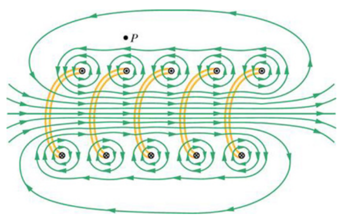
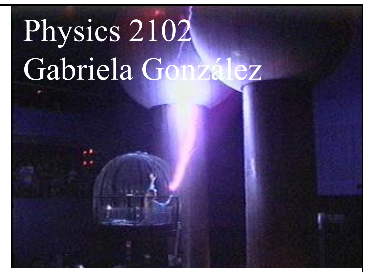
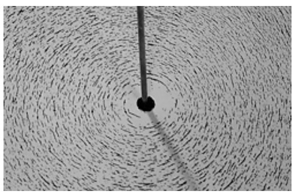
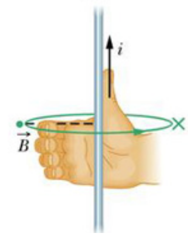


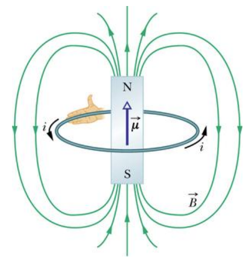


Physics 2102

Magnetic fields produced by currents





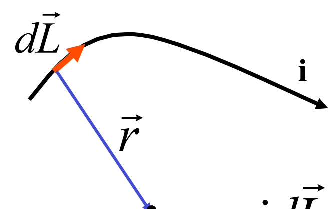


The Biot-Savart Law

?

Jean-Baptiste Biot (1774-1862)

Felix Savart (1791-1841)



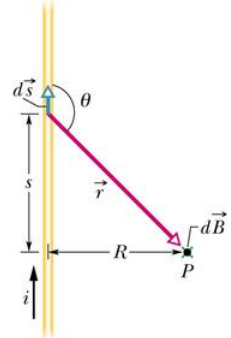
- Quantitative rule for computing the magnetic field from any electric current
- Choose a differential element of wire of length **dL** and carrying a current ***i***
- The field d**B** from this element at a point located by the vector **r** is given by the Biot-Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{L}\times\vec{r}}{r^3}$$

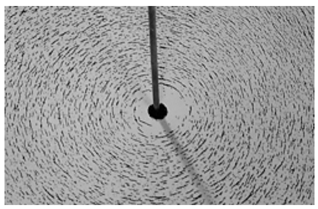
$\mu_0 = 4\pi x 10^{-7}$ T.m/A (permeability constant)

Compare with: $$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq\,\vec{r}}{r^3}$$

Biot-Savart Law



- An infinitely long straight wire carries a current ***i.***
- Determine the magnetic field generated at a point located at a perpendicular distance **R** from the wire.
- Choose an element d**s** as shown
- Biot-Savart Law:
  d**B** ~ ds x **r**
  points INTO the page
- Integrate over all such elements

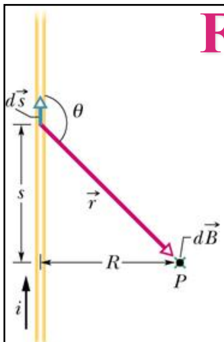


$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s} \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i ds(r\sin\theta)}{r^3}$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{ds(r\sin\theta)}{r^3}$$

Field of a straight wire (continued)



$$\sin\theta = R/r$$

$$r = (s^2 + R^2)^{1/2}$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{ds(r\sin\theta)}{r^3} = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{R\,ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi}\int_{0}^{\infty}\frac{R\,ds}{\left(s^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 i R}{2\pi}\left[\frac{s}{R^2\left(s^2 + R^2\right)^{1/2}}\right]_0^{\infty} = \boxed{\frac{\mu_0 i}{2\pi R}}$$

A Practical Matter

A power line carries a current of 500 A. What is the magnetic field in a house located 100m away from the power line?

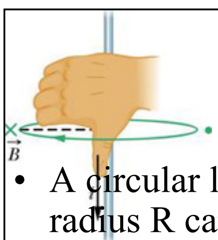
$$B = \frac{\mu_0 i}{2\pi R}$$

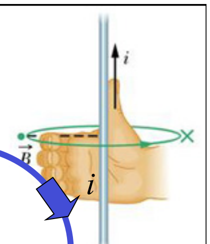
$$= \frac{(4\pi x 10^{-7} T.m/A)(500A)}{2\pi(100m)}$$

= 1 μT!!

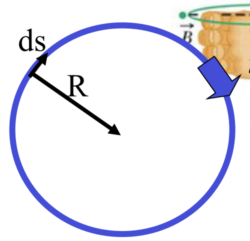
Recall that the earth's magnetic field is ~10^{-4}T = 100 μT

Biot-Savart Law





- A circular loop of wire of radius R carries a current *i*.
- What is the magnetic field at the center of the loop?



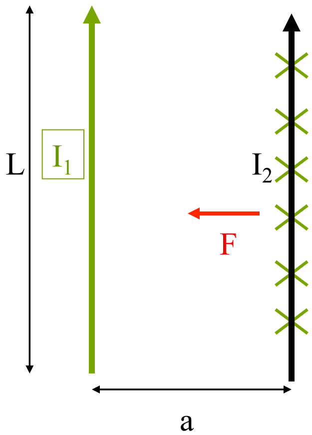
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\overrightarrow{ds}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{idsR}{R^3} = \frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2}$$

$$B = \frac{\mu_0}{4\pi}\int\frac{id\phi}{R} = \frac{\mu_0 i 2\pi}{4\pi R} = \frac{\mu_0 i}{2R}$$

Direction of B?? Not ***another*** right hand rule?! Curl fingers around direction of CURRENT. Thumb points along FIELD! Into page in this case.

Forces between wires

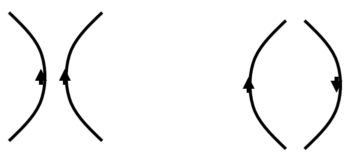


Magnetic field due to wire 1 where the wire 2 is,

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

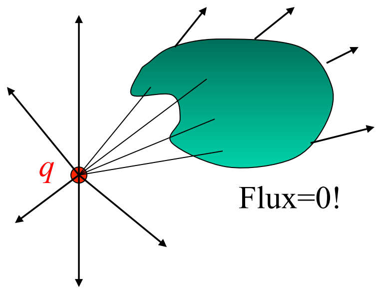
Force on wire 2 due to this field,

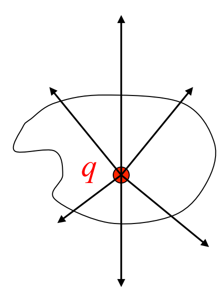
$$F_{21} = L I_2 B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$



Ampere's law: Remember Gauss' law?

Given an arbitrary closed surface, the electric flux through it is proportional to the charge enclosed by the surface.

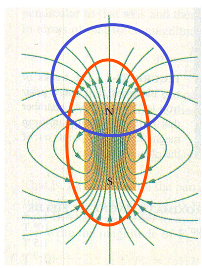




$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Gauss' law for magnetism: simple but useless

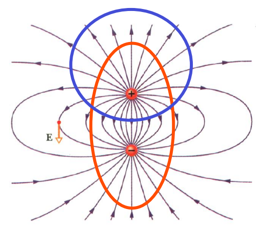
No isolated magnetic poles! The magnetic flux through any closed "Gaussian surface" will be ZERO. This is one of the four "Maxwell's equations".



$$\oint \vec{B} \bullet d\vec{A} = 0$$

Always! No isolated magnetic charges

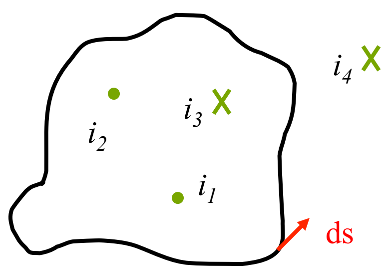
$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$



Ampere's law: a useful version of Gauss' law.

$$\oint_{\text{loop}} \vec{B} \cdot d\vec{s} = \mu_0 I$$

The circulation of B (the integral of B scalar ds) along an imaginary **closed loop** is proportional to the **net amount of current** traversing the loop.



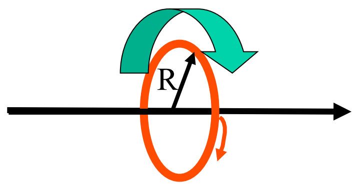
$$\oint_{\text{loop}} \vec{B} \cdot d\vec{s} = \mu_0 (i_1 + i_2 - i_3)$$

Thumb rule for sign; ignore i_4

As was the case for Gauss' law, if you have a lot of symmetry, knowing the circulation of B allows you to know B.

Ampere's Law: Example

- Infinitely long straight wire with current i.
- Symmetry: magnetic field consists of circular loops centered around wire.
- So: choose a circular loop C -- B is tangential to the loop everywhere!
- Angle between B and ds = 0. (Go around loop in same direction as B field lines!)



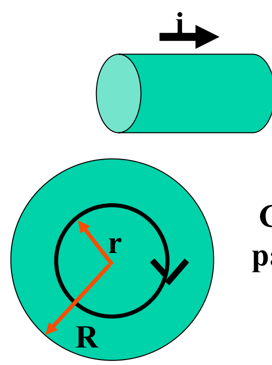
$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$\oint_C B ds = B(2\pi R) = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

Ampere's Law: Example

- Infinitely long cylindrical wire of finite radius **R** carries a total current **i** with uniform current density
- Compute the magnetic field at a distance **r** from cylinder axis for:
  - r < a (inside the wire)
  - r > a (outside the wire)

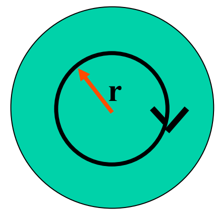


Current into page, circular field lines

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 i$$

Ampere's Law: Example 2 (cont)

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i$$



Current into page, field tangent to the closed amperian loop

$$B(2\pi r) = \mu_0 i_{enclosed}$$

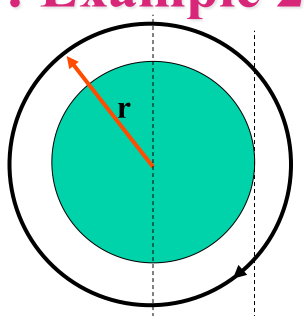
$$B = \frac{\mu_0 i_{enclosed}}{2\pi r}$$

$$i_{enclosed} = J(\pi r^2) = \frac{i}{\pi R^2}\pi r^2 = i\frac{r^2}{R^2}$$

$$B = \frac{\mu_0 i r}{2\pi R^2}$$ **For r < R**

Ampere's Law: Example 2 (cont)

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 i$$



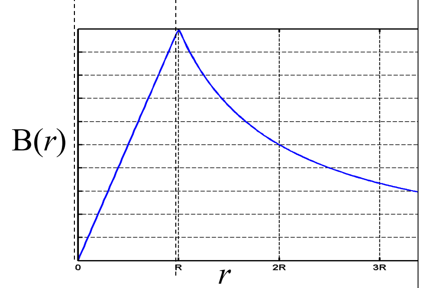
Current into page, field tangent to the closed amperian loop

$$B(2\pi r) = \mu_0 i_{enclosed}$$

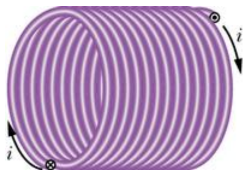
$$B = \frac{\mu_0 i_{enclosed}}{2\pi r} = \frac{\mu_0 i}{2\pi r}$$

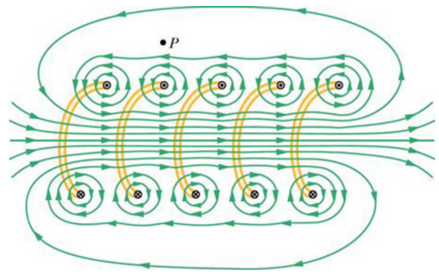
$$B = \frac{\mu_0 i}{2\pi r}$$ **For r > R**

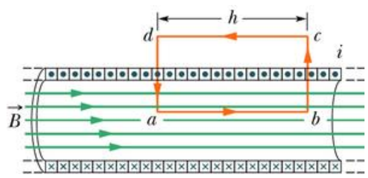
$$B = \frac{\mu_0 i r}{2\pi R^2}$$ **For r < R**



Solenoids







$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc}$$

$$\oint \vec{B} \bullet d\vec{s} = 0 + Bh + 0 + 0$$

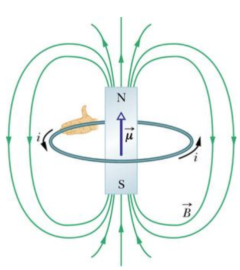
$$i_{enc} = iN_h = i(N/L)h = inh$$

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 i_{enc} \Rightarrow Bh = \mu_0 inh \Rightarrow B = \mu_0 in$$

Magnetic field of a magnetic dipole

A circular loop or a coil currying electrical current is a magnetic dipole, with magnetic dipole moment of magnitude μ=N*i*A.
Since the coil curries a current, it produces a magnetic field, that can be calculated using Biot-Savart's law:

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{(R^2 + z^2)^{3/2}} \approx \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$



All loops in the figure have radius r or 2r. Which of these arrangements produce the largest magnetic field at the point indicated?