Physics 2101, Exam #3, Fall 2009

October 27 , 2009

Name: SOLUTIONS

ID#:______________________

Section: (Circle one)

1 (Chastain, MWF 8:40 AM)

2 (Chastain, MWF 10:40 AM)

3 (Rupnik, MWF 12:40 PM)

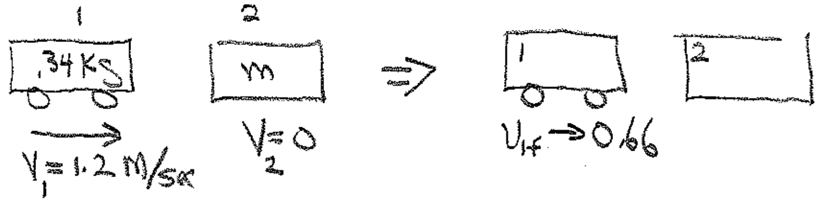
4 (Plummer, TTh 9:10)

5 (Adams, TTh 12:10)

- Please be sure to write (print) your name and circle your section above.
- Please turn OFF your cell phone and MP3 player!
- Feel free to detach, use, and keep the formula sheet. No other reference material is allowed during the exam.
- You may use either a scientific or a graphing calculator...
- GOOD LUCK!

1. (10 pts) A cart with mass 0.34 kg moving on a frictionless air track at an initial speed of 1.2 m/s undergoes an *elastic* collision with a stationary cart on unknown mass. After the collision the first cart continues moving in the original direction with a speed of 0.66 m/s. What is the mass of the second cart?

(a) 1.34 kg
~~(b) 0.34 kg~~
(c) 0.1 kg
~~(d) 0.05 kg~~
(e) zero



$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i} = 0.66 = \frac{.34 - m_2}{.34 + m_2}(1.2)$$

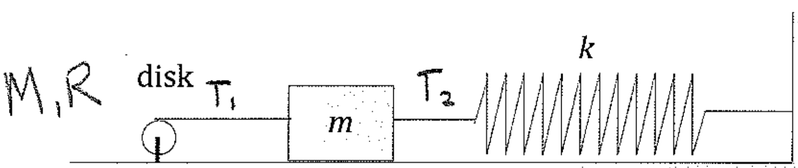
$$m_2 = 0.099 \text{ Kg} = 0.1 \text{ Kg}$$

2. (10 pts) *Show your work on this problem.*
The solid disk shown in the figure has a radius *R* and a mass *M*. One end of the block of mass *m* is connected to a spring of force constant *k*, and the other end is fastened to a chord wrapped around the disk. The disk is initially wound counterclockwise so that the spring stretches a distance *d* from its un-stretched position. The disk is then released. Calculate the acceleration of the block immediately after the disk is released assuming that the table surface is frictionless. Your answer should be in terms of *M*, *m*, *k*, and *d*.

disc

1) $\tau = T_1 R = I\alpha$

$I = \frac{1}{2}MR^2 \quad \alpha = \frac{a}{R}$



2) $T_2 - T_1 = ma$

$T_2 = kd \quad \Rightarrow$

$T_1 = kd - ma$

INTO #1

$$\tau = T_1 R = (kd - ma)R = \frac{1}{2}MR^2\frac{a}{R}$$

$$kd - ma = \frac{1}{2}Ma$$

$$\boxed{a = \frac{kd}{m + \frac{M}{2}} \rightarrow}$$

3. (5pts) Just before a jet shuts down its engines the rotor of one of the engines has an initial angular velocity of 2000 rad/s. The rotor's rotation slows at a rate of 80 rad/s^2. How long does it take for the rotor to come to rest?

(a) 2000 s.
(b) 1000 s.
(c) 200 s.
(d) 100 s
(e) 25 s

$W_f = W_0 - \alpha t$

$\frac{W_f - W_0}{-\alpha} = t = \frac{2000}{80} = 25$ sec

4. The uniform rod in the figure below is released from the horizontal position and then rotates in the plane of the figure about an axis through one end. It has a rotational inertia *I*. As the rod swings through its lowest position, it collides with a putty wad of mass *m* that sticks to the end of the rod. What is the correct answer to the following two questions? (neglect friction)

A. (5 pts) From the instant the rod is released until the instant immediately before the rod hits the putty, which of the following quantities are conserved?

(a) Mechanical Energy

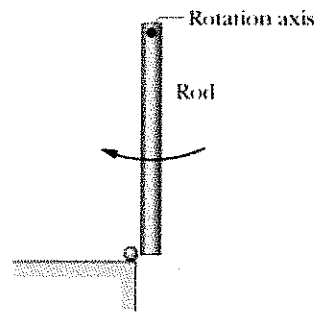
(b) Kinetic energy

(c) Angular Momentum

(d) Both (a) and (c)

(e) (a), (b), and (c)

(f) None of the above are conserved



B. (5 pts) From the instant immediately before the rod hits the putty to the instant just after the putty and rod have stuck together, which of the following quantities are conserved for the system? (Assume the collision occurs over a very short time.)

(a) Kinetic Energy

(b) Torque

(c) Angular Momentum

AND (d) Both (b) and (c)

(e) (a), (b), and (c)

(f) None of the above are conserved

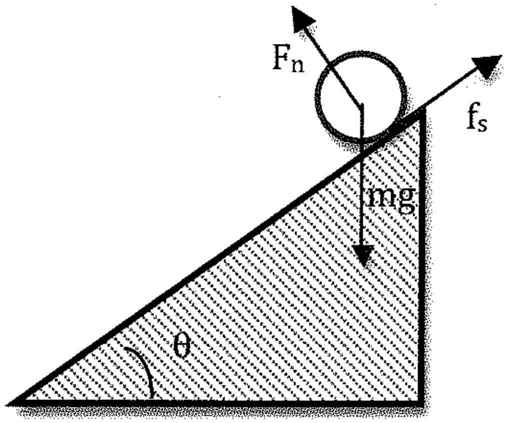
BUT $T = \frac{dL}{dt}$

SO IF $\frac{dL}{dt} = 0$

THEN $T = 0$

SO (d) b+c IS ACCEPTABLE

5. *Show your work.* In the figure below, a solid cylinder of radius R and mass M starts from rest and *rolls without slipping* a distance *L* along the incline.



A) (5pts) What force(s) acting on the cylinder result(s) in a torque about its axis of rotation?

(a) F_n
(b) F_n and mg
(c) f_s
(d) all of them
(e) none of them

B) (15pts) Find translational speed of the cylinder when is reaches the bottom of the incline. Your answer should be in terms of g, L, and θ. (g = acceleration of gravity)

E_{mech} IS CONSERVED

START $E_m = 0$

END $E_m = 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgh$

$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}\right)MR^2\left(\frac{v}{R}\right)^2 - mgL\sin\theta$

$V = \sqrt{\frac{4}{3}gL\sin\theta}$

Rod

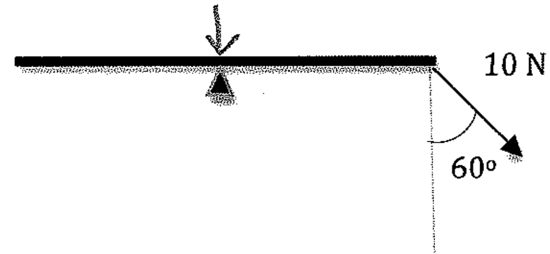
6. *Show your work.* A solid plank is only supported by a centrally placed pivot as shown below. The plank has a length of 1 m and a mass of 5 kg and the pivot is located 0.5 m from the end. A 10 N force is suddenly applied to one end as shown below.

A) (5 pts) What is the net torque on the plank about the pivot point?

$\tau = \vec{r} \times \vec{F}$ INTO PAGE

$= (0.5\text{m})\, 10 \cos 60^\circ$

$= 2.5$ Nm



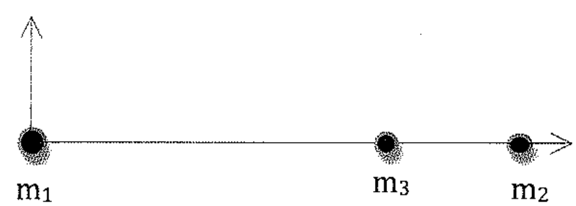
B) (5 pts) What is the angular acceleration of the plank?

$\tau = I\alpha = \frac{1}{12} m L^2 \alpha$

$\alpha = \frac{2.5 \cdot 12}{5\text{kg}\ 1} = 6$ rad sec CLOCK WISE

7. (5 pts) In the figure below $\mathbf{m_1} = 4$ kg, $\mathbf{m_2} = 1$ kg, and $\mathbf{m_3} = 2$ kg. The x-positions are x = 0 ($\mathbf{m_1}$), x = 1.0 m ($\mathbf{m_3}$), and x = 1.5 m ($\mathbf{m_2}$). Find the magnitude of the net force on $\mathbf{m_3}$ due to the gravitation attraction of the other two masses.

(a) zero
~~(b) 5.3 x~~ 10^{-10} N
(c) 10.6 x 10^{-10} N
(d) 6.67 x 10^{-11} N
(e) 2.5 x 10^{-9} N



$F = -\frac{G m_1 m_3}{r_{13}^2} + \frac{G m_2 m_3}{r_{23}^2} = G m_3 \left(\frac{m_2}{(1.5)^2} - \frac{m_1}{1^2} \right)$

$= 0$

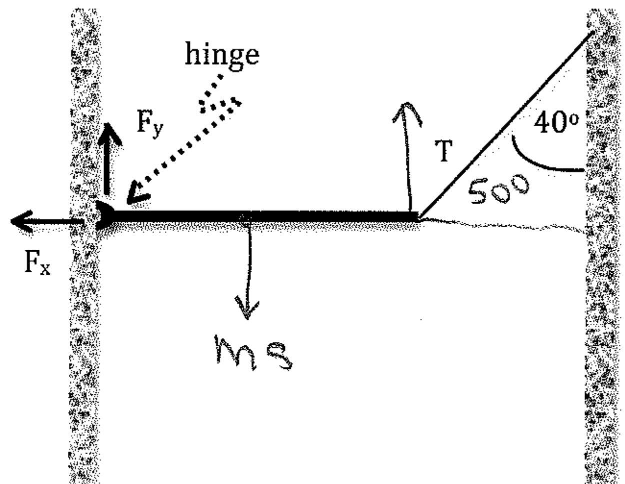
8. (5pts) Which of the following statements is necessarily true for an object in *static equilibrium*?

(a) The net torque on the object is zero.
(b) The net force on the object is zero.
(c) Both (a) and (b)
(d) The frictional forces cancel each other out.
(e) The object is moving in a straight line.

9. A rod of length 2m and mass 5 kg is supported at one end by a hinge and the other end by a cable as shown below. F_x and F_y represent the horizontal and vertical components of the force of the hinge on the rod.

A) (5 pts) Which of the following forces produce a torque about the hinge?

a) mg
b) T
c) F_x
d) F_y
e) both mg and T
f) all of them



B (10pts) What is the magnitude of the tension in the cable?

a) 24.5 N
b) 49 N
c) 32 N
d) 9.8 N
e) none of the above

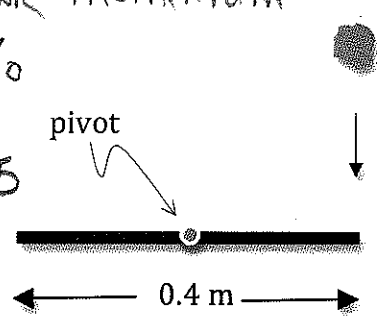
$$2T\cos 40^\circ - mg(1) = 0$$
$$T = \frac{(5kg)(9.8)}{2\cos 40^\circ} = 31.9$$

10. (10 pts) A thin solid rod of length 0.4 m and mass 6 kg is initially at rest, but is free to rotate in a vertical plane without friction about a horizontal axis through its center. A 2 kg wad of wet putty drops onto one end of the rod and sticks to it. The system then begins to rotate with an angular velocity of 5 rad/s. What was the speed of the putty just before it hit the rod?

CONSERVATION ANGULAR MOMENTUM

(a) 4 m/s
(b) 2 m/s
(c) 1 m/s
(d) zero
(e) none of the above

$$I_i = mrv = 2\times 0.2\times V_0$$
$$I_f = I_R\omega + m(.2)^2\omega$$
$$= \left[\frac{1}{12}(0.4)^2 6 + 2(.2)^2\right]5$$
$$I_f = 0.8$$
$$\text{so } V_0 = \frac{I_f}{2(.2)}$$



Formula Sheet for LSU Physics 2101 Exams, Fall '09

Units:

$1\,\mathrm{m} = 39.4\,\mathrm{in} = 3.28\,\mathrm{ft}$ $\quad 1\,\mathrm{mi} = 5280\,\mathrm{ft}$ $\quad 1\,\mathrm{min} = 60\,\mathrm{s}$, $\quad 1\,\mathrm{day} = 24\,\mathrm{h}$ $\quad 1\,\mathrm{rev} = 360° = 2\pi\,\mathrm{rad}$

$1\,\mathrm{atm} = 1.013{\times}10^5\,\mathrm{Pa}$ $\quad 1\,\mathrm{cal} = 4.187\,\mathrm{J}$ $\quad T = \left(\frac{1\,K}{1°\mathrm{C}}\right) T_{\mathrm{C}} + 273.15\,\mathrm{K}$ $\quad T_{\mathrm{F}} = \left(\frac{9\,°\mathrm{F}}{5°\mathrm{C}}\right) T_{\mathrm{C}} + 32°\mathrm{F}$

Constants:

$g = 9.8\,\mathrm{m/s^2}$	$R_{Earth} = 6.37 \times 10^6\,\mathrm{m}$	$M_{Earth} = 5.98{\times}10^{24}\,\mathrm{kg}$
$G = 6.67{\times}10^{-11}\,\mathrm{m^3/(kg{\cdot}s^2)}$	$R_{Moon} = 1.74 \times 10^6\,\mathrm{m}$	$M_{Moon} = 7.36{\times}10^{22}\,\mathrm{kg}$
Earth-Sun distance $= 1.50{\times}10^{11}\,\mathrm{m}$	$M_{Sun} = 1.99{\times}10^{30}\,\mathrm{kg}$	Earth-Moon distance $= 3.82{\times}10^8\,\mathrm{m}$
$k = 1.38 \times 10^{-23}\,\mathrm{J/K}$	$R = 8.31\,\mathrm{J/(mol{\cdot}K)}$	Avogadro's # $= 6.02 \times 10^{23}$ particles/mol

Properties of H_2O:

Density:	$\rho_{\mathrm{water}} = 1000\ \mathrm{kg/m^3}$	
Specific heat:	$c_{\mathrm{water}} = 4187\ \mathrm{J/(kg\ K)}$	$c_{\mathrm{ice}} = 2220\ \mathrm{J/(kg\ K)}$
Heats of transformation:	$L_{\mathrm{vaporization}} = 2.256 \times 10^6\ \mathrm{J/kg}$	$L_{\mathrm{fusion}} = 3.33 \times 10^5\ \mathrm{J/kg}$

Quadratic formula: for $ax^2 + bx + c = 0$, $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Magnitude of a vector: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot Product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = |\vec{a}|\,|\vec{b}|\cos(\phi)$ (ϕ is smaller angle between $\vec{a}$ and $\vec{b}$)

Cross Product: $\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{\imath} + (a_z b_x - a_x b_z)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$, $\quad |\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\sin(\phi)$

Equations of Constant Acceleration:

linear equation along x	missing	missing	rotational equation
$v_x = v_{ox} + a_x t$	$x - x_o$	$\theta - \theta_o$	$\omega = \omega_o + \alpha_x t$
$x - x_o = v_{ox} t + \frac{1}{2} a_x t^2$	v_x	ω	$\theta - \theta_o = \omega_o t + \frac{1}{2}\alpha t^2$
$v_x^2 = v_{ox}^2 + 2a_x(x - x_o)$	t	t	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$
$x - x_o = \frac{1}{2}(v_{ox} + v_x)t$	a_x	α	$\theta - \theta_o = \frac{1}{2}(\omega_o + \omega)t$
$x - x_o = v_x t - \frac{1}{2} a_x t^2$	v_{ox}	ω_o	$\theta - \theta_o = \omega t - \frac{1}{2}\alpha t^2$

Vector Equations of Motion for Constant Acceleration: $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a} t^2$, $\quad \vec{v} = \vec{v}_o + \vec{a} t$

Projectile Motion: (with + direction pointing up from Earth)

$$x - x_o = (v_o \cos\theta_o)t \qquad y - y_o = (v_o \sin\theta_o)t - \frac{1}{2} g t^2$$

$$v_x = v_o \cos\theta_o \qquad v_y = (v_o \sin\theta_o) - gt$$

$$v_y^2 = (v_o \sin\theta_o)^2 - 2g(y - y_o) \qquad y = (\tan\theta_o)x - \frac{g x^2}{2(v_o\cos\theta_o)^2} \qquad R = \frac{v_o^2 \sin(2\theta_o)}{g}$$

Newton's Second Law: $\sum \vec{F} = m\vec{a}$

Uniform circular motion: $F_c = \dfrac{mv^2}{r} = m a_c \quad T = \dfrac{2\,\pi\,r}{v}$

Force of Friction: Static: $f_s \le f_{s,max} = \mu_s F_N$, Kinetic: $f_k = \mu_k F_N$

Elastic (Spring) Force: Hooke's Law $F = -kx$ (k = spring (force) constant)

Kinetic Energy (nonrelativistic): Translational $K = \frac{1}{2} m v^2$

Work:

$W = \vec{F} \cdot \vec{d}$ (constant force), $W = \int_{x_i}^{x_f} F(x)dx$ (variable 1-D force), $W = \int_{r_i}^{r_f} \vec{F}(\vec{r}) \cdot d\vec{r}$ (variable 3-D force)

Work - Kinetic Energy Theorem: $W = \Delta K = K_f - K_i$ where W is the *net* work

Work done by weight (gravity close to the Earth surface): $W = m\,\vec{g}\cdot\vec{d}$

Work done by a spring force ($F = -kx$): $W = -k\int_{x_i}^{x_f} x\,dx = -k\left(\frac{x_f^2}{2} - \frac{x_i^2}{2}\right)$

Power:

Average: $P_{avg} = \frac{W}{\Delta t}$, $P = \vec{F}\cdot\vec{v}_{avg}$ (const. force) Instantaneous: $P = \frac{dW}{dt}$, $P = \vec{F}\cdot\vec{v}$ (const. force)

Potential En. Change: $\Delta U = -W$ (only conservative force) **Potential-Force Relation:** $F(x) = -\frac{dU(x)}{dx}$

Gravitational (near Earth) Potential Energy: $U(y) = mgy$ (at the height y)

Elastic (Spring) Potential Energy: $U = \frac{1}{2}kx^2$ (relative to the relaxed spring)

Mechanical Energy: $E_{mec} = K + U$

Change of Mech. Energy due to non-conservative forces: $W_{\mathrm{nc}} = (K_f + U_f) - (K_i + U_i)$

Conservation of energy: $W_{\mathrm{net,ext}} = \Delta K + \Delta U + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}}$, where $W_{\mathrm{net,ext}}$ is the *net, external* work done on the system, and $\Delta E_{\mathrm{th}} = -W_{f_k} = (f_k d$ for constant friction), or $W_{\mathrm{net,ext}} + K_i + U_i - \Delta E_{th} = K_f + U_f$

Center of mass: $M = \sum_{i=1}^{N} m_i$, $x_{\mathrm{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$, $y_{\mathrm{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$, $z_{\mathrm{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$

$$\vec{r}_{\mathrm{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i \qquad \vec{v}_{\mathrm{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{v}_i \qquad \vec{a}_{\mathrm{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{a}_i = \frac{1}{M}\sum_{i=1}^{N}\vec{F}_i$$

Definition of Linear Momentum: one particle: $\vec{p} = m\vec{v}$, system of particles: $\vec{P} = \sum_{i=1}^{N}\vec{p}_i = M\vec{v}_{\mathrm{com}}$

Newton's 2$^{\mathrm{nd}}$ Law for a System of Particles: $\vec{F}_{\mathrm{net}} = M\vec{a}_{\mathrm{com}} = \frac{d\vec{P}}{dt}$

Conservation of Linear Momentum of an Isolated System: $\sum\vec{p}_i = \sum\vec{p}_f$

Impulse - Linear Momentum Theorem: $\Delta\vec{p}_1 = \vec{J}_{12} = \int_{t_1}^{t_2}\vec{F}_{12}(t)dt = \vec{F}_{\mathrm{avg},12}\Delta t$

Elastic Collision (1 Dim): $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}$ $\quad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$

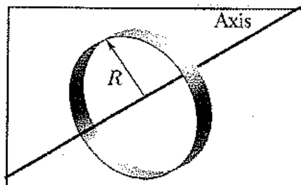
Linear and Angular Variables Related:

$s = r\theta \quad v = \omega r \quad a_t = \alpha r \quad a_r = \frac{v^2}{r} = \omega^2 r$ (magnitude of the radial or centripetal acceleration)

Rotation: Rotational Inertia (I_{com}) for Simple Shapes: see next page

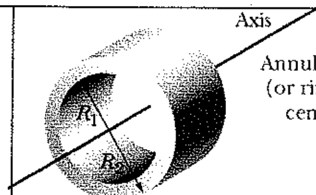
Rotational Inertia:	Descrete particles: $I = \sum_{i=1}^{N} I_i$	Continuous object: $I = \int r^2 dm$
Parallel Axis Theorem:	$I = I_{\mathrm{com}} + Mh^2$	
Torque:	$\vec{\tau} = \vec{r}\times\vec{F}$	$\tau = rF_t = r_{\perp}F = rF\sin\phi$
Angular Momentum:	rigid body, fixed axis: $\vec{L} = I\vec{\omega}$	point-like particle: $\vec{L} = \vec{r}\times\vec{p}$
Newton's 2$^{\mathrm{nd}}$ Law:	$\vec{\tau}_{\mathrm{net}} = I\vec{\alpha}$	$\vec{\tau}_{\mathrm{net}} = \frac{d\vec{L}}{dt}$
Conservation Law	(isolated system, $\sum\tau = 0$):	$\sum\vec{L}_i = \sum\vec{L}_f$
Rotational Work:	$W = \int_{\theta_i}^{\theta_f}\tau d\theta = \tau_{avg}\Delta\theta$	Kinetic Energy: $K = \frac{1}{2}I\omega^2$
Rotational Power:	Instantaneous: $P = \frac{dW}{dt} = \tau\omega$	Average: $P_{avg} = \frac{W}{t} = \tau_{avg}\omega_{avg}$
Rolling:	$v_{\mathrm{com}} = \omega R$	$a_{\mathrm{com}} = \alpha R$
	Kinetic Energy of Rolling:	$K = \frac{1}{2}mv_{\mathrm{com}}^2 + \frac{1}{2}I_{\mathrm{com}}\omega^2$

⇒ *Some Rotational Inertias through COM* ⇐



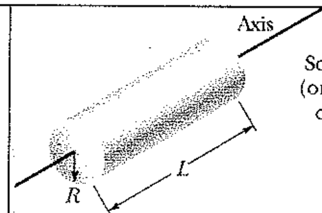
Hoop about central axis

$$I = MR^2$$



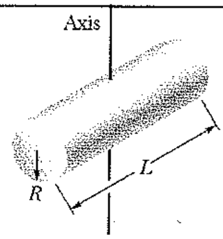
Annular cylinder (or ring) about central axis

$$I = \tfrac{1}{2}M\left(R_1^2 + R_2^2\right)$$



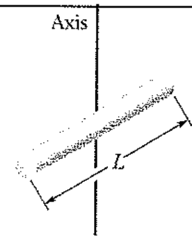
Solid cylinder (or disk) about central axis

$$I = \tfrac{1}{2}MR^2$$



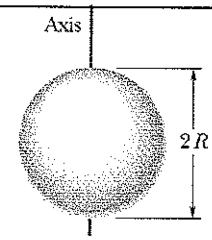
Solid cylinder (or disk) about central diameter

$$I = \tfrac{1}{4}MR^2 + \tfrac{1}{12}ML^2$$



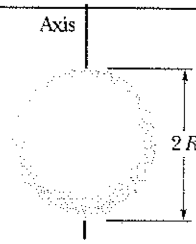
Thin rod about axis through center perpendicular to length

$$I = \tfrac{1}{12}ML^2$$



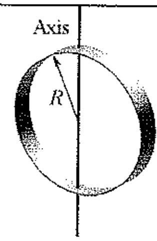
Solid sphere about any diameter

$$I = \tfrac{2}{5}MR^2$$



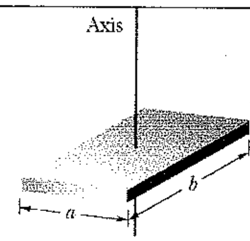
Thin spherical shell about any diameter

$$I = \tfrac{2}{3}MR^2$$



Hoop about any diameter

$$I = \tfrac{1}{2}MR^2$$



Slab about perpendicular axis through center

$$I = \tfrac{1}{12}M\left(a^2 + b^2\right)$$

Static equilibrium: $\vec{F}_{net} = 0 \qquad \vec{\tau}_{net} = 0$

Gravity:

Newton's law: $|\vec{F}| = G\dfrac{m_1 m_2}{r^2}$

Gravitational acceleration (planet of mass M): $a_g = \dfrac{GM}{r^2}$

Law of periods: $T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3$

Potential Energy: $U = -G\dfrac{m_1 m_2}{r}$

Potential Energy of a System (more than 2 masses): $U = -\left(G\dfrac{m_1 m_2}{r_{12}} + G\dfrac{m_1 m_3}{r_{13}} + G\dfrac{m_2 m_3}{r_{23}} + ...\right)$

Static Fluids:

Density: $\rho = \dfrac{\Delta m}{\Delta V}$ Pressure: $p = \dfrac{\Delta F}{\Delta A}$

Hydrostatic Pressure: $p = p_o + \rho g h$

Pressure Variation with Height or Depth: $p_2 = p_1 + \rho g(y_1 - y_2)$

Archimedes' Principle: $F_b = \rho_f V_{\text{displaced}} g = m_f g$

$\text{weight}_{\text{apparent}} = mg - F_b$