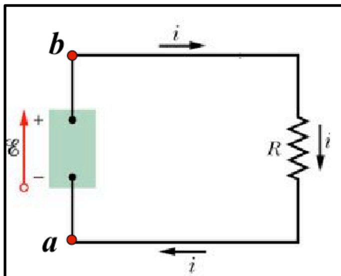
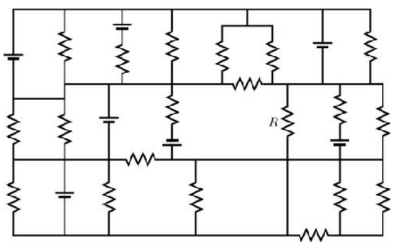
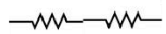
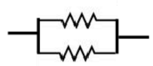
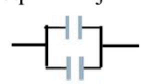


Physics 2102

Circuits

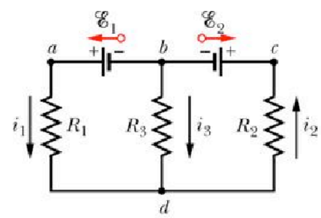


Recall:
Resistors and Capacitors

	Resistors	**Capacitors**
		
<u>Key formula</u>:	$V=iR$	$Q=CV$
<u>In series</u>:	same current	same charge
	$R_{eq}=\sum R_j$	$1/C_{eq}=\sum 1/C_j$
		
<u>In parallel</u>:	same voltage	same voltage
	$1/R_{eq}=\sum 1/R_j$	$C_{eq}=\sum C_j$
		

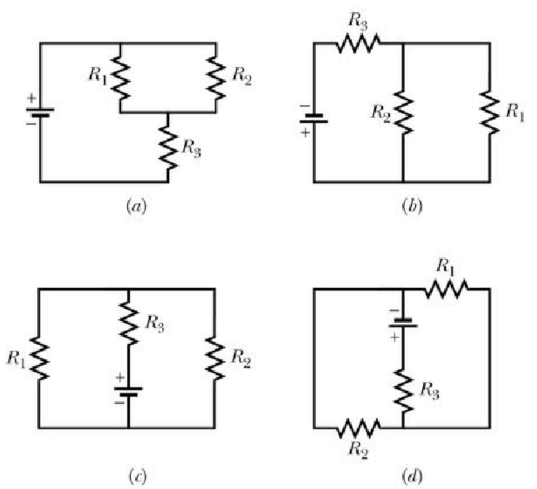
Recall: DC circuits

- **Loop rule**: when walking along a loop, add potential differences across each element, and make the total equal to zero when you come back to the original point.
- **Junction rule**: at every junction, total current is conserved.
- Problem strategy:
  - Replace resistors in series and in parallel with their equivalent resistors
  - Draw currents in every wire, and label them
  - Write the loop rule for each loop (or for the loop that involves your question)
  - Write the junction rules for the currents.
  - Solve the equations for the currents.
  - Answer the question that was asked.



Example

a) Which circuit has the largest equivalent resistance?
b) Assuming that all resistors are the same, which one dissipates more power?
c) Which resistor has the smallest potential difference across it?



(a) All circuits have the same equivalent resistance: In all cases, R1 and R2 are in parallel, and the equivalent resistor R12=1/(1/R1+1/R2)=R1R2/(R1+R2) is in series with R3, so
R123=R3+R1R2/(R1+R2)

(b) If all resistors are the same, we use P=i^2R, so the one with largest current will dissipate more power: that is R3. The current through R1 and R2 is only half, so each of them will dissipate ¼ the power dissipated by tr3.

(c) If the current through the battery and R3 is I, the voltage across R3 is V3=iR. Since only half the current goes through each of R1 and R2, then V1=V2=iR/2. Thus, R1 and R2 have the smallest potential difference across them.

Example

Assume the batteries are ideal, and have emf E_1=12V, E_2=9V, and R=3Ω.

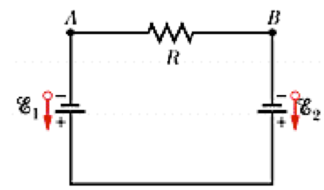
a) Which way will the current flow?

b) Which battery us doing positive work?

c) If the potential at A is 0V, what is the potential at B?

d) How much power is dissipated by the resistor?

e) How much power is delivered (or absorbed) by the batteries?



(a) The current will flow from A to B through the batteries (counterclockwise).
(b) E1 is doing positive work, E2 is doing negative work (as in a car battery getting charged)
(c) Voltage difference between A and B is the voltage difference of the batteries, +3V (B is at a higher potential).
(d) Current is 3Volts/3Ohms=1Amp, so power dissipated by resistor is P=i^2 R = 3 Watts.
(e) E1 delivers P=iV=12 Watts, while E2 absorbs P=iV= 9 Watts.

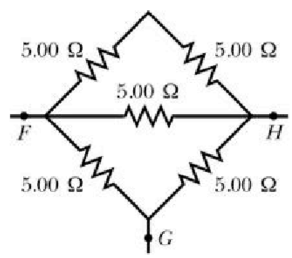
Example

Find the equivalent resistance between points

(a) *F* and *H* and

(b) *F* and *G*.

(*Hint:* For each pair of points, imagine that a battery is connected across the pair.)

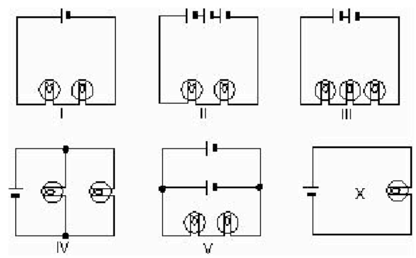


(a) There are three paths for current, or three resistors in parallel;; top and bottom paths have two resistors in series. The three parallel paths have: 10 Ohms on top path, 5 Ohms in middle path, 10 Ohms in bottom path; equivalent resistance is then 1/(1/10+1/5+1/10)=1/(4/10)=10/4=2.5 Ohms.

(b) We first replace top resistors in series with the 10 Ohms equivalent; this 10Ohms is in parallel with 5 Ohms, resulting in 3/10 Ohms. That one is now in series with 5 Ohms between H and G, resulting in 5.3 Ohms, now in parallel with 5 Ohms between F and G. The total equivalent resistance is then 1/(1/5.3+1/5)=2.6 Ohms.

Light bulbs

- If all batteries are ideal, and all batteries and lightbulbs are identical, in which arrangements will the lightbulbs as bright as the one in circuit X?

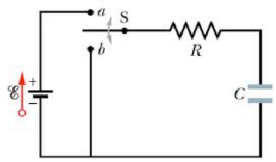


Power through a light bulb is P=i^2R. In X, with i=V/R.
In (i), i=E/2R: dimmer.
In (ii), i=2E/2R=E/R: same as in X.
In (iii), i=2E/3R: dimmer.
In (iv), current through the battery is i=E/(R/2)=2E/R, but current through each light bulb is E/R; same as in X.
In (v), i=E/(2R): dimmer.

So, (ii) and (iv) are as bright as X.

RC circuits: charging a capacitor

In these circuits, current will change for a while, and then stay constant.
We want to solve for current as a function of time *i*(t).
The charge on the capacitor will also be a function of time: *q*(t).
The voltage across the resistor and the capacitor also change with time.
To charge the capacitor, close the switch on *a*.

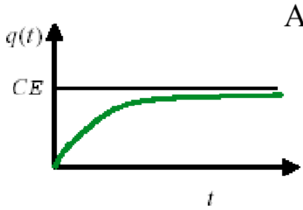


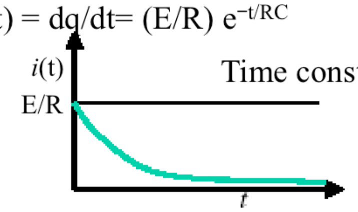
$$E + V_R(t) + V_C(t) = 0$$
$$E - i(t)R - q(t)/C = 0$$
$$E - (dq(t)/dt)\,R - q(t)/C = 0$$

A differential equation for q(t)! The solution is:

$$q(t) = CE(1 - e^{-t/RC})$$

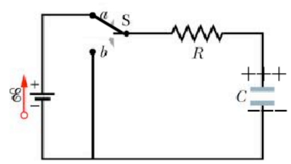
And then $i(t) = dq/dt = (E/R)\,e^{-t/RC}$





Time constant=RC

RC circuits: discharging a capacitor

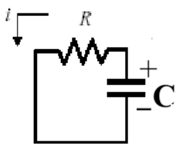


Assume the switch has been closed on a for a long time: the capacitor will be charged with Q=CE.

Then, close the switch on *b*: charges find their way across the circuit, establishing a current.

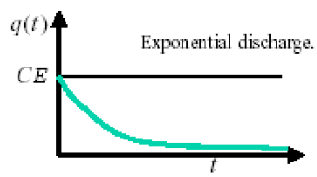
$V_R+V_C=0$

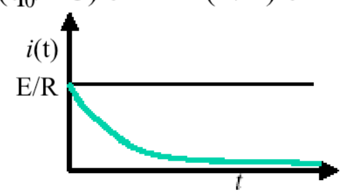
$-i(t)R+q(t)/C=0 \Rightarrow (dq/dt)R+q(t)/C=0$



Solution: $q(t)=q_0e^{-t/RC}=CEe^{-t/RC}$

$i(t) = dq/dt = (q_0/RC)\, e^{-t/RC} = (E/R)\, e^{-t/RC}$





Example

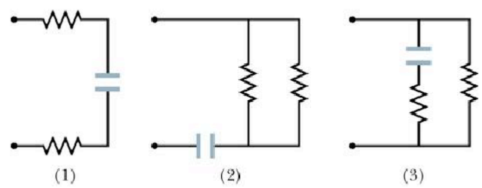
The three circuits below are connected to the same ideal battery with emf E. All resistors have resistance R, and all capacitors have capacitance C.

a) Which capacitor takes the longest in getting charged?

b) Which capacitor ends up with the largest charge?

c) What's the final current delivered by each battery?

d) What happens when we disconnect the battery?



(a) Time constants are 2RC, RC/2, RC, so Capacitor in (1) takes the longest.
(b) Q=CV, and in all cases, V=E at the final equilibrium.
(c) Batteries in (1) and (2) do not deliver any current when the capacitor is charged; Battery in (3) will deliver a current i=E/R.
(d) Capacitor will stay charged in (1) and (2); charge in capacitor (3) will start a current, and will get discharged with a time constant 2RC.

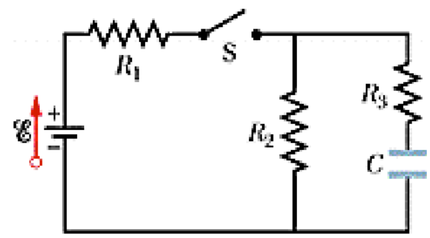
Example

In the figure, E= 1 kV, C = 10 μF, $R_1 = R_2 = R_3 = 1$ MΩ. With C completely uncharged, switch S is suddenly closed (at t = 0).

a) What's the current through each resistor at t=0?

b) What's the current through each resistor after a long time?

c) How long is a long time?



(a) At t=0, there is no voltage across the capacitor, so R2 and R3 are in parallel, with a total equivalent resistance equal to 1.5 MOhm, and the battery delivers a total current i=1kV/1.5MOhm= 0.67 mA.: we have 0.67 mA through R1, and half that (0.33 mA) through each of R2 and R3.

(b) After a long time, the capacitor is charged, no current flows through R3, and R2 and R3 are in series, with a total equivalent resistance equal to 2 MOhm, and a battery delivered by the battery of 1kV/2MOhm=0.5 mA: this current flows through R1 and R2.

(c) The time constant for the capacitor getting charged is 1MOhm x 10 uF = 10 sec : we need to wait tens of seconds.