Physics 2101, Exam 2

February 19, 2008

Name: KEY

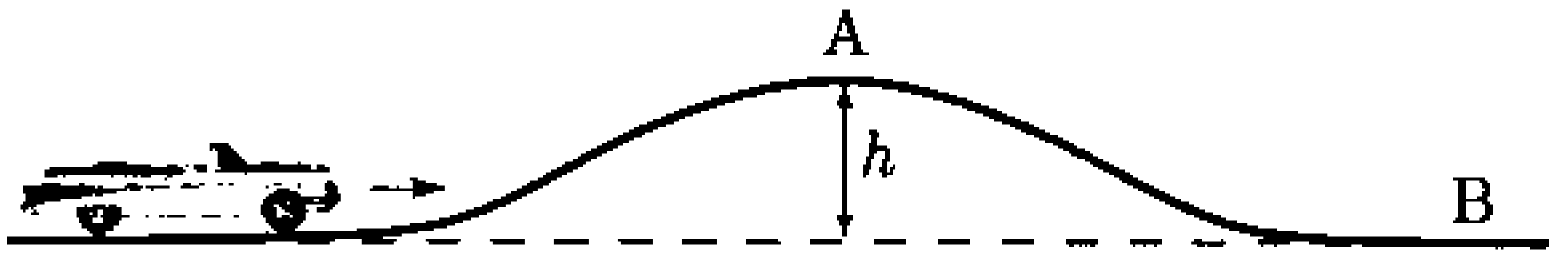
Section: (Circle one)

1 (Rupnik, MWF 8:40am)	4 (Rupnik, MWF 2:40pm)
2 (Giammanco, MWF 10:40am)	5 (Rupnik, TTh 9:10am)
3 (González, MWF 12:40pm)	6 (Sheehy, TTh 12:10pm)

- Please turn OFF your cell phone and MP3 player!
- Feel free to detach, use and keep the formula sheet. No other reference material is allowed during the exam. You may use scientific or graphing calculators.
- For the questions, no work needs to be shown, and there is no partial credit.
- For the problems, please write as much as you can explaining to us your reasoning. Partial credit will be awarded, and correct answers without work shown will not receive full credit.
- Please carry units through your calculations when needed, lack of units will result in a loss of points.
- GOOD LUCK!

Question 1 (10 pts)

The car in the figure is coasting (in neutral, with the engine off) at speed v towards a hill of maximum height h. We are ignoring friction and wind resistance in this problem. The car has mass m, and the acceleration due to gravity is $g = 9.8\frac{m}{s^2}$.



(a) (4 pts) What is the work done by gravity on the car while it travels from its initial position to point A at the top of the hill?

(i) 0.

(ii) mgh.

(iii) $2mgh$.

(iv) $-mgh$.

(v) $-2mgh$

$W_g = -mg\Delta y = -mgh$

$\Delta y = +h$

(b) (3 pts) What is the total work done by gravity on the car while it travels from its initial position to point B on the other side of the hill?

(i) 0.

(ii) mgh.

(iii) $-mgh$.

(iv) $-2mgh$.

(v) $2mgh$

$W_g = 0$ because $\Delta y = 0$

(c) (3 pts) What is the minimum initial speed v such that the car is able to coast to point B?

$mgh = \frac{1}{2}mv_i^2 \Rightarrow v_i = \sqrt{2gh}$

(i) $\sqrt{gh}$.

(ii) $\sqrt{2gh}$.

(iii) $\sqrt{mgh}$.

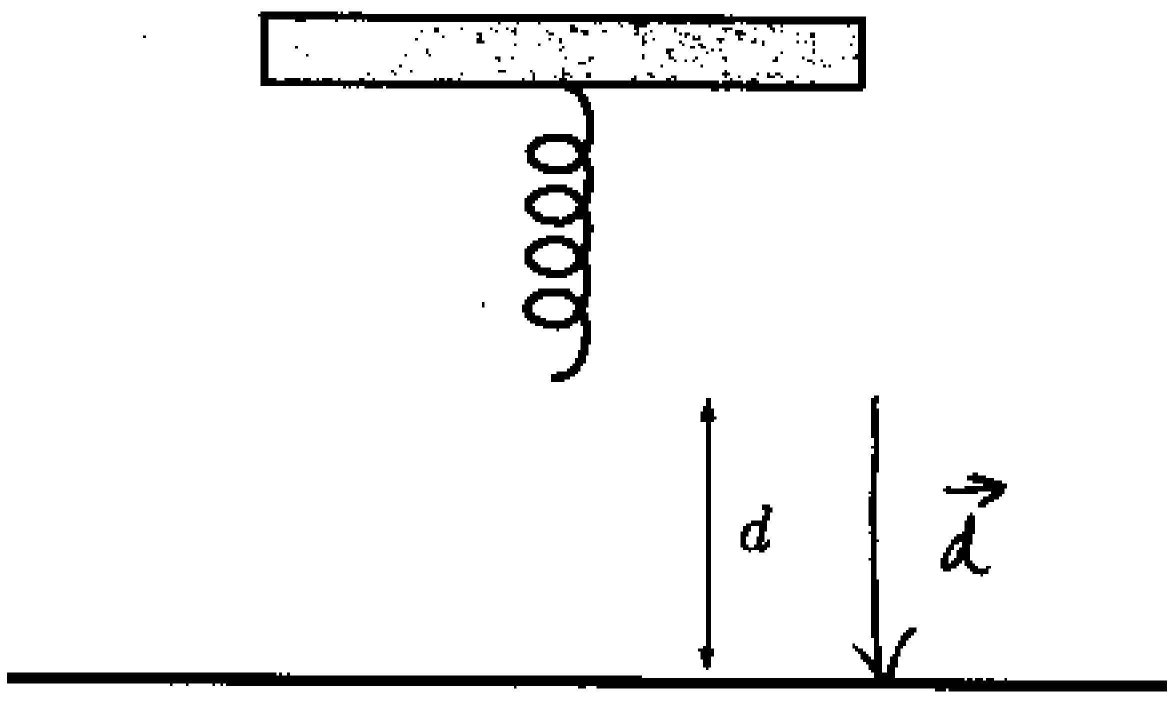
(iv) $\sqrt{4gh}$.

(v) The car will coast to point B with any initial speed.

Problem 1 (20 pts)

The figure shows a hanging spring, with spring constant $k = 23\text{N/m}$. When unstretched, the bottom of the spring is a distance $d = 2.5\text{m}$ from the floor. You gently hang a mass $m = 5\text{kg}$ on the spring and let go.

$k = 23\,N/m$
$d = 2.5\,m$
$m = 5\,kg$
$v_0 = 0$



(a) (5 pts) After you hang the mass, it drops towards, and reaches, the floor. During this motion, what is W_s, the work done by the spring on the mass?

$$W_s = -\frac{k}{2}(x_f^2 - \cancel{x_i^2}^{\,0}) = -\frac{k}{2}(d)^2 = -\frac{23\frac{N}{m}}{2}(2.5m)^2 = -71.875\ J$$

$$W_s \simeq -72\ J$$

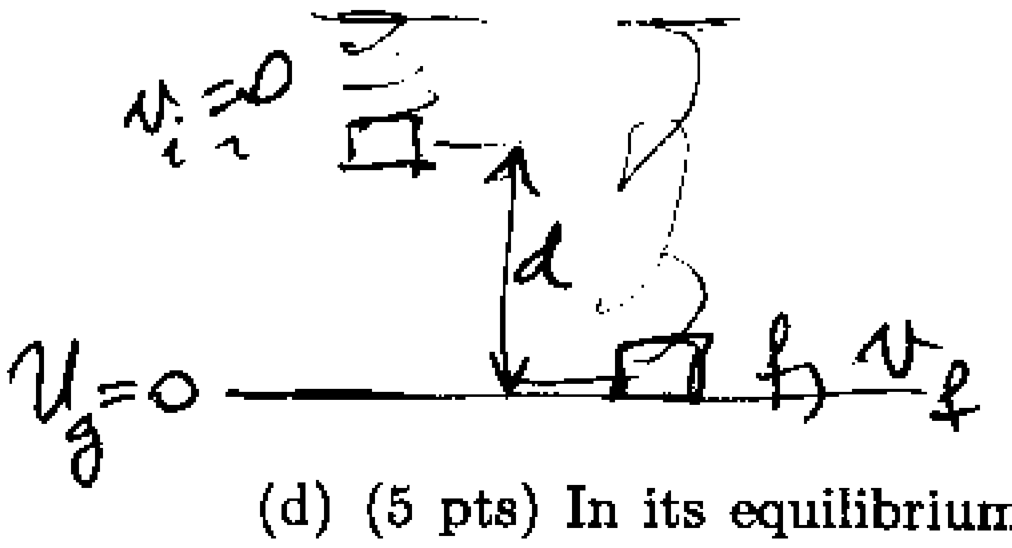
(b) (5 pts) In the motion described in the preceding part, what is W_g, the work done by gravity on the mass?

$$W_g = m\vec{g}\cdot\vec{d} = mgd = (5kg)(9.8\ m/s^2)(2.5m) = +122.5\ J$$

$$W_g \doteq +122.5\ J$$

(c) (5 pts) With what speed does the 5kg mass reach the floor?

$$\boxed{v = 4.5\ m/s}$$

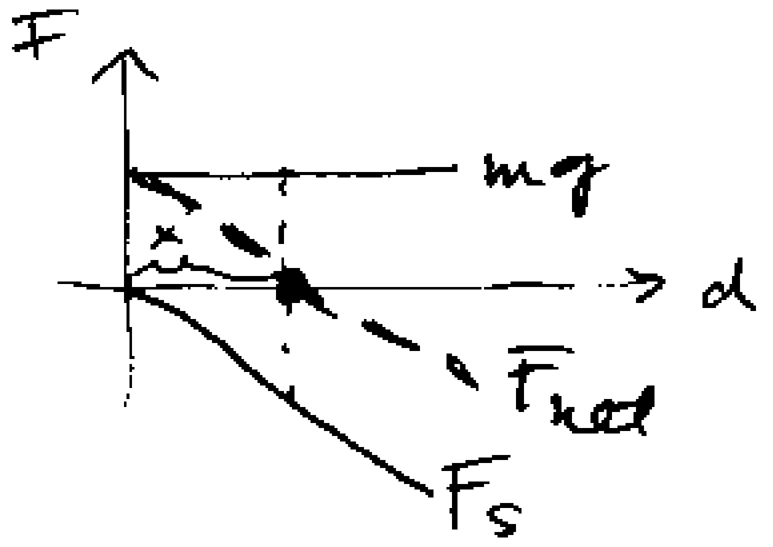


$$(\cancel{K}^0 + U)_i = (K + U)_f$$

$$mgd = \frac{1}{2}mv^2 + \frac{1}{2}kd^2$$

$$v = \sqrt{\frac{2}{m}\underbrace{\left(mgd - \frac{kd^2}{2}\right)}_{(122.5 - 71.875)}} = \sqrt{\frac{2}{2.5}\left[(5)(9.8)(2.5) - \frac{(23)(2.5)^2}{2}\right]} =$$

(d) (5 pts) In its equilibrium position, the 5kg mass stretches the spring a distance x. Determine the distance x.



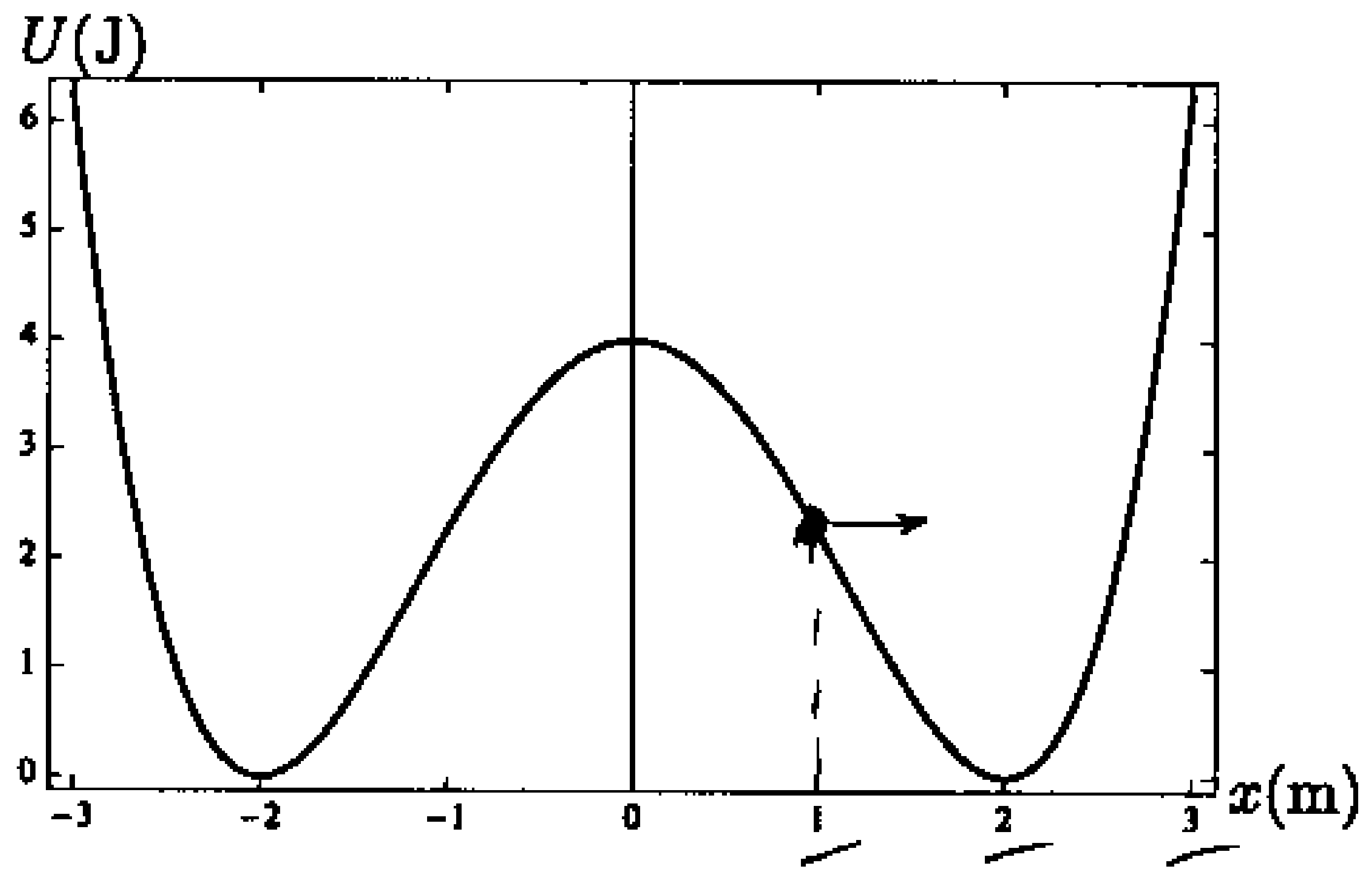
$$mg = kx$$

$$\Rightarrow x = \frac{mg}{k} = \frac{(5kg)(9.8\ m/s^2)}{23\ N/m} = \underline{\underline{2.13\ m}}$$

do not need that!

Question 2 (15 pts)

The potential energy of a particle of mass $m = 3$kg is given by $U(x) = 4 - 2x^2 + \frac{1}{4}x^4$ with U measured in J and x measured in m. $U(x)$ is plotted in the figure.



(a) (5 pts) Which of the following positions is an equilibrium position for the particle?

(i) $x = 1$m
(ii) $x = 2$m
(iii) $x = 3$m
(iv) None of the above.

(b) (5 pts) What is the force on the particle at $x = 0$?

(i) 4N
(ii) -4N
(iii) 0N
(iv) 2N

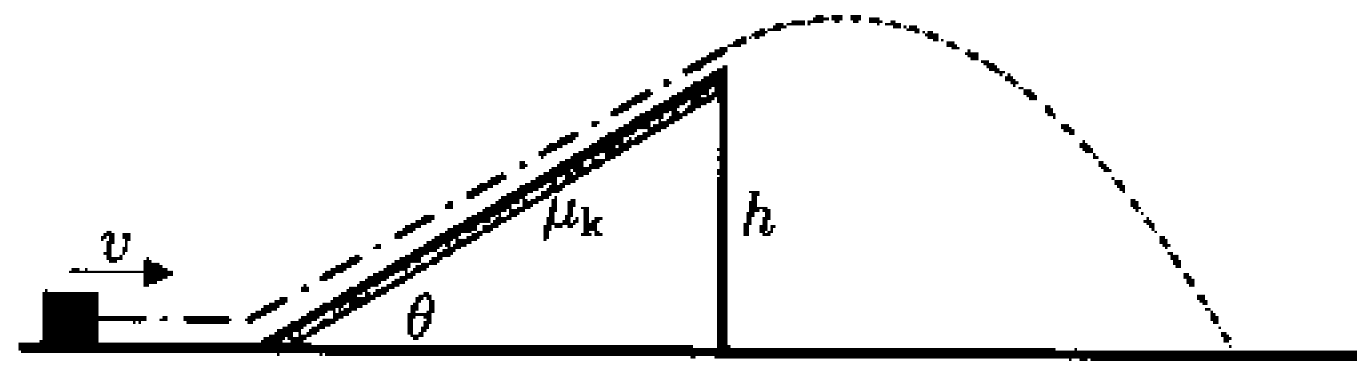
$F = -\frac{dU}{dx} = 0$ for $x = 0$
- slope of a horizontal line

(c) (5 pts) The particle is released from rest at position $x = 1$m. Which of the following statements is *false*:

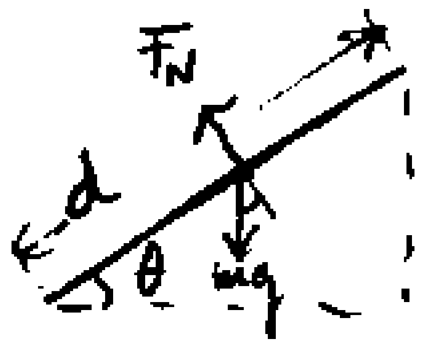
(i) The particle will initially move to the right, but will never reach $x = 3$m. ✓
(ii) The particle will have its maximum speed at $x = 2$m. ✓
(iii) The motion of the particle will have two turning points. ✓
(iv) The particle will always have a positive velocity. FALSE
particle will oscillate back and forth

Problem 2 (20 pts)

The figure shows a block of mass m moving on a horizontal surface towards a stationary triangular wedge. The block reaches the bottom of the wedge with speed v. There is *friction* on the wedge surface, with coefficient of kinetic friction μ_k. The wedge angle is θ and the height from the surface to the upper wedge corner is h. Use work or energy considerations as needed to derive formulas in each part.



(a) (5 pts) Write an expression for the work done by friction on the block during its motion to the top corner of the wedge in terms of the parameters m, μ_k, h, θ and the graviational acceleration constant g.

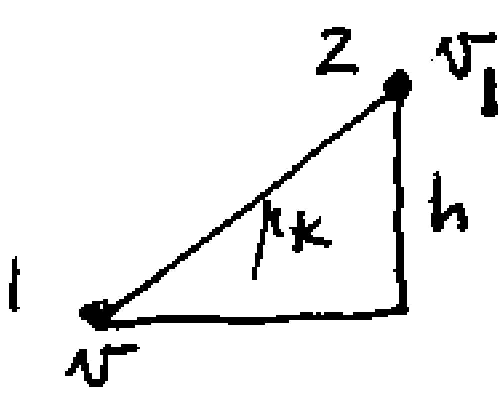


$\sin\theta = \frac{h}{d} \Rightarrow d = \frac{h}{\sin\theta}$ $\quad F_N = mg\cos\theta \quad f_k = \mu_k F_N$

$W_{f_k} = -f_k d = -\mu_k mg\cos\theta \frac{h}{\sin\theta} = -\mu_k mgh\cot\theta$

(b) (5 pts) Assume the block reaches the upper corner of the wedge with speed v_L. Write an expression for v_L in terms of the parameters of the problem, i.e., μ_k, h, θ, v, and g.

using $W = \Delta K$ where $W = W_g + W_{f_k}$ and $W_g = -mgh$ for $1 \to 2$



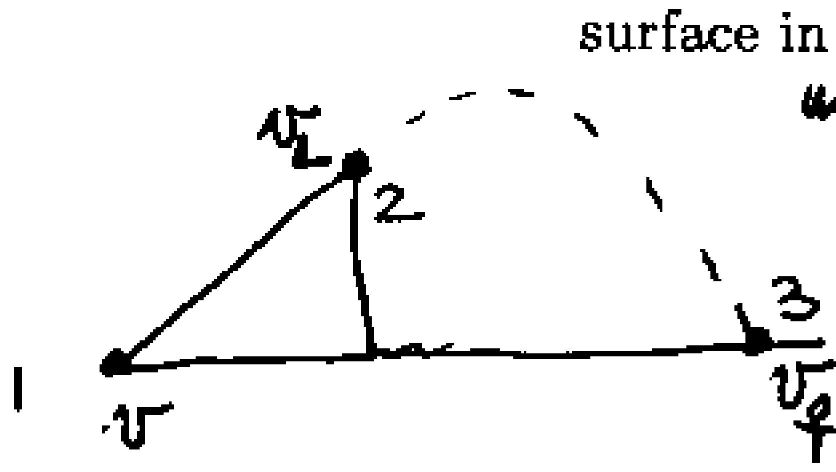
$-mgh - \mu_k mgh\cot\theta = \frac{1}{2} m v_L^2 - \frac{1}{2} m v^2$

$v_L = \sqrt{v^2 - 2gh(1 + \mu_k\cot\theta)}$

$W = \Delta K + \Delta U + \Delta E_{th}$ where $W = 0$, $\Delta U = mgh$ and $\Delta E_{th} = -W_{f_k}$

OR $K_i + \cancel{U_i}^0 + W_{f_k} = K_f + U_f$ where $K_i = \frac{1}{2}mv^2$, $K_f = \frac{1}{2}m v_L^2$ and $U_f = mgh$

(c) (5 pts) The block launches from the upper corner of the wedge with a speed v_L, which is the answer to part (b). It undergoes projectile motion. At a later time, it lands on the horizontal surface. Derive a formula for the speed that it hits the horizontal surface in terms of μ_k, h, θ, v, and g.



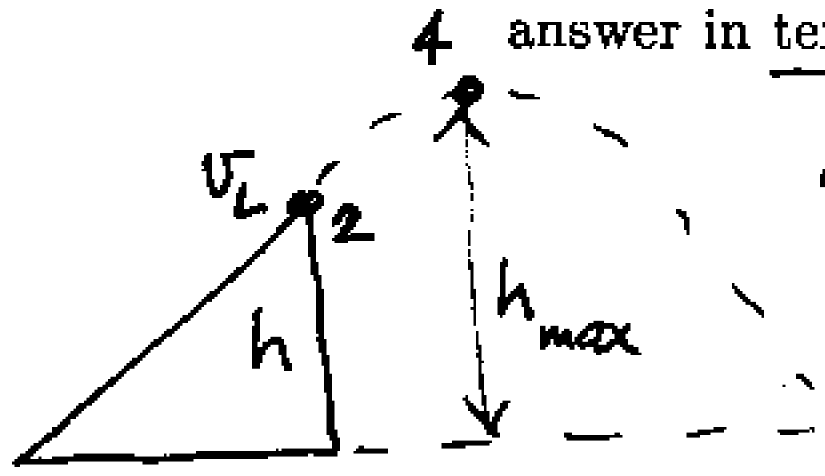
using $W = \Delta K$ where $W = W_{f_k} + W_g$ and $W_g = 0$ for $1 \to 3$

$-\mu_k mgh\cot\theta = \frac{1}{2} m v_f^2 - \frac{1}{2} m v^2$

$\Rightarrow v_f = \sqrt{v^2 - 2gh\mu_k\cot\theta}$

could use $2 \to 3$ and then substitute v_L from (b) into solution for v_f

(d) (5 pts) Derive a formula for the maximum height of the block, measured from the horizontal surface, during its projectile motion. For this part, you may express the answer in terms of the launch velocity v_L and the other parameters of the problem.



• at the top: $v_{top} = v_L\cos\theta$ (only horizontal component)

• $2 \to 4$ ... only mg is doing the work on the block $\Rightarrow$ CME

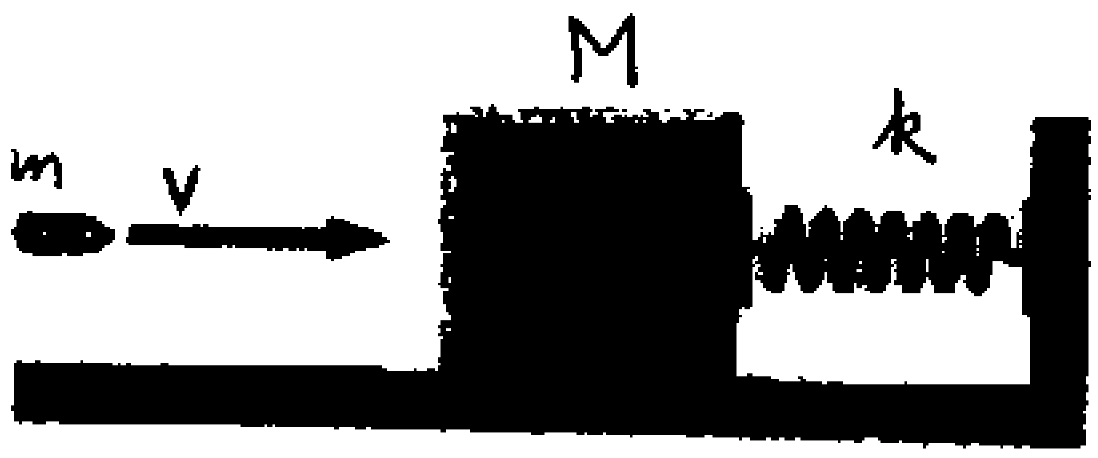
$(K+U)_2 = (K+U)_4$

$\frac{1}{2} m v_L^2 + mgh = \frac{1}{2} m (v_L\cos\theta)^2 + mgh_{max}$

$h_{max} = h + \frac{v_L^2 - v_L^2\cos^2\theta}{2g} = h + \frac{(v_L\sin\theta)^2}{2g}$, $\quad 1 - \cos^2\theta = \sin^2\theta$

Question 3 (15 pts)

The figure shows a bullet of mass m moving with speed v towards a stationary block (with larger mass M) that is attached to a relaxed spring, with spring constant k. It collides with the block and becomes embedded in it, and then the spring becomes compressed. There is no friction between the block and the surface.



(a) (5 pts) Which of the following formulas relates the initial speed of the bullet before the collision (v) to the speed of the block and bullet immediately after the collision (v_f)?

(i) $mv = mv_f$.

(ii) $\frac{1}{2}mv^2 = \frac{1}{2}(m+M)v_f^2$.

(iii) $mv = (m+M)v_f$. CLM ... completelly inelastic collision

(iv) $\frac{1}{2}mv^2 = \frac{1}{2}Mv_f^2$.

(b) (5 pts) The total kinetic energy of the system before the collision is K_i. The total kinetic energy immediately after the collision is K_f. Which expression correctly relates K_i and K_f to the potential energy of the spring, U_s, when it is maximally compressed after the collision?

$K_f < K_i$ (inelastic collision)

$K_f = U_s$ ... block-bullet momentarilly stops when compressed (Kinetic energy transferred to increase in ~~spring~~ energy)

(i) $K_i = K_f = U_s$

(ii) $K_i > K_f > U_s$

(iii) $K_i > K_f = U_s$

(iv) $K_f > K_i > U_s$

(v) None of the above.

(c) (5 pts) Now imagine a different situation, in which the bullet collides elastically with the block. It will therefore bounce backwards. The maximum compression of the spring would be

(i) Smaller than in the previous case.

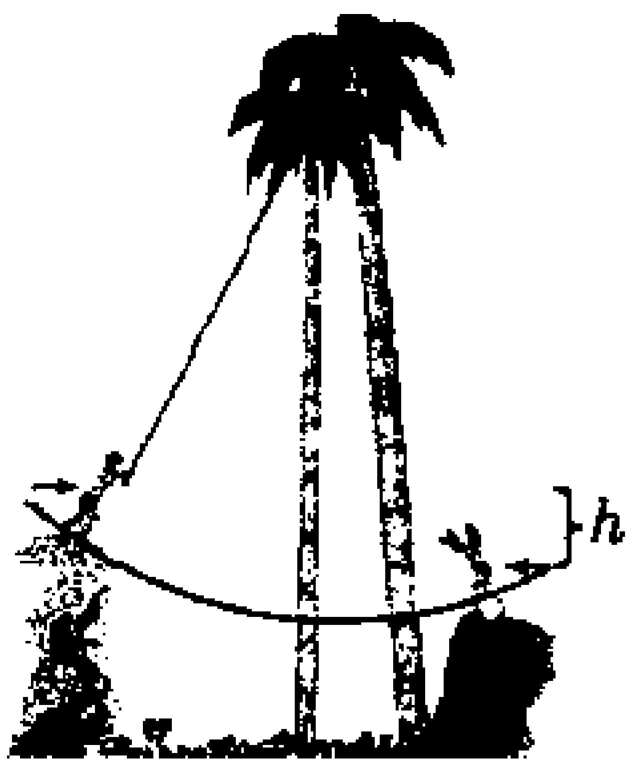
(ii) Larger than in the previous case. Δp_{bullet} = max for elastic bouncing!

(iii) The same as in the previous case.

(iv) Possibly larger or smaller than in the previous case.

Problem 3 (20 pts)

The figure shows Tarzan (mass 75kg), on the left, about to swing on a vine across to Jane (mass 60kg), on the right. The initial height difference between the centers of mass of Tarzan and Jane is $h = 6\text{m}$, indicated in the figure.



(a) (5 pts) What is the change in Tarzan's potential energy as he swings from his initial position to Jane?

$$\Delta U = \Delta U_g = U_{gf} - U_{gi} = -mgh = -(75\text{kg})(9.8\text{ m/s}^2)(6\text{m})$$

$$\Delta U_g = \underline{-4410\text{ J}}$$

(b) (5 pts) What is Tarzan's speed immediately before colliding with Jane?

$$(K+U)_i = (K+U)_f$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(6)} = 10.844\text{ m/s} \simeq 10.8\text{ m/s}$$

(c) (5 pts) Assume Tarzan catches Jane as he collides with her. What is their speed immediately after the collision?

$$m_T v = (m_T + m_J) v_f$$

$$v_f = \frac{m_T}{m_T + m_J} v = \frac{75}{75+60} 10.844 = 6.0246\text{ m/s} \simeq 6.02\text{ m/s}$$

(d) (5 pts) The collision between Tarzan and Jane is inelastic. What is the total change in kinetic energy?

$$\Delta K = K_f - K_i = \frac{1}{2}(m_T + m_J) v_f^2 - \overbrace{\frac{1}{2} m_T v^2}^{mgh}$$

$$= \frac{1}{2}(75+60)\, 6.02^2 - \frac{1}{2}(75)\, 10.844^2 = 2450 - 4410 = -1960\text{ J}$$

$$\Delta K = -1960\text{ J}$$