30^o))$
$=22^o$



Example: Solar panels



Semiconductors such a silicon are used to build solar cells. They are coated with a transparent thin film, whose index of refraction is 1.45, in order to minimize reflected light. If the index of refraction of silicon is 3.5, what is the minimum width of the coating that will produce the least reflection at a wavelength of 552nm?



Both rays undergo 180 phase changes at reflection, therefore for destructive interference (no reflection), the distance travelled (twice the thickness) should be equal to half a wavelength in the coating

$$2t = \frac{\lambda}{2n} \Rightarrow 95.1nm$$

Example: the stealth planes

Radar waves have a wavelength of 3cm. Suppose the plane is made of metal (speed of propagation=0, n is infinite and reflection on the polymer-metal surface therefore has a 180 degree phase change). The polymer has n=1.5. Same calculation as in previous example gives,

$$t = \frac{\lambda}{4n} = \frac{3cm}{4 \times 1.5} \Rightarrow 0.5cm$$

On the other hand, if one coated a plane with the same polymer (for instance to prevent rust) and for safety reasons wanted to maximize radar visibility (safety!), one would have

$$t = \frac{\lambda}{2n} = \frac{3cm}{2 \times 1.5} \Rightarrow 1cm$$