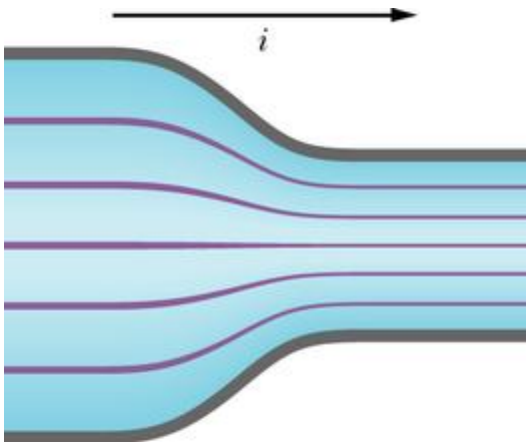
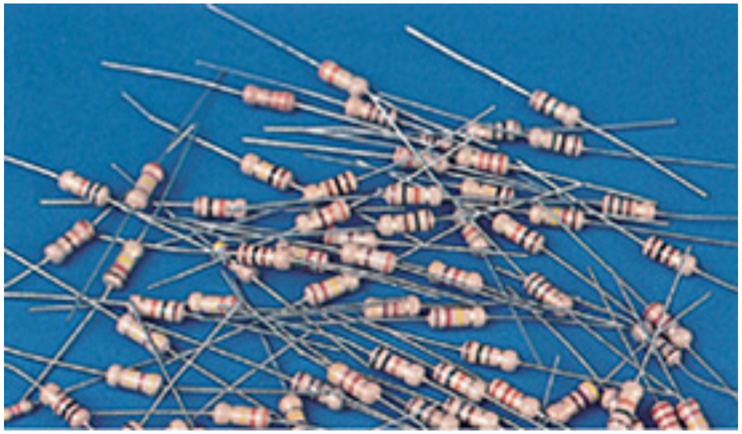


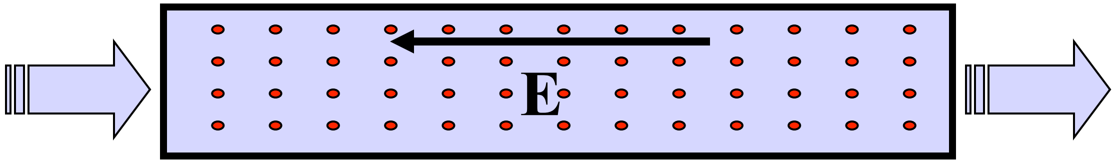
Physics 2102

Current and resistance



Georg Simon Ohm
(1789-1854)

Electrical current



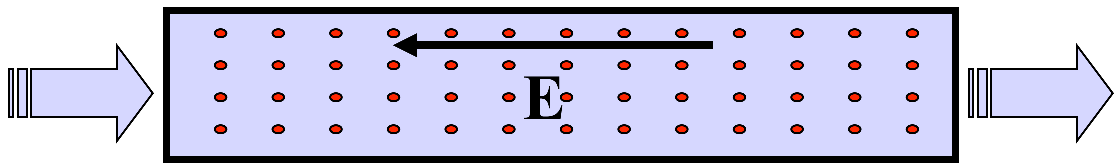
In a conductor, electrons are free to move. If there is a field E inside the conductor, F=qE means the electrons move in a direction opposite to the field: this is an electrical current.

We think of current as motion of imaginary positive charges along the field directions.

$$i = \frac{dq}{dt}, \qquad q = \int i\, dt \quad \text{Units}: [\mathrm{i}] = \frac{\text{Coulomb}}{\text{second}} \equiv \text{Ampere}$$

Andre-Marie Ampere 1775-1836

Electrical current



Wasn't the field supposed to be zero inside conductors?

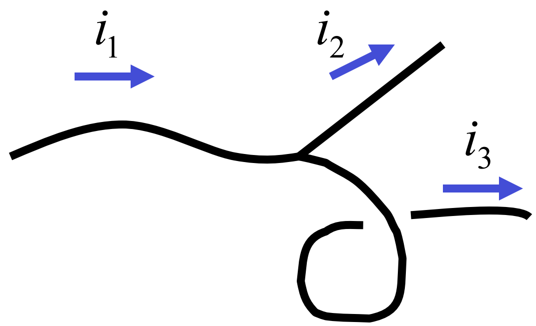
Yes, if the charges were in *equilibrium*. The reasoning was "electrons move until they cancel out the field". If the situation is not static, that is, if electrons are moving, then the field can be nonzero in a conductor, and the potential is not constant across it!

However, "somebody" has to be pumping the electrons: this is the job of the battery we put across a circuit. If there is no source creating the electric field, the charges reach equilibrium at E=0.

Electrical current: Conservation

Current is a scalar, NOT a vector, although we use arrows to indicate direction of propagation.

Current is conserved, because charge is conserved!



$$i_1 = i_2 + i_3$$

“junction rule”: everything that comes in, must go out.

Resistance

Electrons are not "completely free to move" in a conductor. They move erratically, colliding with the nuclei all the time: this is what we call "resistance".

The resistance is related to the potential we need to apply to a device to drive a given current **through** it. The larger the resistance, the larger the potential we need to drive the same current.

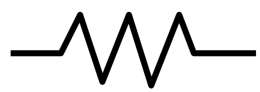
Ohm's laws

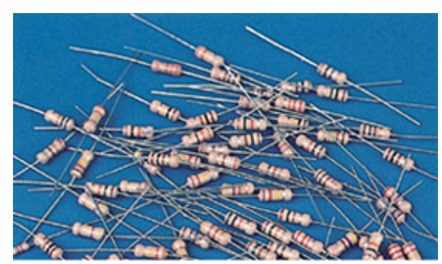
$$R \equiv \frac{V}{i} \qquad \text{and therefore: } i = \frac{V}{R} \quad \text{and } V = iR$$

Georg Simon Ohm (1789-1854)

"a professor who preaches such heresies is unworthy to teach science." Prussian minister of education 1830

$$\text{Units: [R]} = \frac{\text{Volt}}{\text{Ampere}} \equiv \text{Ohm (abbr. } \Omega)$$

Devices specifically designed to have a constant value of R are called resistors, and symbolized by 

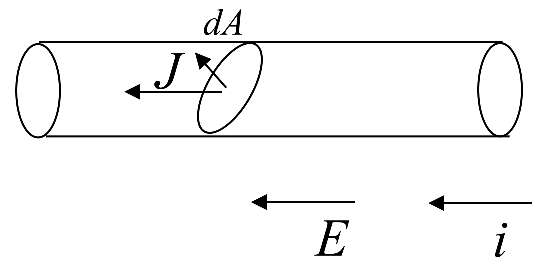


Current density and drift speed

Vector : $\vec{J}$ Same direction as $\vec{E}$ such that $i = \int \vec{J} \cdot d\vec{A}$

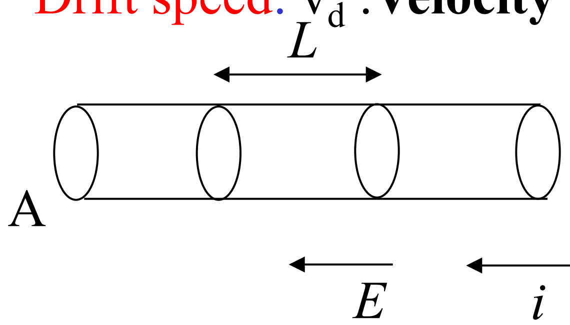
The current is the flux of the current density!

If surface is perpendicular to a constant electric field, then i=JA, or J=i/A



Units: $[J] = \dfrac{\text{Ampere}}{\text{m}^2}$

Drift speed: v_d :**Velocity at which electrons move in order to establish a current.**



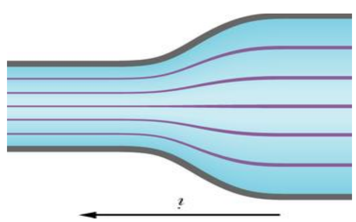
Charge *q* in the length *L* of conductor: $q = (nAL)e$

n =density of electrons, e =electric charge

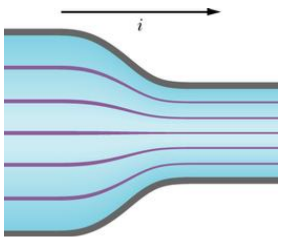
$$t = \frac{L}{v_d} \qquad i = \frac{q}{t} = \frac{nALe}{\frac{L}{v_d}} = nAev_d$$

$$v_d = \frac{i}{nAe} = \frac{J}{ne}$$

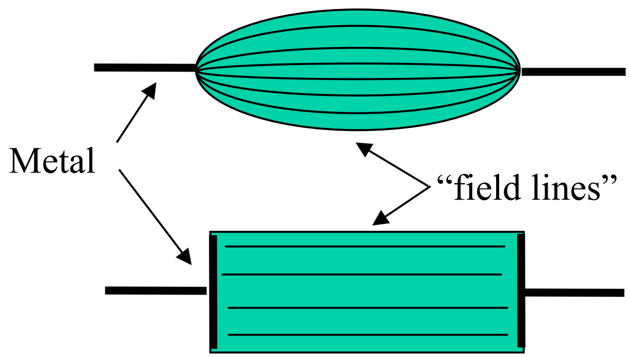
$$\vec{J} = ne\vec{v}_d$$



Where is the (current, current density, electron density, drift velocity, electric field) largest?



Resistivity and resistance



These two devices could have the same resistance R, when measured on the outgoing metal leads. However, it is obvious that inside of them different things go on.

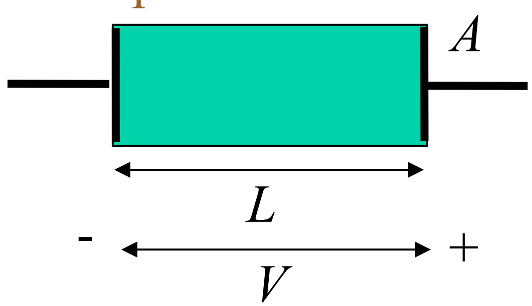
resistivity: $\rho = \dfrac{E}{J}$ or, as vectors, $\vec{E} = \rho \vec{J}$

Resistivity is associated with a **material**, **resistance** with respect to a **device** constructed with the material.

(resistance: $R = V/I$ **)**

Conductivity : $\sigma = \dfrac{1}{\rho}$

Example:



$$E = \frac{V}{L}, \quad J = \frac{i}{A} \quad \rho = \frac{V/L}{i/A} = R\frac{A}{L}$$

$$\boxed{R = \rho \frac{L}{A}}$$

Makes sense!

For a given material:

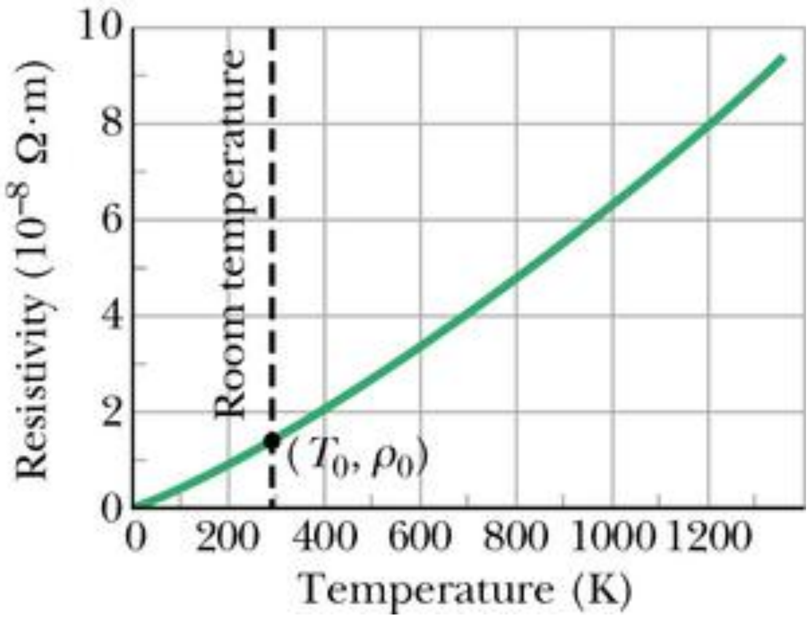
Longer → More resistance

Thicker → Less resistance

Resistivity and Temperature

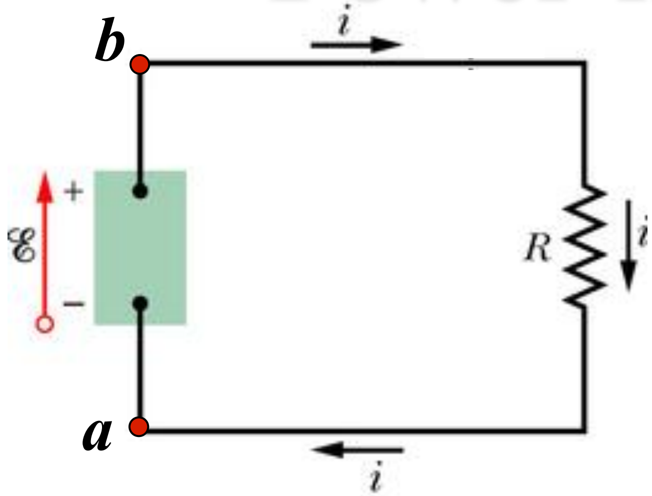
Resistivity depends on temperature:

$$\rho = \rho_0(1+\alpha\,(T-T_0)\,)$$



- At what temperature would the resistance of a copper conductor be double its resistance at 20.0°C?
- Does this same "doubling temperature" hold for all copper conductors, regardless of shape or size?

Power in electrical circuits



A battery "pumps" charges through the resistor (or any device), by producing a potential difference V between points a and b. How much work does the battery do to move a small amount of charge dq from b to a?

$dW = -dU = -dq\ V = (dq/dt)\ dt\ V = iV\ dt$

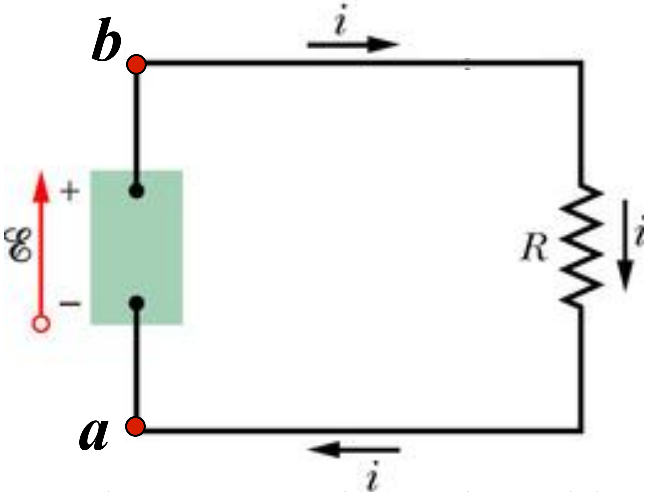
The battery "power" is the work it does <u>per unit time</u>:

$$P = dW/dt = iV$$

$P = iV$ is true for the battery pumping charges through any device. If the device follows Ohm's law (i.e., it is a resistor), then $V = iR$ and

$$P = iV = i^2R = V^2/R$$

Emf devices and single loop circuits

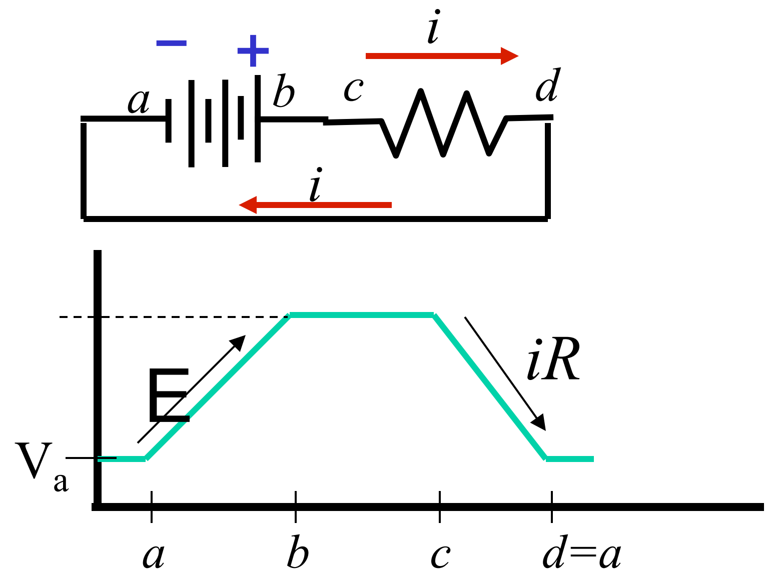


The battery operates as a "pump" that moves positive charges from lower to higher electric potential. A battery is an example of an "electromotive force" (EMF) device.

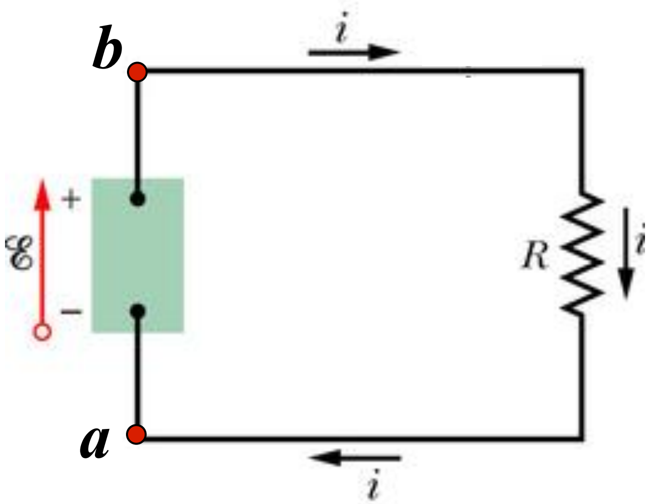
These come in various kinds, and all transform one source of energy into electrical energy. A battery uses chemical energy, a generator mechanical energy, a solar cell energy from light, etc.

The difference in potential energy that the device establishes is called the EMF and denoted by E.

$$V_a + E - iR = V_a \qquad E = iR$$



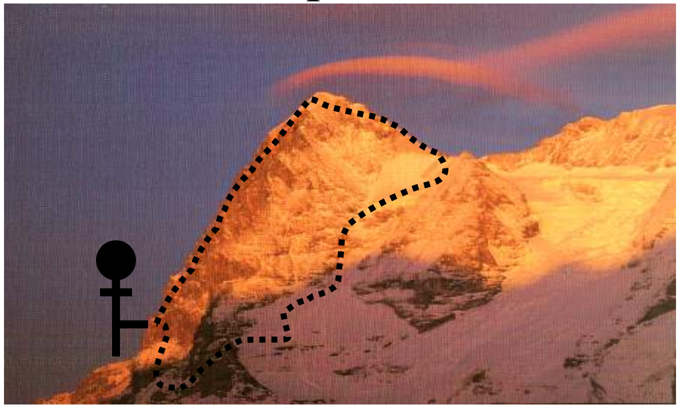
Circuit problems



Given the emf devices and resistors in a circuit, we want to calculate the circulating currents. Circuit solving consists in "taking a walk" along the wires. As one "walks" through the circuit (in any direction) one needs to follow two rules:

When walking through an EMF, add +E if you flow with the current or − E otherwise. How to remember: current "gains" potential in a battery.

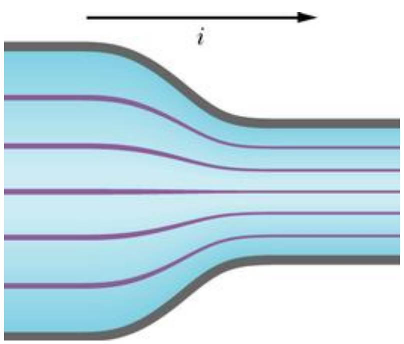
When walking through a resistor, add $-iR$, if flowing with the current or $+iR$ otherwise. How to remember: resistors are passive, current flows "potential down".



Example:
Walking clockwise from a: $+ E - iR = 0$.
Walking counter-clockwise from a: $- E + iR = 0$.

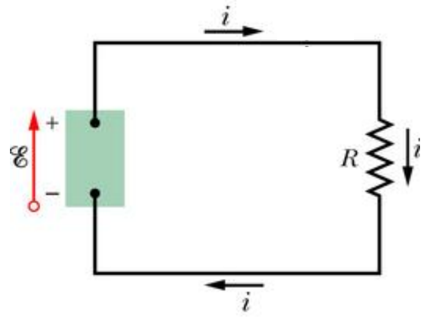
Summary

- Current and current density:
  i=dq/dt; $i=\int \mathbf{J}\cdot d\mathbf{A}$; $J=nev_d$



- Resistance and resistivity:
  $V= iR$; $E=J\rho$; $R=\rho\, L/A$; $\rho=\rho_0(1+\alpha(T-T_0))$

- Power: $P=iV= (V^2/R = i^2R)$



- Walking a circuit: $\mathcal{E}-iR=0$

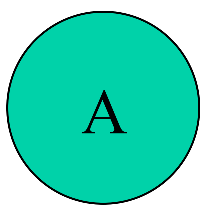
Example

A human being can be electrocuted if a current as small as 50 mA passes near the heart. An electrician working with sweaty hands makes good contact with the two conductors he is holding. If his resistance is 1500 Ω , what might the fatal voltage be?

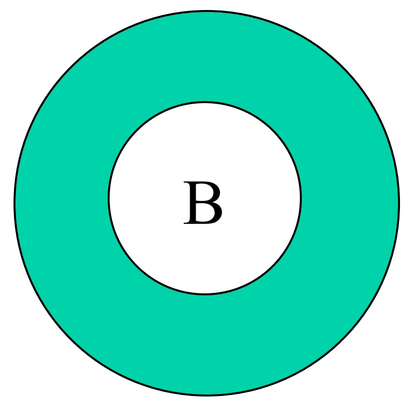
(Ans: 75 V)

Example

Two conductors are made of the same material and have the same length. Conductor *A* is a solid wire of diameter 1.0 mm. Conductor *B* is a hollow tube of outside diameter 2.0 mm and inside diameter 1.0 mm. What is the resistance ratio R_A/R_B, measured between their ends?



$R = \rho L/A$



$A_A = \pi r^2$

$A_B = \pi\left((2r)^2 - r^2\right) = 3\pi r^2$

$R_A/R_B = A_B/A_A = 3$

Example

A 1250 W radiant heater is constructed to operate at 115 V.

(a) What will be the current in the heater?
(b) What is the resistance of the heating coil?
(c) How much thermal energy is produced in 1.0 h by the heater?

- Formulas: $P=i^2R=V^2/R$; $V=iR$
- Know P, V; need R to calculate current!
- $P=1250W$; $V=115V \Rightarrow R=V^2/P=(115V)^2/1250W=10.6\ \Omega$
- $i=V/R= 115V/10.6\ \Omega=10.8\ A$
- Energy? $P=dU/dt \Rightarrow dU=P\ dt = 1250W\times 3600\ sec= 4.5\ MJ$

Example

A 100 W lightbulb is plugged into a standard 120 V outlet.

(a) What is the resistance of the bulb?

(b) What is the current in the bulb?

(c) How much does it cost per month to leave the light turned on continuously? Assume electric energy costs 6¢/kW·h.

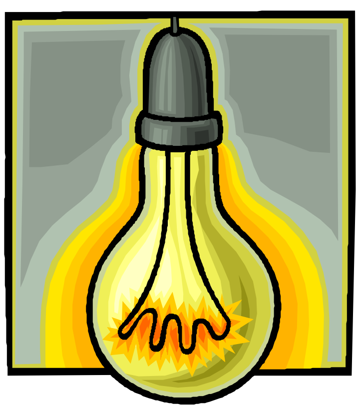
(d) Is the resistance different when the bulb is turned off?

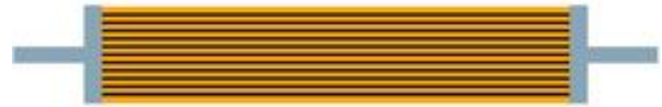
- Resistance: same as before, $R=V^2/P=144\ \Omega$
- Current, same as before, $i=V/R=0.83$ A
- We pay for energy used (kW h):

  $U=Pt=0.1\text{kW} \times (30\times 24)\text{ h} = 72\text{ kW h} \Rightarrow \$4.32$
- (d): Resistance should be the same, but it's not: **resistivity and resistance increase with temperature**. When the bulb is turned off, it is colder than when it is turned on, so the resistance is lower.

Incandescent light bulbs

(a) Which light bulb has a smaller resistance: a 60W, or a 100W one?
(b) Is the resistance of a light bulb different when it is on and off?
(c) Which light bulb has a larger current through its filament: a 60W one, or a 100 W one?
(d) Would a light bulb be any brighter if used in Europe, using 240 V outlets?
(e) Would a US light bulb used in Europe last more or less time?
(f) Why do light bulbs mostly burn out when switched on?





Example

An electrical cable consists of 105 strands of fine wire, each having 2.35 Ω resistance. The same potential difference is applied between the ends of all the strands and results in a total current of 0.720 A.

(a) What is the current in each strand?
[0.00686] A

(b) What is the applied potential difference?
[1.61e-08] V

(c) What is the resistance of the cable?
[2.24e-08]