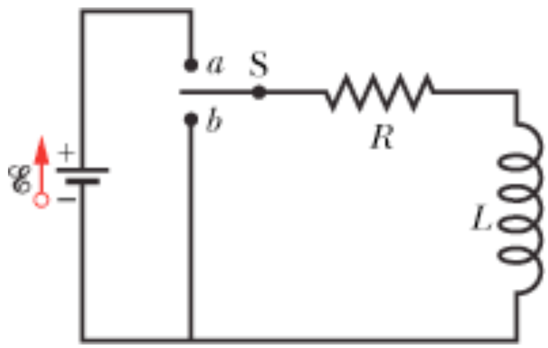
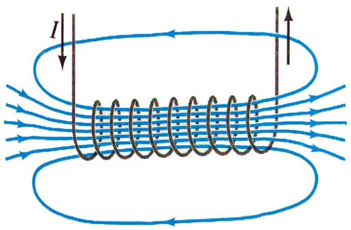
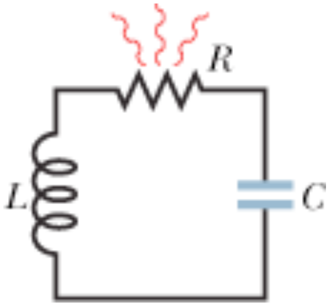
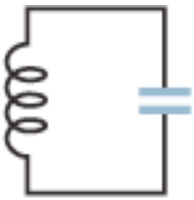


Physics 2102





Inductors,
RL circuits,
LC circuits



(a)

What are we going to learn?
A road map

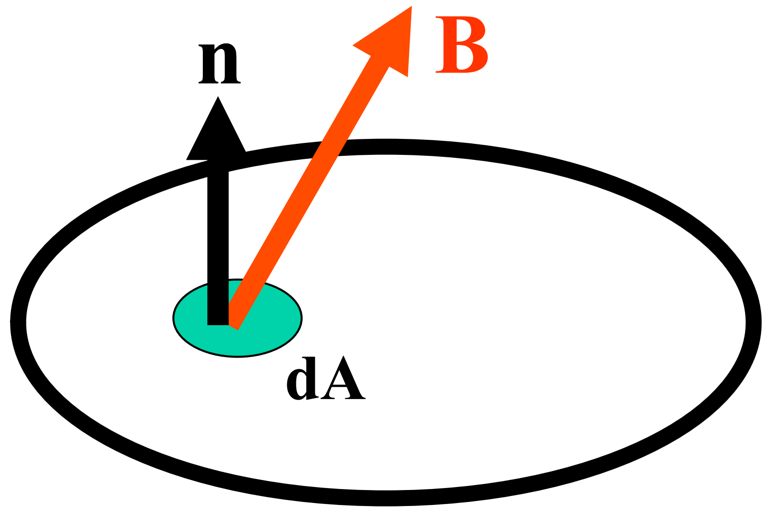
- Electric *charge*
  - Electric *force* on other electric charges
  - Electric *field*, and electric *potential*
- Moving electric charges : current
- Electronic circuit components: batteries, resistors, capacitors
- Electric currents
  - Magnetic field
  - Magnetic force on moving charges
- Time-varying magnetic field
  - Electric Field
- More circuit components: inductors
- All together: Maxwell's equations
- Electromagnetic waves
- Matter waves

Faraday's Law

Magnetic Flux:

$$\Phi_B = \int_S \vec{B} \cdot \hat{n} dA$$

A <u>time varying</u> magnetic flux creates an electric field, which induces an EMF (and a current if the edge of the surface is a conductor)



$$EMF = \oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

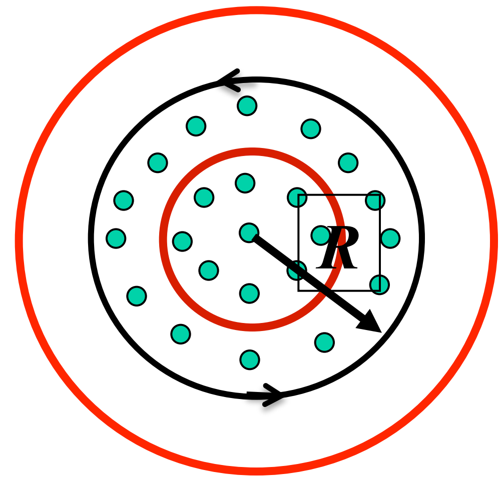
"Lenz's Law"

Notice that the electric field has closed field lines, and is *not* pointing towards "lower" electric potential – this is only true for fields produced by electric charges.

$$EMF = \oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Example

- Q: A long solenoid has a circular cross-section of radius ***R***, carrying a current ***i*** clockwise. What's the direction and magnitude of the magnetic field produced inside the solenoid?
  Ans: $B = \mu_0 in$, out of the page.
- The current through the solenoid is increasing at a steady rate ***di/dt***. What will be the direction of the electric field lines produced?
- Compute the variation of the electric field as a function of the distance ***r*** from the axis of the solenoid.
  Ans: from symmetry, we know the magnitude of E depends only on r.



First, let's look at ***r < R:***

$$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

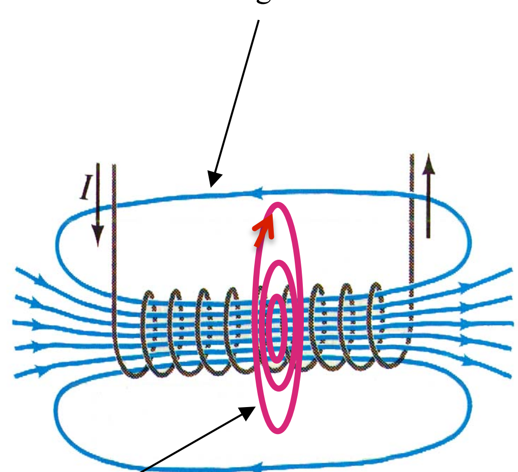
$$|E|(2\pi r) = \left|\frac{d}{dt}\left(B\pi r^2\right)\right| = \pi r^2 \frac{d}{dt}\left(\mu_0 in\right)$$

$$E(r) = \frac{\mu_0 n}{2}\frac{di}{dt} r$$

Next, let's look at ***r > R:***

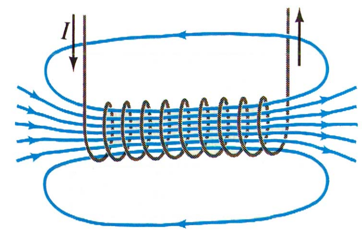
$$|E|(2\pi r) = (\pi R^2)\frac{dB}{dt}$$

$$E(r) = \frac{\mu_0 n}{2}\frac{di}{dt}\frac{R^2}{r}$$



Inductors: solenoids

Inductors are with respect to the magnetic field what capacitors are with respect to the electric field. They “pack a lot of field in a small region”. Also, the higher the current, the higher the magnetic field they produce.



Capacitance → how much **potential** for a given charge: **Q=CV**

Inductance → how much **magnetic flux** for a given current: **Φ=L*i***

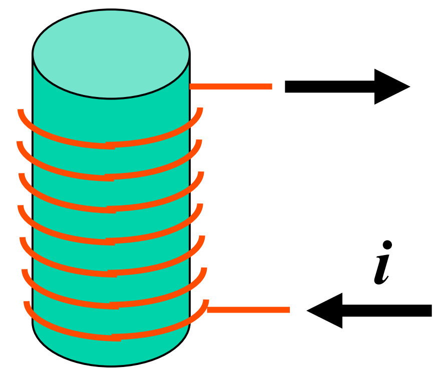
Using Faraday’s law: $$EMF = -L\frac{di}{dt}$$

$$\text{Units}: \quad [L] = \frac{\text{Tesla}\cdot\text{m}^2}{\text{Ampere}} \equiv \text{H} \quad \text{(Henry)}$$

Joseph Henry
(1799-1878)

Self-inductance of a solenoid

- Solenoid of cross-sectional area ***A***, length ***l***, total number of turns ***N***, turns per unit length ***n***
- Field inside solenoid = $\mu_0 n i$
- Field outside ~ 0

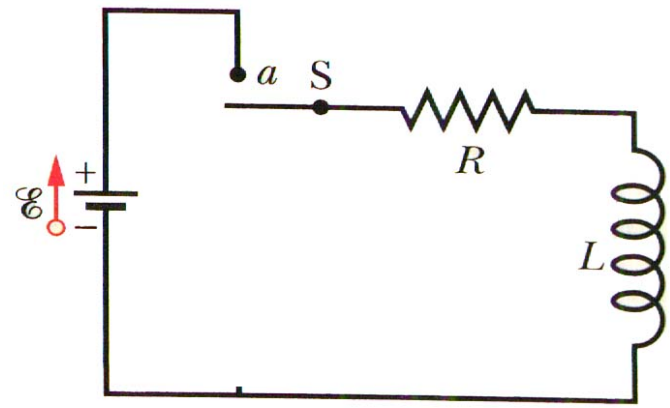


$$\Phi_B = NAB = NA\mu_0 ni = Li$$

$$L = \text{"inductance"} = \mu_0 NAn = \mu_0 \frac{N^2}{l} A$$

The RL circuit

- Set up a single loop series circuit with a battery, a resistor, a solenoid and a switch.
- Describe what happens when the switch is closed.
- Key processes to understand:
  – What happens JUST AFTER the switch is closed?
  – What happens a LONG TIME after switch has been closed?
  – What happens in between?



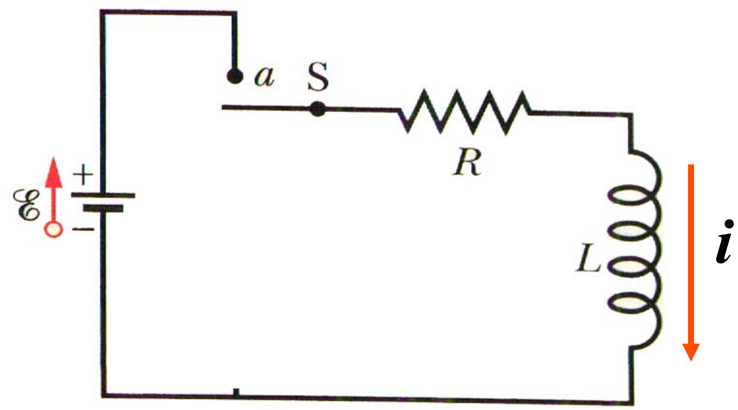
<u>Key insights:</u>
- If a circuit is not broken, one cannot change the CURRENT in an inductor instantaneously!
- If you wait long enough, the current in an RL circuit stops changing!

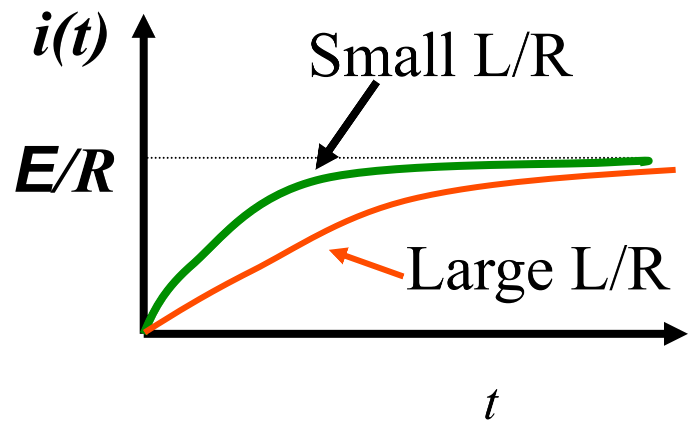
"Charging" an inductor

Loop rule:

$$-iR + \mathsf{E} - L\frac{di}{dt} = 0$$

$$i = \frac{\mathsf{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

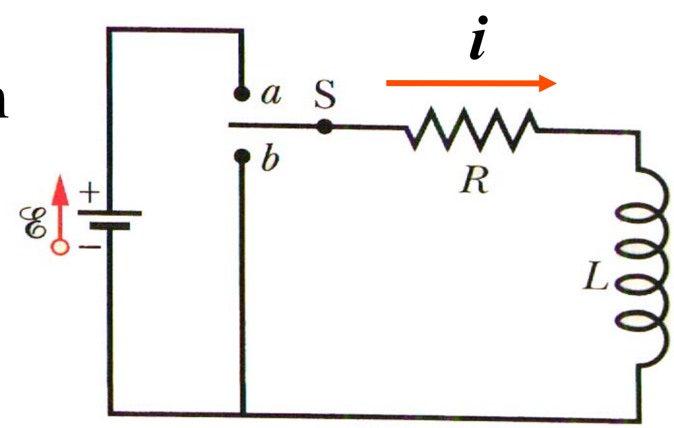




"Time constant" of RL circuit = L/R

“Discharging” an inductor

The switch is in a for a long time, until the inductor is charged. Then, the switch is closed to b.

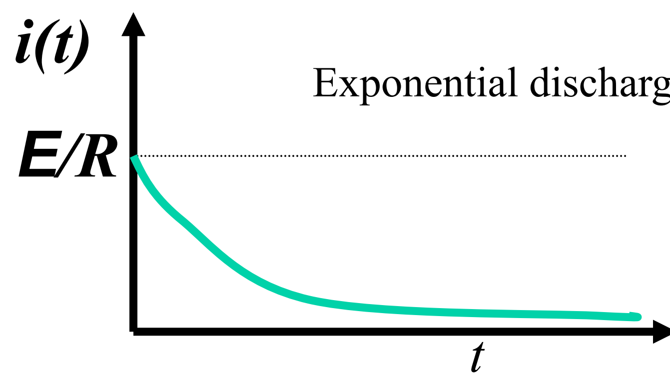


What is the current in the circuit?

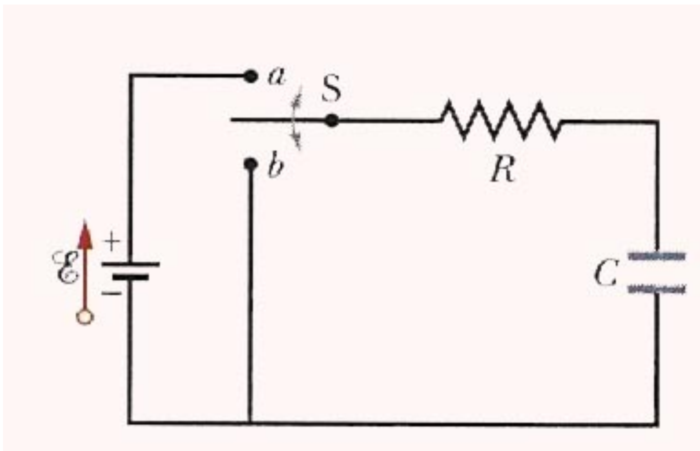
Loop rule around the new circuit:

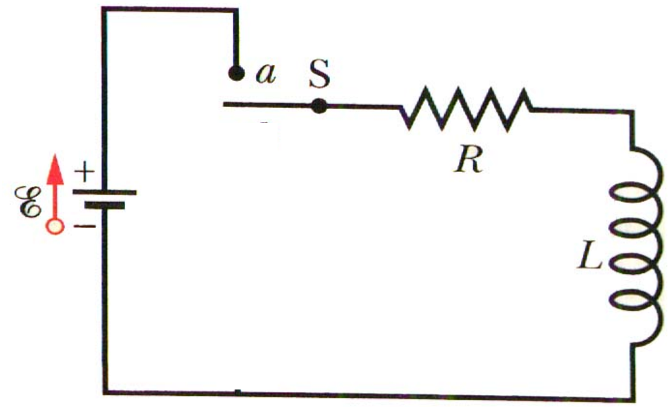
$$iR + L\frac{di}{dt} = 0$$

$$i = \frac{\mathbf{E}}{R}e^{-\frac{Rt}{L}}$$



RL circuits





In an RC circuit, while charging, Q = CV and the loop rule mean:

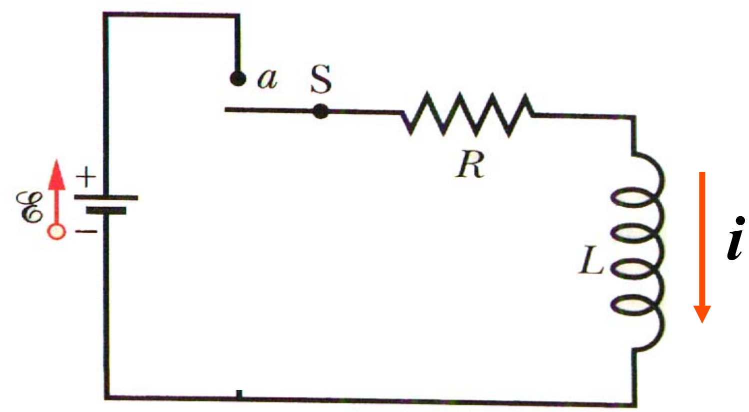
- **charge** *increases* from 0 to CE
- **current** *decreases* from E/R to 0
- **voltage** across capacitor *increases* from 0 to E

In an RL circuit, while charging, emf = L*di*/dt and the loop rule mean:

- **magnetic field** *increases* from 0 to B
- **current** *increases* from 0 to E/R
- **voltage** across inductor *decreases* from −E to 0

Inductors & Energy

- Recall that **capacitors** store energy in an **electric** field
- **Inductors** store energy in a **magnetic** field.



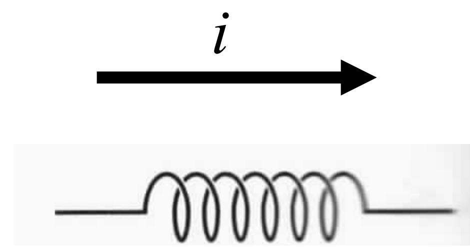
$$\mathcal{E} = iR + L\frac{di}{dt}$$

$$(i\mathcal{E}) = (i^2R) + Li\frac{di}{dt} \Rightarrow (i\mathcal{E}) = (i^2R) + \frac{d}{dt}\left(\frac{Li^2}{2}\right)$$

Power delivered by battery = power dissipated by R

+ energy stored in L

Example



- The current in a 10 H inductor is decreasing at a steady rate of 5 A/s.
- If the current is as shown at some instant in time, what is the induced EMF?

(a) 50 V 

(b) 50 V 

- Current is <u>decreasing</u>
- Induced emf must be in a direction that OPPOSES this change.
- So, induced emf must be in same direction as current
- Magnitude = (10 H)(5 A/s) = 50 V

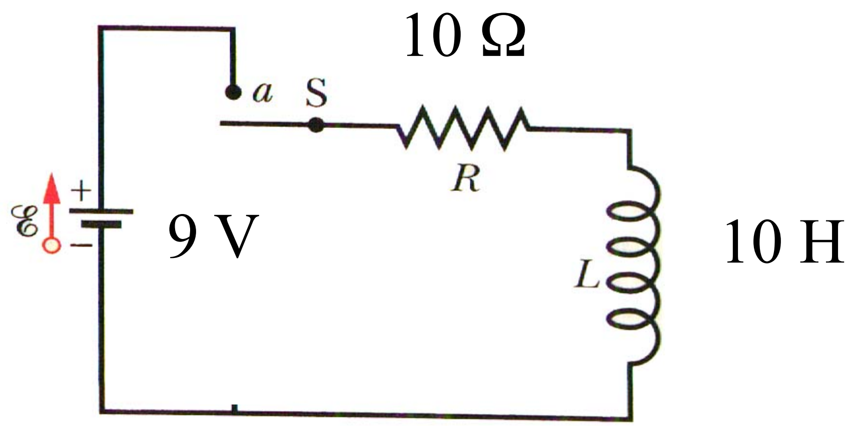
Example

Immediately after the switch is closed, what is the potential difference across the inductor?

(a) 0 V

(b) 9 V

(c) 0.9 V



- Immediately after the switch, current in circuit = 0.
- So, potential difference across the resistor = 0!
- So, the potential difference across the inductor = **E** = 9 V!

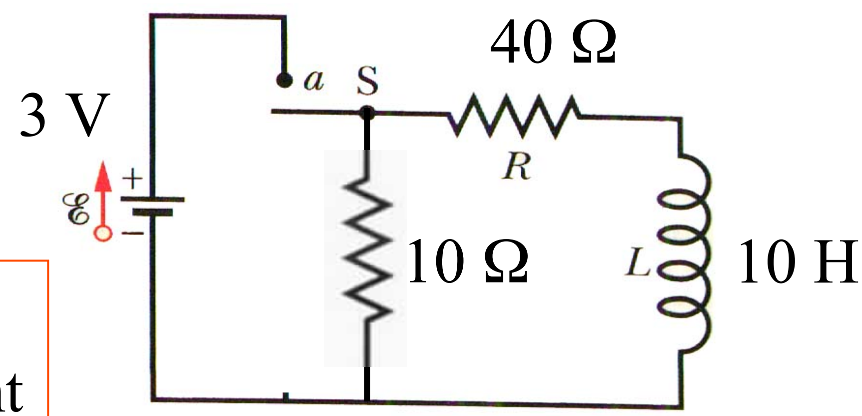
Example

- Immediately after the switch is closed, what is the current ***i*** through the 10 Ω resistor?

(a) 0.375 A

(b) 0.3 A

(c) 0



• Immediately after switch is closed, current through inductor = ***0.***
• Hence,
i **= (3 V)/(10Ω) = 0.3 A**

- Long after the switch has been closed, what is the current in the 40Ω resistor?

(a) 0.375 A

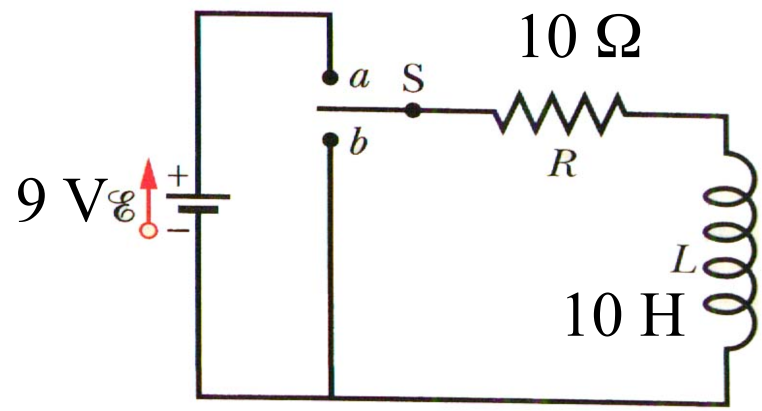
(b) 0.3 A

(c) 0.075 A

- Long after switch is closed, potential across inductor = 0**.**
- Hence, current through 40**Ω** resistor = (3 V)/(40Ω) = 0.075 A

Example

- The switch has been in position "a" for a long time.
- It is now moved to position "b" without breaking the circuit.
- What is the total energy dissipated by the resistor until the circuit reaches equilibrium?



- When switch has been in position "a" for long time, current through inductor = (9V)/(10Ω) = 0.9A.
- Energy stored in inductor = $(0.5)(10H)(0.9A)^2 = 4.05$ J
- When inductor "discharges" through the resistor, all this stored energy is dissipated as heat = 4.05 J.

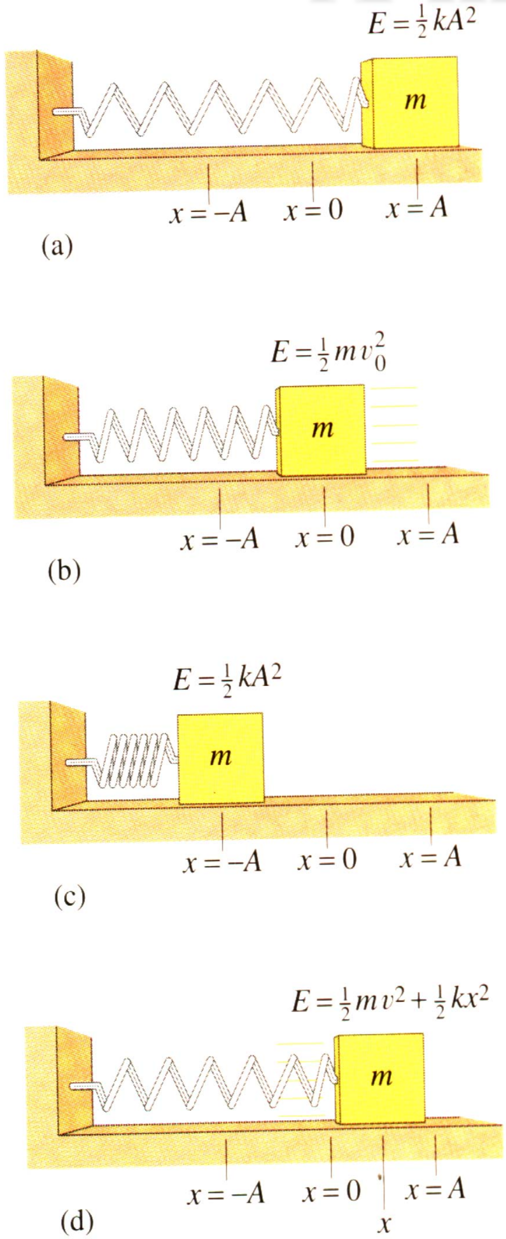
Oscillators in Physics

Oscillators are very useful in practical applications, for instance, to keep time, or to focus energy in a system.

All oscillators operate along the same principle: they are systems that can store energy in more than one way and exchange it back and forth between the different storage possibilities. For instance, in pendulums (and swings) one **exchanges energy** between **kinetic and potential** form.

In this course we have studied that **coils and capacitors** are devices that can store **electromagnetic energy**. In one case it is stored in a magnetic field, in the other in an electric field.

A mechanical oscillator



$$E_{tot} = E_{kin} + E_{pot} \qquad E_{tot} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

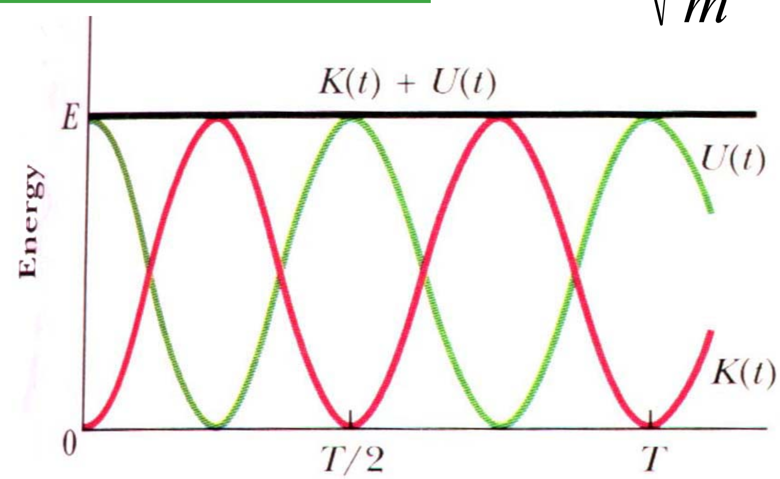
$$\frac{dE_{tot}}{dt} = 0 = \frac{1}{2} m\left(2v\frac{dv}{dt}\right) + \frac{1}{2} k\left(2x\frac{dx}{dt}\right) \qquad v = \frac{dx}{dt}$$

$$m\frac{dv}{dt} + k x = 0 \qquad m\frac{d^2x}{dt^2} + k x = 0$$

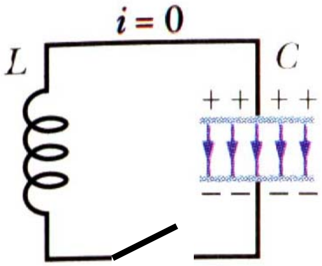
Newton's law
F=ma!

$$\text{Solution}: \quad x(t) = x_0 \cos(\omega t + \phi_0) \qquad \omega = \sqrt{\frac{k}{m}}$$

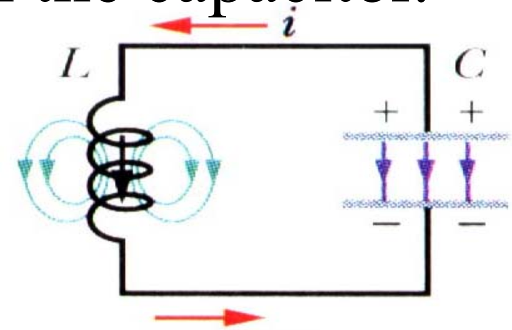
x_0 : amplitude
ω : frequency
ϕ_0 : phase



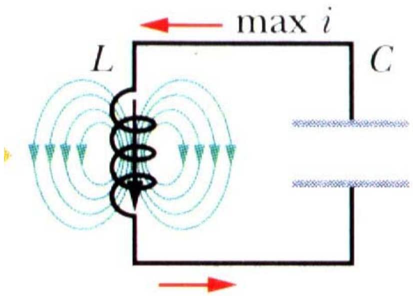
An electromagnetic oscillator



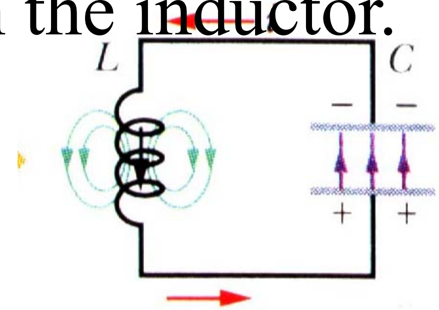
Capacitor initially charged. Initially, current is zero, energy is all stored in the capacitor.



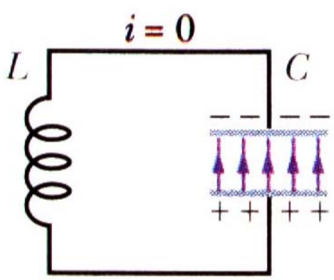
A current gets going, energy gets split between the capacitor and the inductor.



Capacitor discharges completely, yet current keeps going. Energy is all in the inductor.

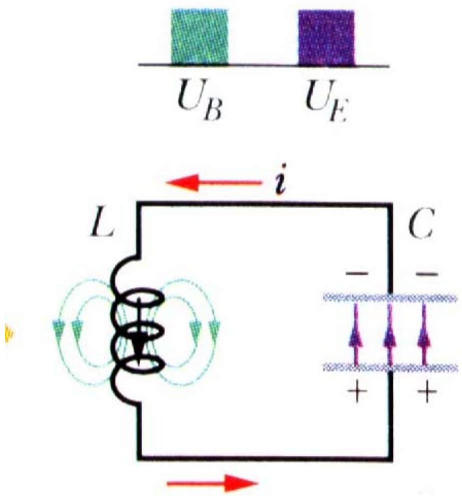


The magnetic field on the coil starts to collapse, which will start to recharge the



Finally, we reach the same state we started with (with opposite polarity) and the cycle restarts.

EM Oscillators: the math



$$E_{tot} = E_{mag} + E_{elec} \qquad E_{tot} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$$

$$\frac{dE_{tot}}{dt} = 0 = \frac{1}{2}L\left(2i\frac{di}{dt}\right) + \frac{1}{2C}\left(2q\frac{dq}{dt}\right) \qquad i = \frac{dq}{dt}$$

$$0 = L\left(\frac{di}{dt}\right) + \frac{1}{C}(q) \quad \text{(the loop rule!)}$$

$$\boxed{0 = L\frac{d^2q}{dt^2} + \frac{q}{C}}$$

Compare with: $\boxed{m\frac{d^2x}{dt^2} + kx = 0}$

Analogy between electrical and mechanical oscillations:

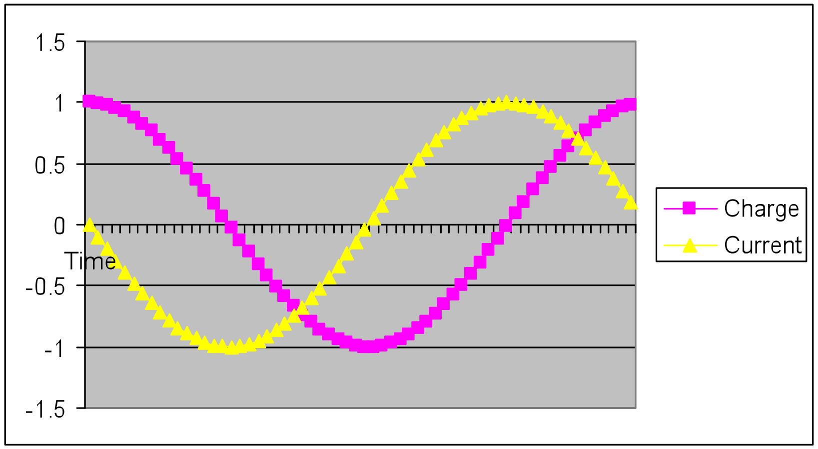
$q \to x \qquad 1/C \to k$

$i \to v \qquad L \to M$

$$x(t) = x_0\cos(\omega t + \phi_0) \qquad \boxed{q = q_0\cos(\omega t + \phi_0)}$$

$$\omega = \sqrt{\frac{k}{m}} \qquad \boxed{\omega = \sqrt{\frac{1}{LC}}}$$

Electric Oscillators: the math

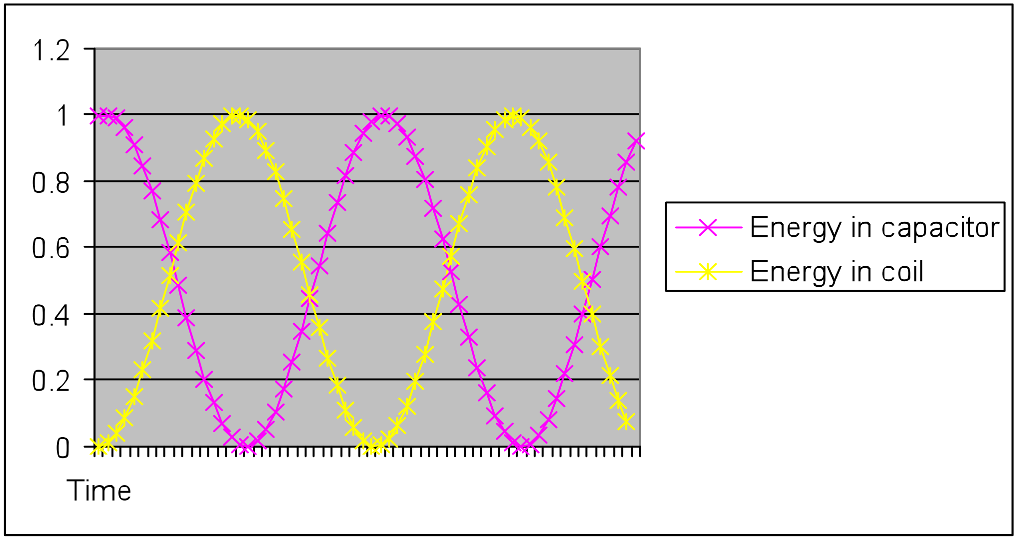


$$q = q_0 \cos(\omega t + \phi_0)$$

$$i = \frac{dq}{dt} = -\omega q_0 \sin(\omega t + \phi_0)$$

$$E_{mag} = \frac{1}{2} L i^2 = \frac{1}{2} L \omega^2 q_0^{\,2} \sin^2(\omega t + \phi_0)$$

$$E_{ele} = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2C} q_0^{\,2} \cos^2(\omega t + \phi_0)$$



And remembering that,

$$\cos^2 x + \sin^2 x = 1, \text{ and } \omega = \sqrt{\frac{1}{LC}}$$

$$E_{tot} = E_{mag} + E_{ele} = \frac{1}{2C} q_0^{\,2}$$

The energy is constant and equal to what we started with.

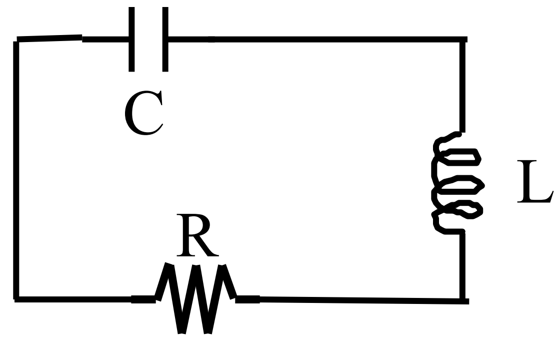
Damped LC Oscillator

Ideal LC circuit without resistance:
  oscillations go on for ever;
  $\omega = (LC)^{-1/2}$

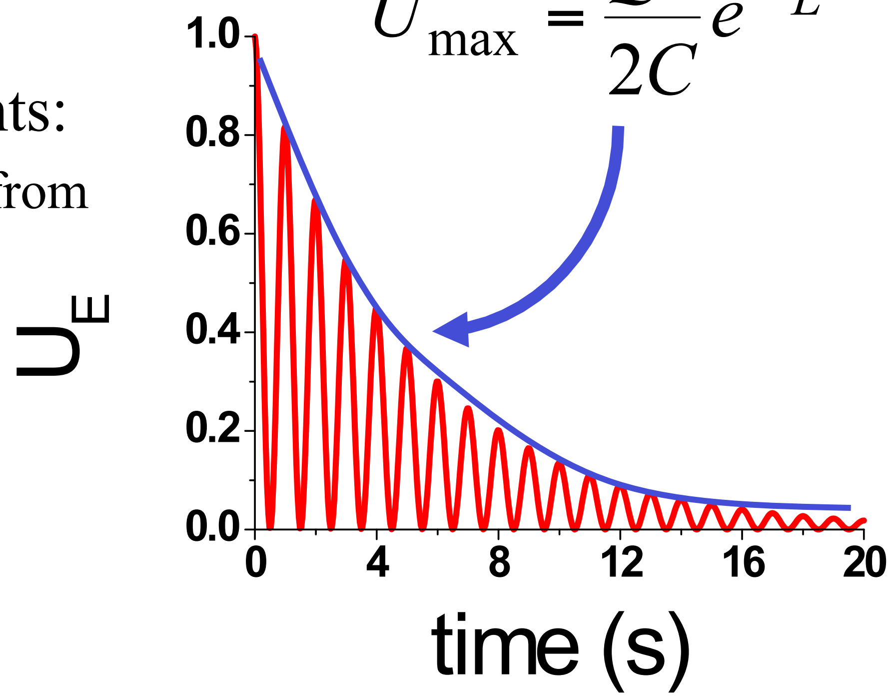
Real circuit has resistance, dissipates
  energy: oscillations die out, or are
  "damped"

Math is complicated! Important points:

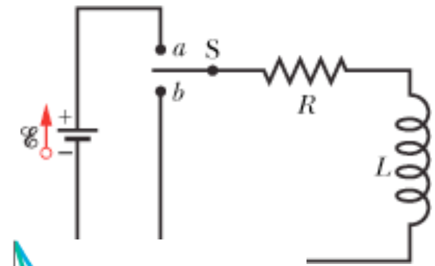
- Frequency of oscillator shifts away from $\omega = (LC)^{-1/2}$
- Peak CHARGE decays with time constant = 2L/R
- For small damping, peak ENERGY decays with time constant = L/R

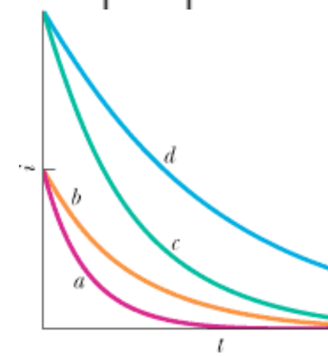


$$U_{\max} = \frac{Q^2}{2C} e^{-\frac{Rt}{L}}$$

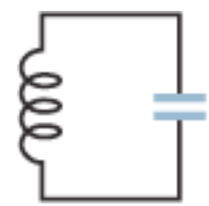


Summary





- In an RL circuit, we can "charge" the inductor with a battery until there is a constant current, or "discharge" the inductor through the resistor. Time constant is L/R.
- An LC combination produces an electrical oscillator, natural frequency of oscillator is $\omega = 1/\sqrt{LC}$



(a)

- Total energy in circuit is conserved: switches between capacitor (electric field) and inductor (magnetic field).
- If a resistor is included in the circuit, the total energy decays (is dissipated by R).