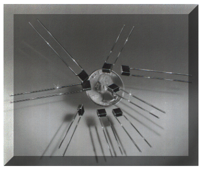
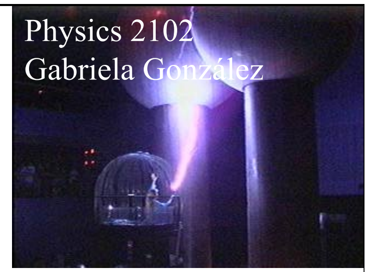




Physics 2102

Capacitors

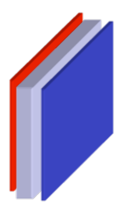
Summary (last class)

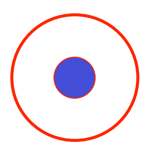
- Any two charged conductors form a capacitor.
- Capacitance : C= Q/V
- Simple Capacitors:

  Parallel plates: $C = \varepsilon_0 A/d$

  Spherical : $C = \varepsilon_0 4\pi ab/(b-a)$

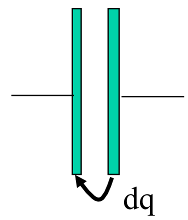
  Cylindrical: $C = \varepsilon_0 2\pi L/\ln(b/a)$





Energy Stored in a Capacitor

- **Start out with uncharged capacitor**
- **Transfer small amount of charge dq from one plate to the other until charge on each plate has magnitude Q**
- **How much work was needed?**



$$U = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \boxed{\frac{Q^2}{2C} = \frac{CV^2}{2}}$$

Energy Stored in Electric Field

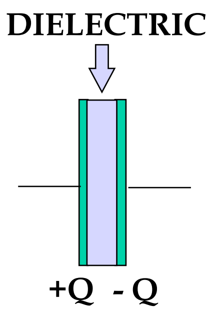
- **Energy stored in capacitor: $U = Q^2/(2C) = CV^2/2$**
- **View the energy as stored in ELECTRIC FIELD**
- **For example, parallel plate capacitor:**
  Energy DENSITY = energy/volume = u =

$$U = \frac{Q^2}{2CAd} = \frac{Q^2}{2\left(\frac{\varepsilon_0 A}{d}\right)Ad} = \frac{Q^2}{2\varepsilon_0 A^2} = \frac{\varepsilon_0}{2}\left(\frac{Q}{\varepsilon_0 A}\right)^2 = \boxed{\frac{\varepsilon_0 E^2}{2}}$$

volume = Ad

General expression for any region with vacuum (or air)

Dielectric Constant

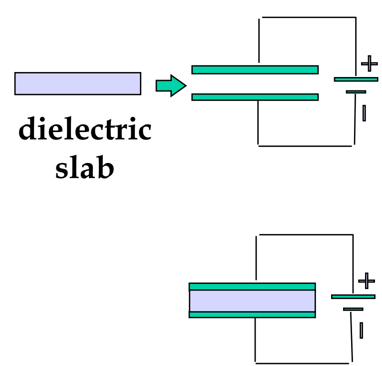


$C = \kappa\varepsilon_0 A/d$

- If the space between capacitor plates is filled by a dielectric, the capacitance **INCREASES** by a factor κ
- This is a useful, working definition for dielectric constant.
- Typical values of κ: 10 - 200

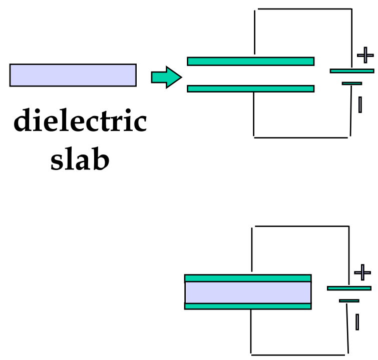
Example

- Capacitor has charge Q, voltage V
- Battery remains connected while dielectric slab is inserted.
- Do the following increase, decrease or stay the same:
  - Potential difference?
  - Capacitance?
  - Charge?
  - Electric field?



Example (soln)

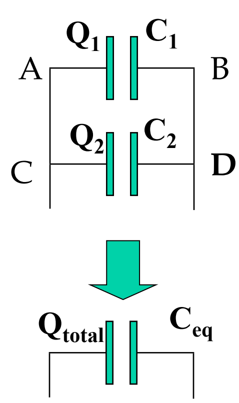
- Initial values:
  capacitance = **C**; charge = **Q**;
  potential difference = **V**;
  electric field = **E**;
- Battery remains connected
- V is FIXED; $\mathbf{V_{new} = V}$ (**same**)
- $\mathbf{C_{new} = \kappa C}$ (**increases**)
- $\mathbf{Q_{new} = (\kappa C)V = \kappa Q}$ (**increases**).
- Since $\mathbf{V_{new} = V, E_{new} = E}$ (**same**)



Energy stored? $u = \varepsilon_0 E^2/2 \Rightarrow u = \kappa\varepsilon_0 E^2/2 = \varepsilon E^2/2$

Capacitors in Parallel

- A wire is a conductor, so it is an equipotential.
- Capacitors in parallel have SAME potential difference but NOT ALWAYS same charge.
- $V_{AB} = V_{CD} = V$
- $Q_{total} = Q_1 + Q_2$
- $C_{eq}V = C_1V + C_2V$
- $\mathbf{C_{eq} = C_1 + C_2}$
- **Equivalent parallel capacitance = sum of capacitances**



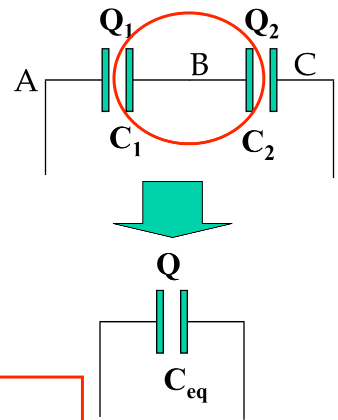
PARALLEL:
- V is same for all capacitors
- Total charge in C_{eq} = sum of charges

Capacitors in series

- $Q_1 = Q_2 = Q$ (WHY??)
- $V_{AC} = V_{AB} + V_{BC}$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

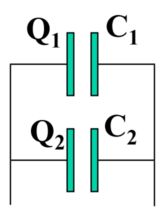
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

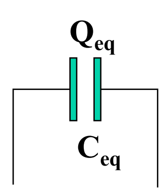


SERIES:
- Q is same for all capacitors
- Total potential difference in C_{eq} = sum of V

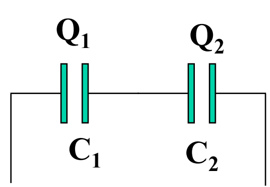
Capacitors in parallel and in series

- **In parallel :**
  - $C_{eq} = C_1 + C_2$
  - $V_{eq} = V_1 = V_2$
  - $Q_{eq} = Q_1 + Q_2$





- **In series :**
  - $1/C_{eq} = 1/C_1 + 1/C_2$
  - $V_{eq} = V_1 + V_2$
  - $Q_{eq} = Q_1 = Q_2$



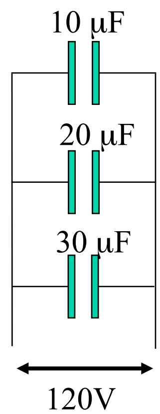
Summary

- Capacitors in series and in parallel:
  - in series: charge is the same, potential adds, equivalent capacitance is given by $1/C=1/C_1+1/C_2$
  - in parallel: charge adds, potential is the same, equivalent capaciatnce is given by $C=C_1+C_2$.
- Energy in a capacitor: $U=Q^2/2C=CV^2/2$; energy density $u=\varepsilon_0 E^2/2$
- Capacitor with a dielectric: capacitance increases $C'=\kappa C$

Example 1

What is the charge on each capacitor?

- $Q = CV$; $V = 120$ V
- $Q_1 = (10\ \mu F)(120V) = 1200\ \mu C$
- $Q_2 = (20\ \mu F)(120V) = 2400\ \mu C$
- $Q_3 = (30\ \mu F)(120V) = 3600\ \mu C$

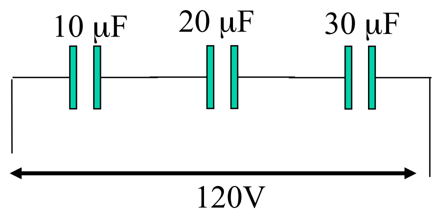


Note that:
- Total charge (7200 μC) is shared between the 3 capacitors in the ratio $C_1:C_2:C_3$ -- i.e. 1:2:3

Example 2

What is the potential difference across each capacitor?

- $Q = CV$; Q is same for all capacitors
- Combined C is given by:



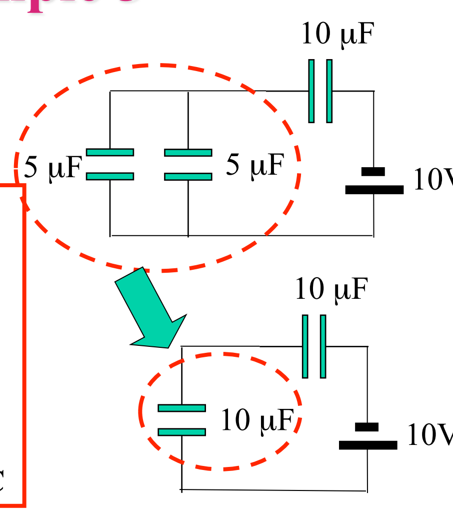
$$\frac{1}{C_{eq}} = \frac{1}{(10\mu F)} + \frac{1}{(20\mu F)} + \frac{1}{(30\mu F)}$$

- $C_{eq} = 5.46\ \mu F$
- $Q = CV = (5.46\ \mu F)(120V) = 655\ \mu C$
- $V_1 = Q/C_1 = (655\ \mu C)/(10\ \mu F) = 65.5\ V$
- $V_2 = Q/C_2 = (655\ \mu C)/(20\ \mu F) = 32.75\ V$
- $V_3 = Q/C_3 = (655\ \mu C)/(30\ \mu F) = 21.8\ V$

Note: 120V is shared in the ratio of INVERSE capacitances i.e.1:(1/2):(1/3)
(largest C gets smallest V)

Example 3

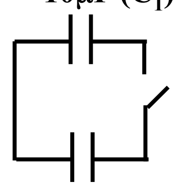
In the circuit shown, what is the charge on the 10μF capacitor?



- The two **5μF** capacitors are in parallel
- Replace by **10μF**
- Then, we have two **10μF** capacitors in series
- So, there is 5V across the **10μF** capacitor of interest
- Hence, Q = (**10μF**)(5V) = **50μC**

Example

- **10μF capacitor is initially charged to 120V. 20μF capacitor is initially uncharged.**
- **Switch is closed, equilibrium is reached.**
- **How much energy is dissipated in the process?**



Initial charge on 10μF = (10μF)(120V)= 1200μC

After switch is closed, let charges = Q_1 and Q_2.

Charge is conserved: $Q_1 + Q_2 = 1200\mu C$

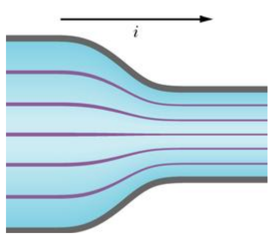
Also, V_{final} is same: $\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ⇨ $Q_1 = \frac{Q_2}{2}$ ⇨

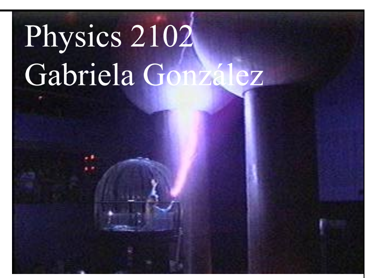
- $Q_1 = 400\mu C$
- $Q_2 = 800\mu C$
- $V_{final} = Q_1/C_1 = 40$ V

Initial energy stored = $(1/2)C_1V_{initial}^2 = (0.5)(10\mu F)(120)^2 = 72mJ$

Final energy stored = $(1/2)(C_1 + C_2)V_{final}^2 = (0.5)(30\mu F)(40)^2 = 24mJ$

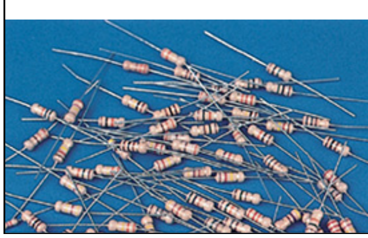
Energy lost (dissipated) = 48mJ





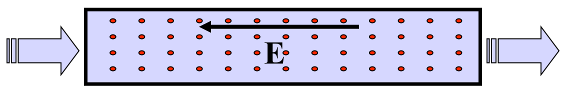
Physics 2102

Current and resistance



Georg Simon Ohm
(1789-1854)

Electrical current



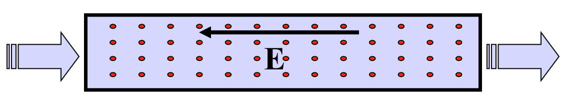
In a conductor, electrons are free to move. If there is a field E inside the conductor, F=qE means the electrons move in a direction opposite to the field: this is an electrical current.

We think of current as motion of imaginary positive charges along the field directions.

$$i = \frac{dq}{dt}, \qquad q = \int i \, dt \quad \textbf{Units : [i]} = \frac{\textbf{Coulomb}}{\textbf{second}} \equiv \textbf{Ampere}$$

Andre-Marie Ampere 1775-1836

Electrical current



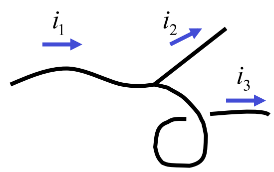
Wasn't the field supposed to be zero inside conductors?

Yes, if the charges were in *equilibrium*. The reasoning was "electrons move until they cancel out the field". If the situation is not static, that is, if electrons are moving, then the field can be nonzero in a conductor, and the potential is not constant across it!

However, "somebody" has to be pumping the electrons: this is the job of the battery we put across a circuit. If there is no source creating the electric field, the charges reach equilibrium at E=0.

Electrical current: Conservation

Current is a scalar, NOT a vector, although we use arrows to indicate direction of propagation.
Current is conserved, because charge is conserved!



$$i_1 = i_2 + i_3$$

"junction rule": everything that comes in, must go out.

Resistance

Electrons are not "completely free to move" in a conductor. They move erratically, colliding with the nuclei all the time: this is what we call "resistance".

The resistance is related to the potential we need to apply to a device to drive a given current **through** it. The larger the resistance, the larger the potential we need to drive the same current.

Ohm's laws

$$R \equiv \frac{V}{i} \quad \text{and therefore: } i = \frac{V}{R} \quad \text{and } V = iR$$

Georg Simon Ohm (1789-1854)

"a professor who preaches such heresies is unworthy to teach science." Prussian minister of education 1830

$$\text{Units: } [\text{R}] = \frac{\text{Volt}}{\text{Ampere}} \equiv \text{Ohm (abbr. } \Omega)$$

Devices specifically designed to have a constant value of R are called resistors, and symbolized by