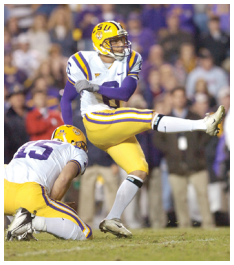
Lecture 2: Motion in 2-D Newton's laws

Phys 2101
Gabriela González



Motion in ~~one~~ two, three… dimension

- Position is a vector, function of time:

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

- Velocity:

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} \quad \Rightarrow \quad \vec{r}(t) = \int_0^t \vec{v}(t)\,dt$$

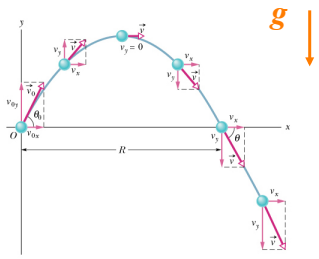
- Acceleration:

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2} \quad \Rightarrow \quad \vec{v}(t) = \int_0^t \vec{a}(t)\,dt$$

2

2-D motion: Projectile motion

Motion in two dimensions, with constant acceleration:



$$\vec{a}(t) = -g\,\hat{j}$$

$$\vec{v}(t) = \vec{v}_0 - gt\hat{j}$$

$$\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t - \frac{1}{2}gt^2\hat{j}$$

3

Projectile motion

A football player punts the football so that it will have a "hang time" (time of flight) of 5 s and land 50m away. If the ball leaves the player's foot 150cm above the ground, what must be the magnitude and direction of the ball's initial velocity?

A football player punts the football so that it will have a “hang time” (time of flight) of 5 s and land 50m away. If the ball leaves the player’s foot 150cm above the ground, what must be the magnitude and direction of the ball’s initial velocity?

Newton’s laws

Physics having predictive value!

Newton’s laws: two qualitative, one quantitative:

1. If no net force, no acceleration (either at rest, or moving with constant velocity)
2. $\vec{F} = m\,\vec{a}$
3. Action and reaction forces are equal and opposite.

6

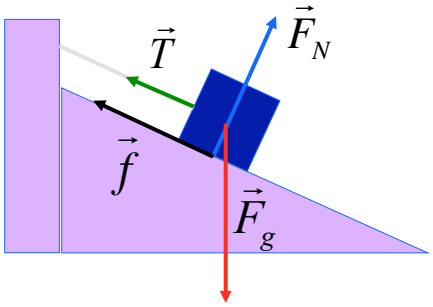
Some special forces

Gravitational force: vertical, pointing down, from center of gravity.

Normal force: perpendicular to the surface, making total force into the surface zero.

Tension: reaction force produced by a string

Friction force: produced by the surface, parallel to the surface, opposing motion.

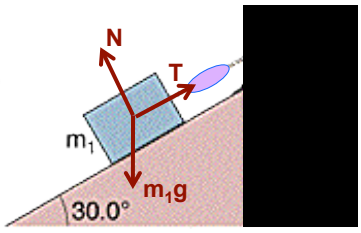


7

$\vec{F} = m\vec{a}$ is all you need!

The block m_1 does not move.

- If m_1=10 kg, what does the spring scale read?
- What are the magnitude and direction of the normal force?

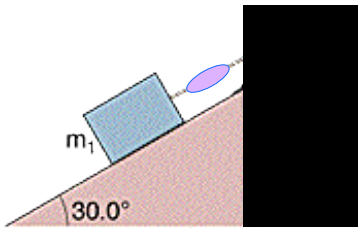


8

The block m_1 does not move.
a) If m_1=10 kg, what does the spring scale read?
b) What are the magnitude and direction of the normal force?

$\vec{F}= m\vec{a}$ is all you need!

The block m_1=10 kg is held in equilibrium by the rope. What is the tension if μ_s=0.6? And if μ_s=0.1?

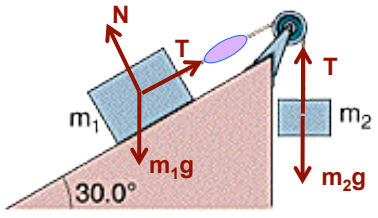


If there is friction, it has be to static, so it is whatever it needs to be to keep the body at rest, up to a maximum value of $\mu_s N$.
If there is no friction and no tension, the body wants to go down the incline, so the friction force will oppose attempted motion, and will point **up** the incline.

10

The block m_1=10 kg is held in equilibrium by the rope. What is the tension if μ_s=0.6? And if μ_s=0.1?

$\vec{F}= m\vec{a}$ is all you need!



- If m_1=m_2 =10kg, and there is no friction, do the objects move?
- What is now the reading in the spring scale?
- If there is friction, do the answers change?

12

a) If $m_1=m_2$ =10kg, and there is no friction, do the objects move?
b) What is now the reading in the spring scale?
c) What is the tension if μ_s=0.6? And if it is 0.05?

a) If $m_1=m_2$ =10kg, and there is no friction, do the objects move?
b) What is now the reading in the spring scale?
c) What is the tension if μ_s=0.6? And if it is 0.05?

Free body diagrams! First, assume a direction for a and solve for its value: is it ipositive, negative or zero?

The distance one mass moves up the incline is the same distance the other mass moves down, so the magnitude (but not direction!) of the acceleration of the two masses is the same.

-- First, no friction: Assume m_1 moves down the incline, m_2 moves up.
F=ma for block on the incline, forces along the incline, down positive

$$-T+m_1 g \sin\theta=m_1 a \quad (1)$$

Forces perpendicular to the incline, up positive:

$$N-m_1 g \cos\theta=0 \quad (2)$$

(no acceleration in this direction!), so N=85N like before.

F=ma for block that moves up and down, with *up* as positive (since we took down the incline as positive!):

$$-m_2 g+T=m_2 a \quad (3)$$

If we add (1) and (3), we can eliminate the tension:

$$(m_1 \sin\theta-m_2)\, g=(m_1+m_2)a \rightarrow a=g(m_1 \sin\theta - m_2)/(m_1+m_2) \quad (4)$$

So a=g(5kg-10kg)/(10kg+5kg)=-g/3=-3.3m/s^2, not zero! The blocks are moving, but in the opposite direction than we assumed: m_1 moves up the incline, m_2 moves down.
The tension can now be solved from (1) or (3): $T=m_2(a+g)=m2*(2g/3)=65N$.

Note: If $m_1=m_2$, according to (4) the acceleration is never zero, unless $\sin\theta=1$, or $\theta=90^o$: the two masses are hanging vertically. For the acceleration to be zero, m_1 has to be larger than m_2, *and* the incline has to have an angle $\sin\theta=m_2/m_1$. If the angle is larger than that, the hanging block moves up; if the angle is smaller than that, the hanging block moves down. Our formula for acceleration changes sign for different angles (or masses) to indicate the different directions.

a) If $m_1=m_2$ =10kg, and there is no friction, do the objects move?
b) What is now the reading in the spring scale?
c) What is the tension if μ_s=0.5 and μ_k=0.3? And if μ_s=0.05 and μ_k=0.03?

--Now with friction:
In the absence of friction, the block along the incline is moving up, so friction force will point down the incline, same as the gravity component. We choose up the incline as positive now, so a is zero or positive.

$$(1) \quad T-f-m_1 g\sin\theta=m_1 a$$

We still have the same equation for mass m_2:

$$(2) \quad m_2 g-T=m_2 a$$

We cannot solve this, we have three unknowns (f, T, a) and two equations… we also need to know whether frictions is static or kinetic. Can the system be in equilibrium? If so, a=0, and the friction force is static.

If a=0, $T=m_2 g$=9.8N from (2), and static friction force from (1) is $f_s=T-m_1 g \sin\theta$=4.9N.
The maximum possible value for the static friction force is $f_s=\mu_s N=\mu_s$ 85N, so for μ_s=0.5, this is 42.5N, and for μ_s=0.05, it is 4.25: the larger friction will hold the system in equilibrium, the smaller friction will let it slide.

If μ_s=0.5 the system is then in equilibrium, a=0, and T=98N.

If μ_s=0.05, the system is not in equilibrium, and the friction is kinetic friction, so $f=\mu_k N=\mu_k m_1 g\cos\theta$, so equations (1) and (2) are now:

$$(1) \quad T-m_1 g(\mu_k\cos\theta+\sin\theta)=m_1 a$$
$$(2) \quad m_2 g-T=m_2 a$$

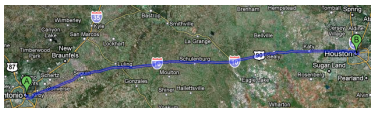
From which we can solve for tension and acceleration:

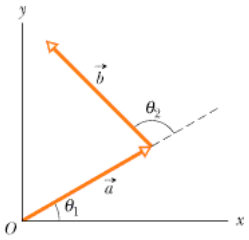
$$a=g(m_2-m_1(\mu_k\cos\theta+\sin\theta))/(m_1+m_2)=0.24\ g = 2.32\ m/s^2$$
$$T=m_2(g-a)=74\ N$$

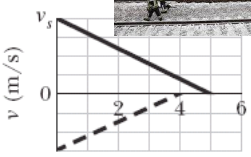
So, m1 moves up the incline with a=3.3 m/s² if there is no friction, with smaller a=2.32 m/s² if there is a little friction, and not at all (a=0) if there is enough friction: the more friction, the less acceleration.
The tension is T=65N if there is no friction, 74 N if there is a little friction, and 98 N if there is enough friction to keep the system in equilibrium: the more friction, the larger the tension.

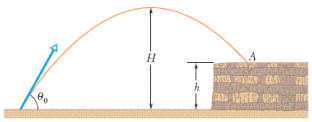
15

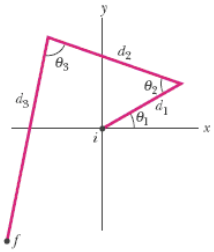
HOMEWORK #1 : CH 2, 3, 4







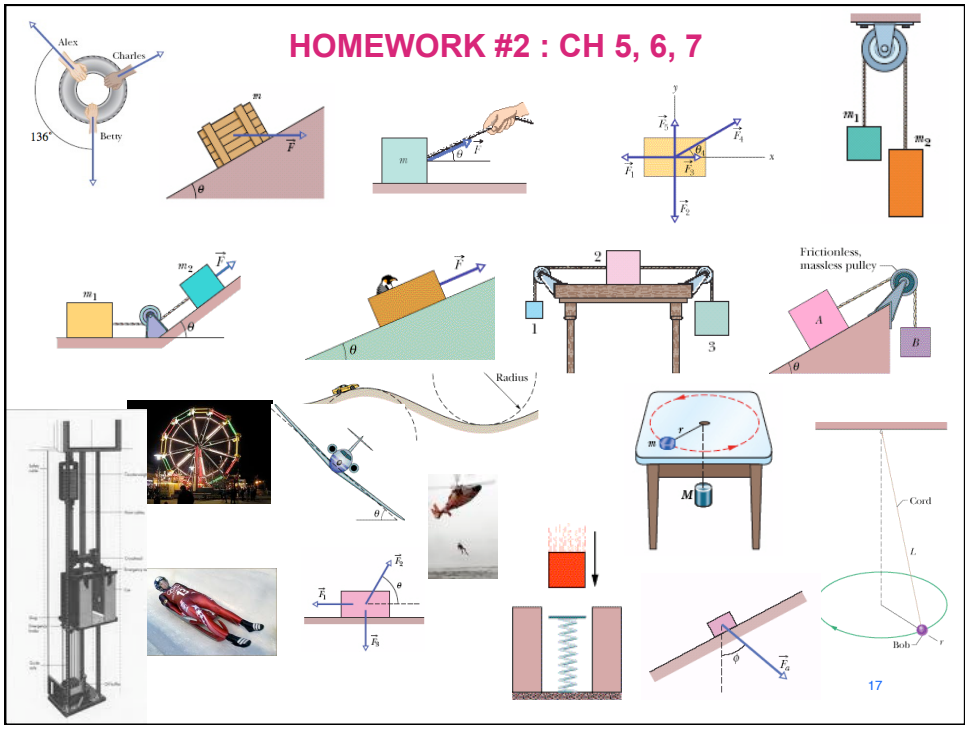




A
B

FIG. 4-61 Problem 119.

16


HOMEWORK #2 : CH 5, 6, 7
Alex
Charles
Betty
136°
Frictionless,
massless pulley
Radius
Cord
Bob
17