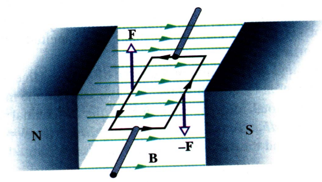
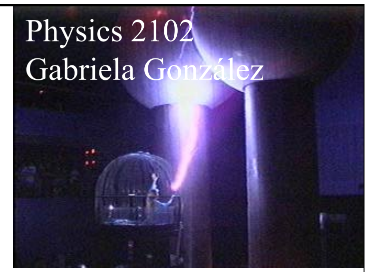
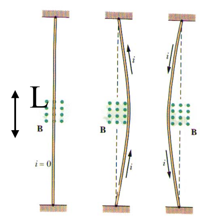


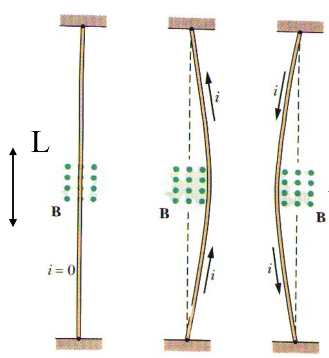


Physics 2102

Magnetic fields



Magnetic force on a wire



$$q = i\,t = i\frac{L}{v_d}$$

$$\vec{F} = q\,\vec{v}_d \times \vec{B}$$

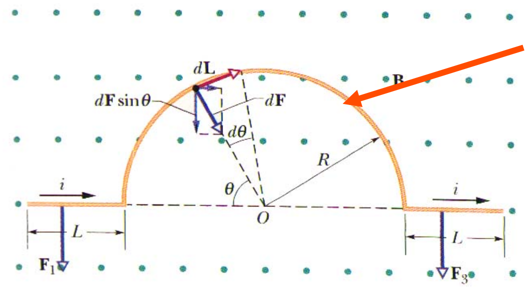
$$\vec{F} = q\frac{i\vec{L}}{q}\times\vec{B} = i\vec{L}\times\vec{B}$$

$$\vec{F} = i\vec{L}\times\vec{B}$$

Note: If wire is not straight, compute force on differential elements and integrate:

$$d\vec{F} = i\,d\vec{L}\times\vec{B}$$

Example



Wire with current i.
Magnetic field out of page.
What is net force on wire?

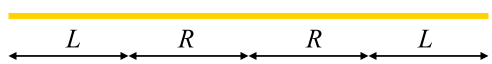
$$F_1 = F_3 = iLB$$
$$dF = iBdL = iBRd\theta$$

By symmetry, F_2 will only have a vertical component,

$$F_2 = \int_0^{\pi} \sin(\theta)dF = iBR\int_0^{\pi} \sin(\theta)d\theta = 2iBR$$

$$F_{\text{total}} = F_1 + F_2 + F_3 = iLB + 2iRB + iLB = 2iB(L+R)$$

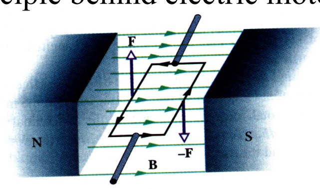
Notice that the force is the same as that for a straight wire,



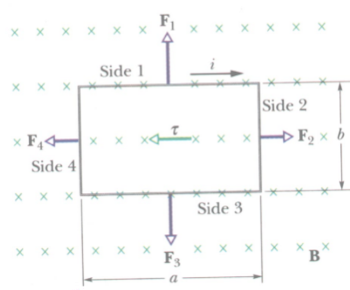
and this would be true no matter what the shape of the central segment!.

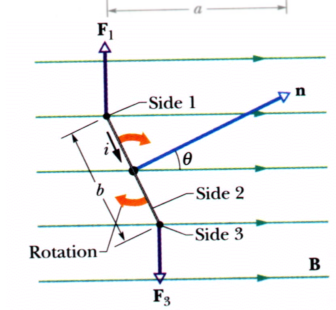
Torque on a current loop:

Principle behind electric motors.



Rectangular coil: a x b, current = i





Net force on current loop = 0

But: Net torque is NOT zero!

$$F_1 = F_3 = iaB$$

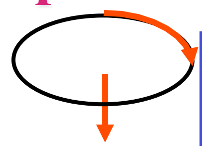
$$F_\perp = F_1 \sin(\theta)$$

$$Torque = |\tau| = F_\perp b = iabB\sin(\theta)$$

For a coil with **N** turns,
τ= **N** $I\,A$ B sinθ,
where A is the area of coil

Magnetic Dipole Moment

We just showed: τ = NiABsinθ
N=number of turns in coil
A=area of coil.



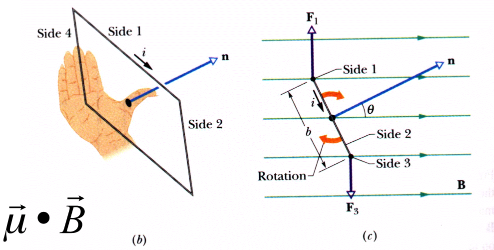
Right hand rule:
curl fingers in direction of current;
thumb points along μ

Define: magnetic dipole moment μ

$\vec{\mu}, \hat{n}$

$$\vec{\mu} = (NiA)\hat{n}$$

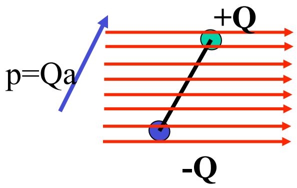
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

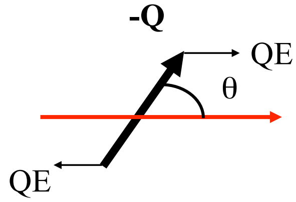


Potential energy: $U = -\vec{\mu} \bullet \vec{B}$

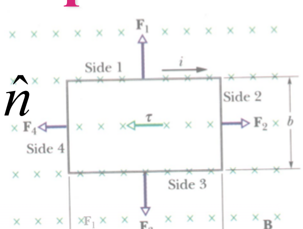
As in the case of electric dipoles, magnetic dipoles tend to align with the magnetic field (where potential energy is a minimum).

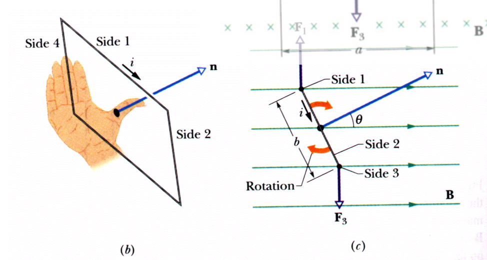
Electric and magnetic dipoles





$$\vec{\mu} = (NiA)\hat{n}$$





$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \bullet \vec{E}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \bullet \vec{B}$$

Magnetic forces: summary

- Magnetic force on a moving electric charge:
  $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$
- Magnetic Force on a wire:
  $\mathbf{F} = i\,\mathbf{L} \times \mathbf{B}$
- Magnetic Torque on a magnetic dipole:
  $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ (potential energy: $U = -\boldsymbol{\mu}\cdot\mathbf{B}$)

Example

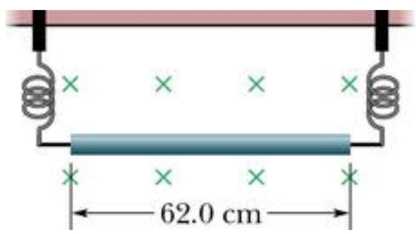
A magnetic dipole with a dipole moment of magnitude 0.02 J/T is released from rest in a uniform magnetic field of magnitude 50 mT. The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientation where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.70 mJ.

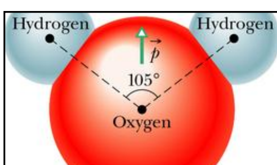
(a) What is the initial angle between the dipole moment and the magnetic field?

(b) What is the angle when the dipole is next (momentarily) at rest?

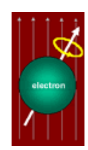
Example

A wire of 62.0 cm length and 13 g mass is suspended by a pair of flexible leads in a magnetic field of 0.440 T. What are the magnitude and direction of the current required to remove the tension in the supporting leads?



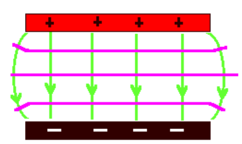


Magnetic fields



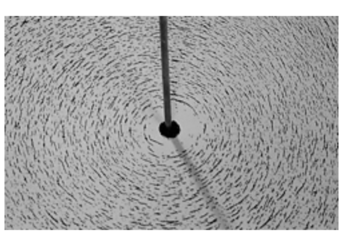
Electric fields are created:

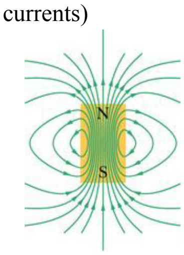
- *microscopically*, by electric charges (fields) of elementary particles (electrons, protons)
- *macroscopically*, by adding the field of many elementary charges of the same sign



Magnetic fields are created :

- *microscopically*, by magnetic "moments" of elementary particles (electrons, protons, neutrons)
- *macroscopically*, by
  - adding many microscopic magnetic moments (magnetic materials); or by
  - electric charges that move (electric currents)



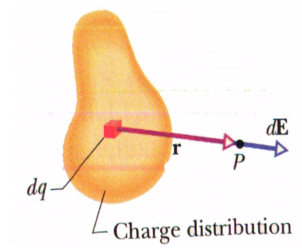


(b)

Currents producing magnetic fields: Biot-Savart Law

When we computed the electric field due to charges we used Coulomb's law. If one had a large irregular object, one broke it into infinitesimal pieces and computed,

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$



Which we can write as,

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

Magnetic fields are produced by electrical currents. If you wish to compute the magnetic field due to a current in a wire, you use the law of Biot and Savart.

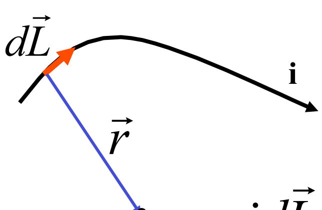
Jean-Baptiste Biot (1774-1862)

The Biot-Savart Law

?

Felix Savart (1791-1841)

- Quantitative rule for computing the magnetic field from any electric current
- Choose a differential element of wire of length **dL** and carrying a current ***i***
- The field d**B** from this element at a point located by the vector **r** is given by the Biot-Savart Law:



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{L}\times\vec{r}}{r^3}$$

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A (permeability constant)

Compare with: $$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq\,\vec{r}}{r^3}$$