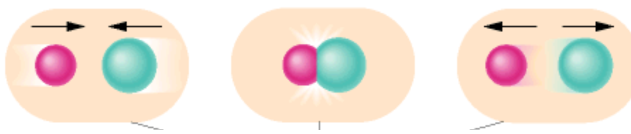
Conservation of momentum
Collisions

Phys 2101

Gabriela González



Linear momentum

- Single particle: $\vec{p} = m\vec{v}$ $\quad \dfrac{d\vec{p}}{dt} = \vec{F}_{net} = m\vec{a}$
- Several particles: $\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$

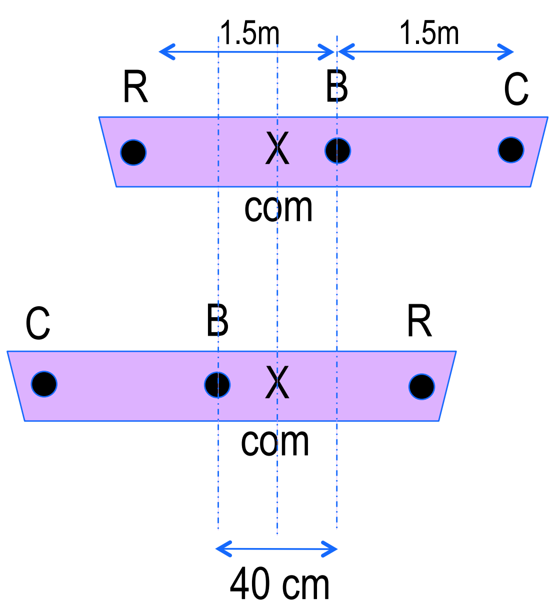
$$\frac{d\vec{P}}{dt} = M\vec{a}_{com} = \vec{F}_{ext}$$

If F_{net} =0, momentum is <u>conserved</u>

Example

Ricardo, mass 95 kg, and Carmelita, who is lighter, are enjoying Lake Merced at dusk in a 35 kg canoe. When the canoe is at rest in the placid water, they exchange seats, which are 3.0 m apart and symmetrically located with respect to the canoe's center. Ricardo notices that the canoe moved 40 cm relative to a submerged log during the exchange and calculates Carmelita's weight, which she has not told him. What is it?

Ricardo, mass 95 kg, and Carmelita, who is lighter, are enjoying Lake Merced at dusk in a 35 kg canoe. When the canoe is at rest in the placid water, they exchange seats, which are 3.0 m apart and symmetrically located with respect to the canoe's center. Ricardo notices that the canoe moved 40 cm relative to a submerged log during the exchange and calculates Carmelita's weight, which she has not told him. What is it?



We know that the center of mass is 20cm from the center of the boat, closer to Ricardo.
Choosing a coordinate system with the origin at the center of the boat in the original position, the coordinate of the center of mass is:

$$x_{com} = \frac{m_R x_R + m_B x_B + m_C x_C}{m_R + m_B + m_C}$$

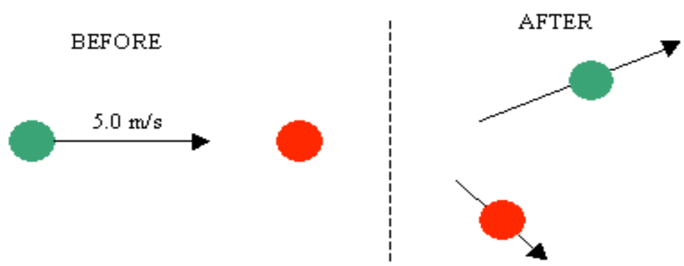
$$-0.2m = \frac{95kg \times (-1.5m) + 30kg \times (0m) + m_C(1.5m)}{95kg + 30kg + m_C}$$

$$-0.2m \times (125kg + m_C) = -142.5kgm + 1.5m \times m_C$$

$$142.5kgm - 25kgm = (1.5m + 0.2m)m_C$$

$$m_C = \frac{117.5kgm}{1.7m} = 69kg$$

A 2.0 kg ball is moving east at 5.0 m/s. It collides with an identical ball at rest, and the first ball moves off at 30° N of E. the second ball moves away 50° S of E. Find the magnitudes of the velocities of each ball after the collision.



$\vec{p}_i = 10 \text{ kgm/s } \hat{i}$

$\vec{p}_f = 10\text{kg}v_1\left(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}\right) + 10\text{kg}v_2\left(\cos(-50^\circ)\hat{i} + \sin(-50^\circ)\hat{j}\right)$

$\vec{p}_i = \vec{p}_f \Rightarrow$

$$0 = v_1 \sin 30^\circ - v_2 \sin 50^\circ$$

$$1 \text{ m/s} = v_1 \cos 30^\circ + v_2 \cos 50^\circ$$

$$v_2 = v_1 \frac{\sin 30^\circ}{\sin 50^\circ}$$

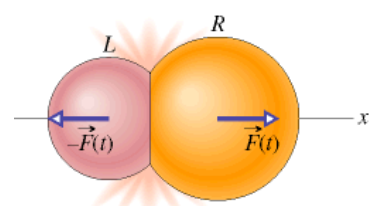
$$1 \text{ m/s} = v_1\left(\cos 30^\circ + \frac{\sin 30^\circ}{\sin 50^\circ}\cos 50^\circ\right)$$

$$v_1 = \frac{1 \text{ m/s}}{\cos 30^\circ + \dfrac{\sin 30^\circ}{\sin 50^\circ}\cos 50^\circ} = 0.78 \text{ m/s}$$

$$v_2 = v_1 \frac{\sin 30^\circ}{\sin 50^\circ} = 0.51 \text{ m/s}$$

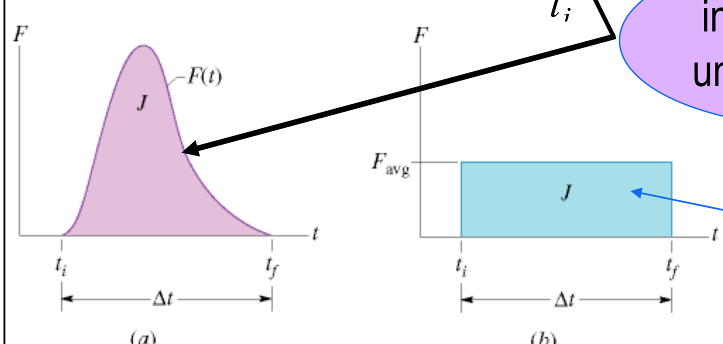
Momentum, force and impulse

Change in momentum = "impulse"



$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\, dt = \vec{J}$$

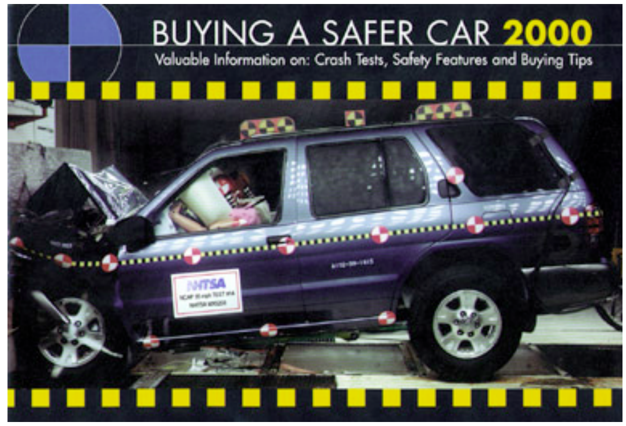
integral = area under the curve



$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\, dt \Rightarrow J = F_{avg}\Delta t$$

Example

The National Transportation Safety Board is testing the crash-worthiness of a new car. The 2300 kg vehicle, moving at 15 m/s, is allowed to collide with a bridge abutment, which stops it in 0.56 s. What is the magnitude of the average force that acts on the car during the impact?

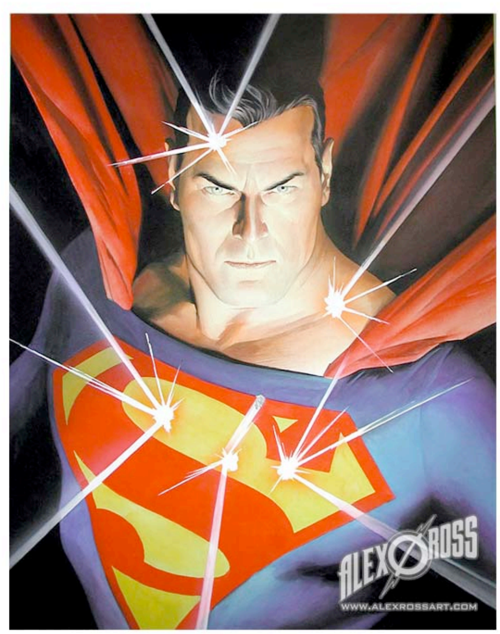


National Highway Traffic Safety Administration:

http://www.safercar.gov/

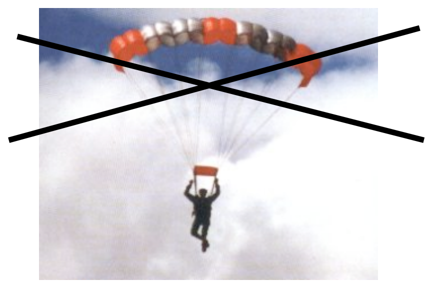
Superman

It is well known that bullets and other missiles fired at Superman simply bounce off his chest. Suppose that a gangster sprays Superman's chest with 10 g bullets at the rate of 100 bullets/min, the speed of each bullet being 700 m/s. Suppose too that the bullets rebound straight back with no change in speed. What is the magnitude of the average force on Superman's chest from the stream of bullets?



Example

In February 1955, a paratrooper fell 370 m from an airplane without being able to open his chute but happened to land in snow, suffering only minor injuries. Assume that his speed at impact was 56 m/s (terminal speed), that his mass (including gear) was 85 kg, and that the magnitude of the force on him from the snow was at the survivable limit of 1.2×10^5 N. What are (a) the minimum depth of snow that would have stopped him safely and (b) the magnitude of the impulse on him from the snow?

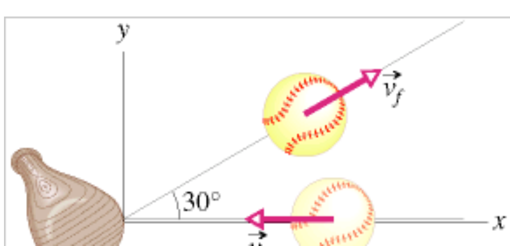


Example

A pitched 140 g baseball, in horizontal flight with a speed v_i of 39.0 m/s, is struck by a bat. After leaving the bat, the ball has a speed v_f of 45.0 m/s at an upward angle of 30.0° .

(a) What impulse J acts on the ball while it is in contact with the bat during the collision?

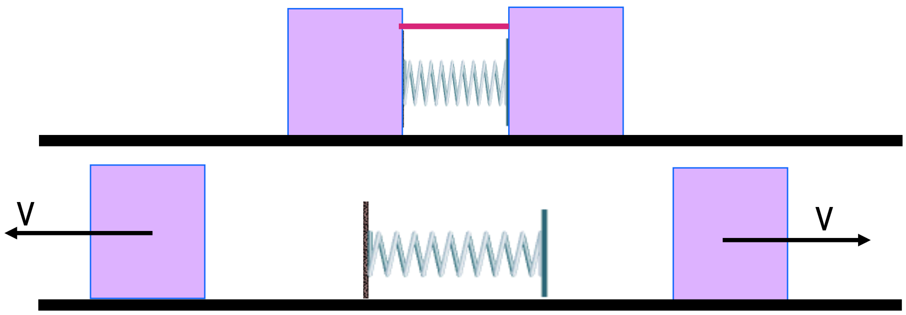
(b) The impact time Δt for the baseball–bat collision is 1.20 ms. What average force acts on the baseball?



Energy conservation

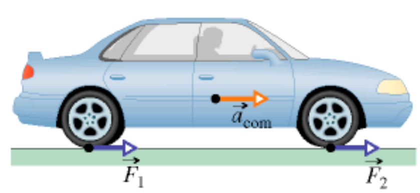
Consider two equal masses, at rest, held by a compressed spring. If the clamp holding is broken, the two masses fly apart from each other with the same speed.

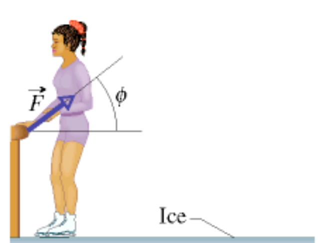
- Is momentum conserved?
- Is kinetic energy conserved?
- Is total energy conserved?

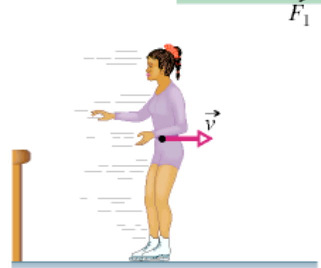


Conservation of energy

In general, when the spring is “hidden”, we call the equivalent of its stored potential energy, the “internal energy”. Sometimes, the work of the force on one of the objects in the system is easier to identify, and then it is an “external” force (to one of the objects). In general, there must be forces working if energy is changing!



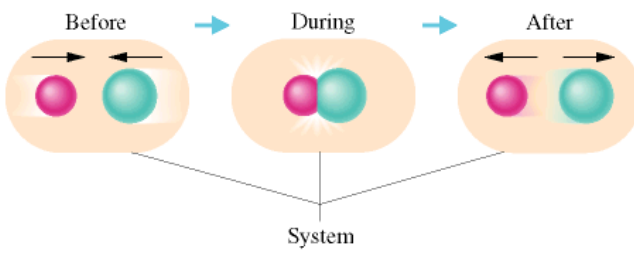




Collisions

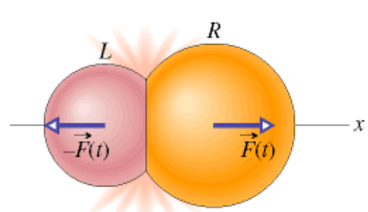
A **collision** is an isolated event in which two or more bodies (the colliding bodies) exert relatively strong forces on each other for a relatively short time.

The rules of the game are the laws of conservation of momentum and of energy.



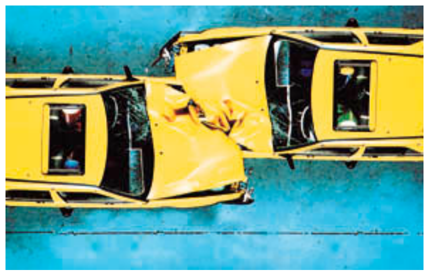
Collisions

In a collision, the system has a certain kinetic energy and a certain linear momentum before the collision. During the collision, the kinetic energy and linear momentum of **each body** are changed by the impulse from **the other body**. We will only consider collisions in systems that are **closed** (no mass enters or leaves them) and **isolated** (no net external forces act on the bodies within the system).



Elastic and Inelastic collisions

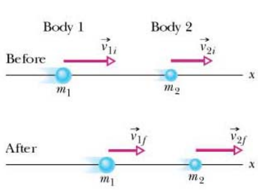
- Elastic collisions: kinetic energy is conserved
- Inelastic collisions: kinetic energy is not conserved
- The **total** linear momentum is always conserved!
- Before and after the collision, problems use conservation of energy or work-energy theorem as before.

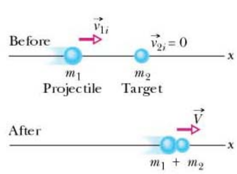


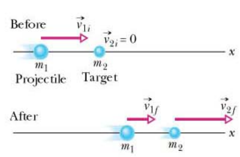
Inelastic collisions

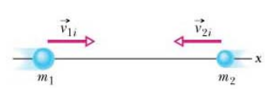
Momentum is conserved:

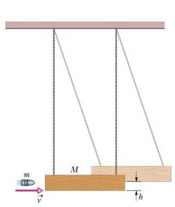
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

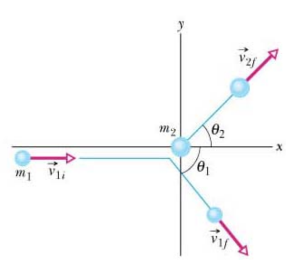












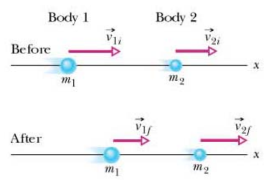
Elastic collisions: a special case

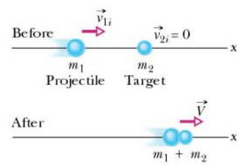
Momentum AND Energy is conserved:

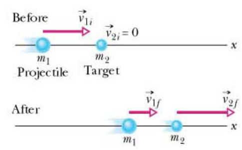
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

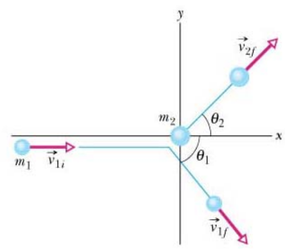
Collisions in a horizontal plane (energy=kinetic energy):

$$m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

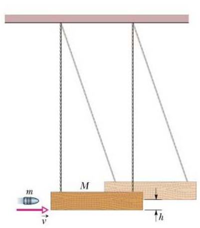








Example: ballistic pendulum

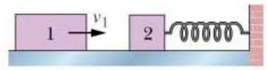


$$mv = (m+M)V$$

$$\frac{1}{2}(m+M)V^2 = (m+M)gh \Rightarrow V = \sqrt{2gh}$$

$$v = \frac{(m+M)V}{m} = \frac{(m+M)}{m}\sqrt{2gh}$$

Example



Initially, block 2 is at rest, spring is unstretched.
Block 1 collides with Block 2, and they stick together.
When the blocks momentarily stop, by what distance is the spring compressed?

$$m_1 v_i = (m_1 + m_2) v_f$$

$$v_f = v_{\max}$$

$$\frac{1}{2}(m_1 + m_2) v_{\max}^2 = \frac{1}{2} k x_{\max}^2$$

$$\Rightarrow x_{\max} = \sqrt{\frac{(m_1 + m_2)}{k}} v_{\max} = \sqrt{\frac{(m_1 + m_2)}{k}} \frac{m_1}{(m_1 + m_2)} v_i = \sqrt{\frac{m_1^2}{k(m_1 + m_2)}} v_i$$