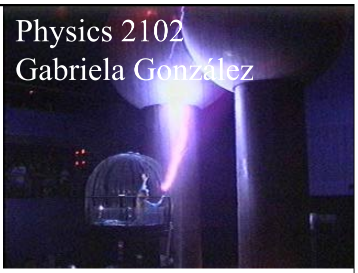
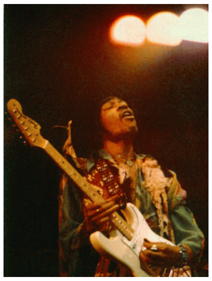


Physics 2102
Gabriela González

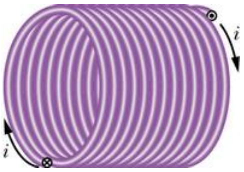
Physics 2102

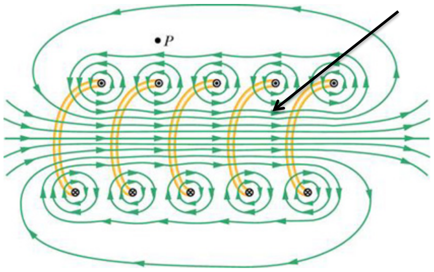
Faraday's law



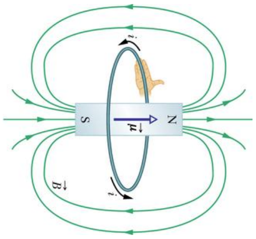
Solenoids, dipoles

An electrical current produces a magnetic field.





$B = \mu_0 i n$



μ=NiA

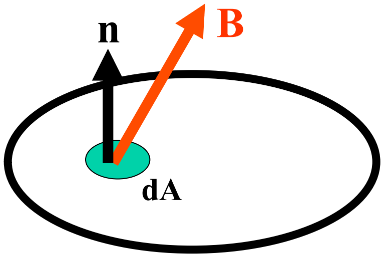
$$\vec{B}(z) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$

Faraday's Law

A magnetic field can create a en electrical current too!

If we define magnetic flux, similar to definition of electric flux, but for an open surface with an edge:

$$\Phi_B = \int_S \vec{B} \cdot \hat{n} dA$$



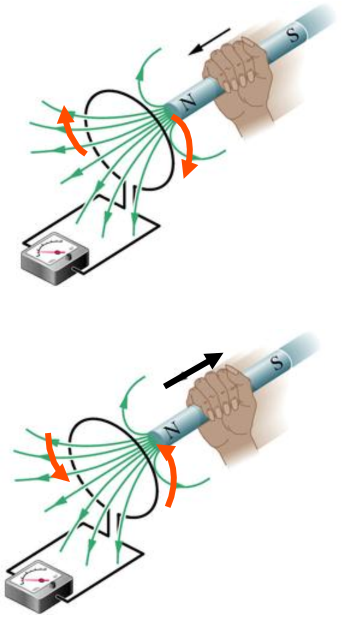
Then a <u>time varying</u> magnetic FLUX creates an induced EMF, and thus an electrical current if the edge is a wire!:

$$EMF = -\frac{d\Phi_B}{dt}$$

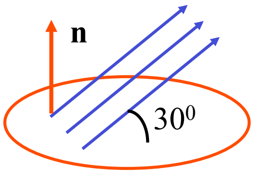
- Take note of the MINUS sign!!
- The induced EMF acts in such a way that it OPPOSES the change in magnetic flux ("Lenz's Law").

Example

- When the N pole approaches the loop, the flux "into" the loop ("downwards") increases
- The loop can "oppose" this change if a current were to flow clockwise, hence creating a magnetic flux "upwards."
- So, the induced EMF is in a direction that makes a current flow clockwise.
- If the N pole moves AWAY, the flux "downwards" DECREASES, so the loop has a counter clockwise current!



Example



- A closed loop of wire encloses an area of 1 m^2 in which in a uniform magnetic field exists at 30^0 to the PLANE of the loop. The magnetic field is DECREASING at a rate of 1T/s. The resistance of the wire is 10 Ω.
- What is the induced current?

$$\Phi_B = \int_S \vec{B}\cdot\hat{n}\,dA$$

$$= BA\cos(60^0) = \frac{BA}{2}$$

$$|EMF| = \frac{d\Phi_B}{dt} = \frac{A}{2}\frac{dB}{dt}$$

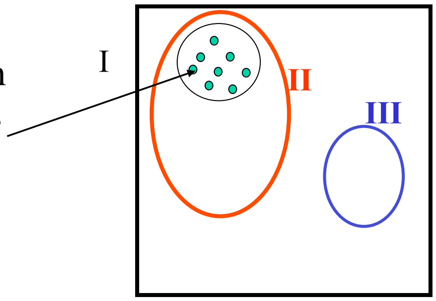
$$i = \frac{EMF}{R} = \frac{A}{2R}\frac{dB}{dt}$$

$$i = \frac{(1m^2)}{2(10\Omega)}(1T/s) = 0.05A$$

Is it
…clockwise or
…counterclockwise?

Example

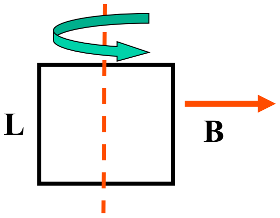
- 3 loops are shown.
- B = 0 everywhere except in the circular region where B is uniform, pointing out of the page and is **increasing at a steady rate.**
- Rank the 3 loops in order of increasing induced EMF.
  - (a) III, II, I
  - (b) III, (I & II are same)
  - (c) ALL SAME.

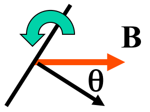


- Just look at the rate of change of ENCLOSED flux
- III encloses no flux and it does not change.
- I and II enclose same flux and it changes at same rate.

Example : the Generator

- A square loop of wire of side ***L*** is rotated at a uniform frequency ***f*** in the presence of a uniform magnetic field ***B*** as shown.
- Describe the EMF induced in the loop.





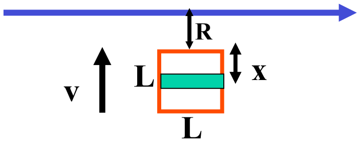
$$\Phi_B = \int_S \vec{B} \cdot \hat{n} dA$$

$$= BL^2 \cos(\theta)$$

$$EMF = -\frac{d\Phi_B}{dt} = BL^2 \frac{d\theta}{dt} \sin(\theta) = BL^2 (2\pi f) \sin(2\pi f t)$$

Example

- An infinitely long wire carries a constant current ***i*** as shown
- A square loop of side ***L*** is moving towards the wire with a constant velocity ***v***.
- What is the EMF induced in the loop when it is a distance R from the loop?



Choose a "strip" of width *dx* located as shown.
Flux thru this "strip"

$$= BL(dx) = \frac{\mu_0 i L(dx)}{2\pi(R+x)}$$

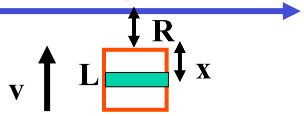
$$\Phi_B = \int_0^L \frac{\mu_0 i L(dx)}{2\pi(R+x)}$$

$$= \left[\frac{\mu_0 i L}{2\pi} \ln(R+x)\right]_0^L$$

$$= \frac{\mu_0 i L}{2\pi} \ln\left[\frac{R+L}{R}\right]$$

$$\boxed{\begin{aligned} EMF &= -\frac{d\Phi_B}{dt} \\ &= -\frac{\mu_0 L i}{2\pi} \frac{d}{dt}\left\{\ln\left[1+\frac{L}{R}\right]\right\} \end{aligned}}$$

Example (continued)



$$EMF = -\frac{d\Phi_B}{dt}$$

$$= -\frac{\mu_0 Li}{2\pi}\frac{d}{dt}\left\{\ln\left[1+\frac{L}{R}\right]\right\}$$

$$= \frac{\mu_0 Li}{2\pi}\frac{dR}{dt}\left[\frac{R}{R+L}\right]\frac{L}{R^2}$$

$$= \frac{\mu_0 i}{2\pi}v\left[\frac{L^2}{(R+L)R}\right]$$

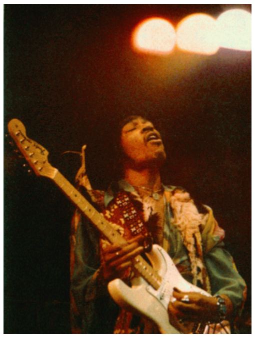
What is the DIRECTION of the induced current?

- Magnetic field due to wire points INTO page and gets stronger as you get closer to wire
- So, flux into page is INCREASING
- Hence, current induced must be counter clockwise to oppose this increase in flux.

Some interesting applications

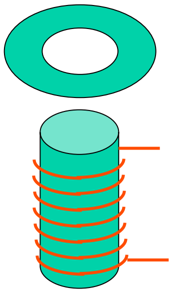
MagLev train relies on Faraday's Law: currents induced in non-magnetic rail tracks repel the moving magnets creating the induction; result: levitation!

Guitar pickups also use Faraday's Law -- a vibrating string modulates the flux through a coil hence creating an electrical signal at the same frequency.



Faraday's law: Eddy Currents

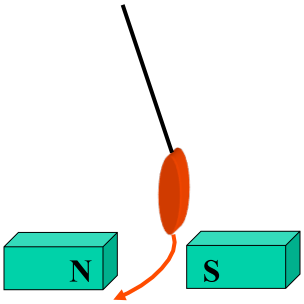
$$\Phi_B = \int_S \vec{B}\cdot\hat{n}\,dA \qquad EMF = -\frac{d\Phi_B}{dt}$$



- A non-magnetic (e.g. copper, aluminum) ring is placed near a solenoid.
- What happens if:
  - There is a steady current in the solenoid?
  - The current in the solenoid is suddenly changed?
  - The ring has a "cut" in it?
  - The ring is extremely cold?

Another Experimental Observation

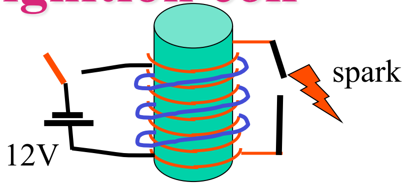
- Drop a non-magnetic pendulum (copper or aluminum) through an inhomogeneous magnetic field
- What do you observe? Why? (Think about energy conservation!)



Pendulum had kinetic energy
What happened to it?
Isn't energy conserved??

Example : the ignition coil

- The gap between the spark plug in a combustion engine needs an electric field of ~10^7 V/m in order to ignite the air-fuel mixture. For a typical spark plug gap, one needs to generate a potential difference > 10^4 V!
- But, the typical EMF of a car battery is 12 V. So, how does a spark plug work??



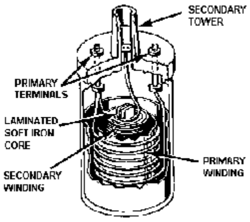
•Breaking the circuit changes the current through "primary coil"

• Result: LARGE change in flux thru secondary -- large induced EMF!

The "ignition coil" is a double layer solenoid:

• Primary: small number of turns -- 12 V

• Secondary: MANY turns -- spark plug



http://www.familycar.com/Classroom/ignition.htm